FHSST Authors

# The Free High School Science Texts: Textbooks for High School Students Studying the Sciences
# Physics
# Grades 10 - 12

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# Contents

# Part III

# Grade 11 - Physics

# Chapter 11

# Vectors

## 11.1 Introduction

This chapter focuses on vectors. We will learn what is a vector, how it differs from everyday numbers, how to add, subtract and multiply them and where they appear in Physics.

Are vectors Physics? No, vectors themselves are not Physics. Physics is just a description of the world around us. To describe something we need to use a language. The most common language used to describe Physics is Mathematics. Vectors form a very important part of the mathematical description of Physics, so much so that it is absolutely essential to master the use of vectors.

## 11.2 Scalars and Vectors

In Mathematics, you learned that a number is something that represents a quantity. For example if you have 5 books, 6 apples and 1 bicycle, the 5, 6, and 1 represent how many of each item you have.

These kinds of numbers are known as *scalars*.

**Definition: Scalar**
A scalar is a quantity that has only magnitude (size).

An extension to a scalar is a vector, which is a scalar with a direction. For example, if you travel 1 km down Main Road to school, the quantity **1 km down Main Road** is a vector. The **1 km** is the quantity (or scalar) and the **down Main Road** gives a direction.

In Physics we use the word *magnitude* to refer to the scalar part of the vector.

**Definition: Vectors**
A vector is a quantity that has both magnitude and direction.

A vector should tell you *how much* and *which way*.

For example, a man is driving his car east along a freeway at 100 km·hr$^{-1}$. What we have given here is a vector – the velocity. The car is moving at 100 km·hr$^{-1}$(this is the magnitude) and we know where it is going – east (this is the direction). Thus, we know the speed and direction of the car. These two quantities, a magnitude and a direction, form a vector we call velocity.

## 11.3 Notation

Vectors are different to scalars and therefore has its own notation.

### 11.3.1 Mathematical Representation

There are many ways of writing the symbol for a vector. Vectors are denoted by symbols with an arrow pointing to the right above it. For example, $\vec{a}$, $\vec{v}$ and $\vec{F}$ represent the vectors acceleration, velocity and force, meaning they have both a magnitude and a direction.

Sometimes just the magnitude of a vector is needed. In this case, the arrow is omitted. In other words, $F$ denotes the magnitude of vector $\vec{F}$. $|\vec{F}|$ is another way of representing the magnitude of a vector.

### 11.3.2 Graphical Representation

Vectors are drawn as arrows. An arrow has both a magnitude (how long it is) and a direction (the direction in which it points). The starting point of a vector is known as the *tail* and the end point is known as the *head*.

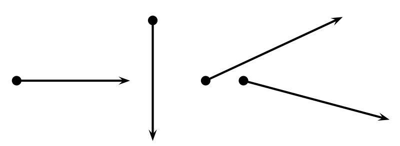


Figure 11.1: Examples of vectors

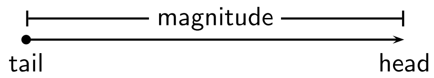



Figure 11.2: Parts of a vector

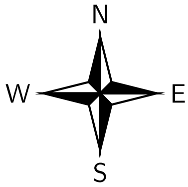
## 11.4 Directions

There are many acceptable methods of writing vectors. As long as the vector has a magnitude and a direction, it is most likely acceptable. These different methods come from the different methods of expressing a direction for a vector.

### 11.4.1 Relative Directions

The simplest method of expressing direction is relative directions: to the left, to the right, forward, backward, up and down.

### 11.4.2 Compass Directions

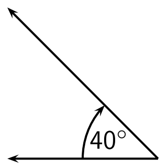



Another common method of expressing directions is to use the points of a compass: North, South, East, and West.

If a vector does not point exactly in one of the compass directions, then we use an angle. For example, we can have a vector pointing 40° North of West. Start with the vector pointing along the West direction:



Then rotate the vector towards the north until there is a 40° angle between the vector and the West.

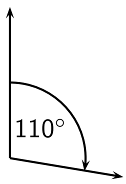



The direction of this vector can also be described as: W 40° N (West 40° North); or N 50° W (North 50° West)

### 11.4.3 Bearing

The final method of expressing direction is to use a *bearing*. A bearing is a direction relative to a fixed point.

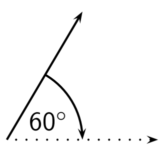
Given just an angle, the convention is to define the angle with respect to the North. So, a vector with a direction of 110° has been rotated clockwise 110° relative to the North. A bearing is always written as a three digit number, for example 275° or 080° (for 80°).

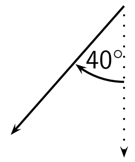



**Exercise: Scalars and Vectors**

1. Classify the following quantities as scalars or vectors:
   1.1 12 km
   1.2 1 m south
   1.3 2 m·s$^{-1}$, 45°
   1.4 075°, 2 cm
   1.5 100 km·hr$^{-1}$, 0°
2. Use two different notations to write down the direction of the vector in each of the following diagrams:

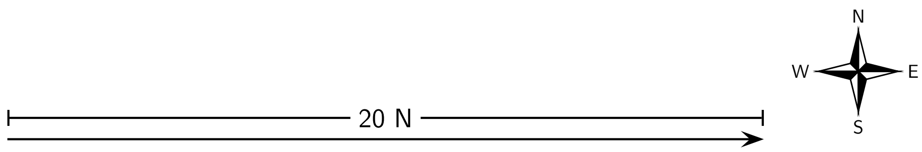
2.1


2.2


2.3 


## 11.5 Drawing Vectors

In order to draw a vector accurately we must specify a scale and include a reference direction in the diagram. A scale allows us to translate the length of the arrow into the vector's magnitude. For instance if one chose a scale of 1 cm = 2 N (1 cm represents 2 N), a force of 20 N towards the East would be represented as an arrow 10 cm long. A reference direction may be a line representing a horizontal surface or the points of a compass.




**Method: Drawing Vectors**

1. Decide upon a scale and write it down.
2. Determine the length of the arrow representing the vector, by using the scale.
3. Draw the vector as an arrow. Make sure that you fill in the arrow head.
4. Fill in the magnitude of the vector.

**Worked Example 49: Drawing vectors**

**Question:** Represent the following vector quantities:

1. 6 $m \cdot s^{-1}$north
2. 16 m east

**Answer**
**Step 1 : Decide upon a scale and write it down.**

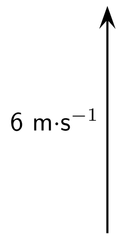
1. 1 cm = 2 $m \cdot s^{-1}$
2. 1 cm = 4 m

**Step 2 : Determine the length of the arrow at the specific scale.**

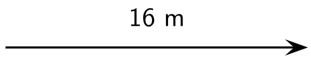
1. If 1 cm = 2 $m \cdot s^{-1}$, then 6 $m \cdot s^{-1}$= 3 cm
2. If 1 cm = 4 m, then 16 m = 4 cm

**Step 3 : Draw the vectors as arrows.**

1. Scale used: 1 cm = 2 $m \cdot s^{-1}$
   Direction = North




2. Scale used: 1 cm = 4 m
   Direction = East




**Exercise: Drawing Vectors**

Draw each of the following vectors to scale. Indicate the scale that you have used:

1. 12 km south
2. 1,5 m N 45° W
3. 1 m·s$^{-1}$, 20° East of North
4. 50 km·hr$^{-1}$, 085°
5. 5 mm, 225°

## 11.6 Mathematical Properties of Vectors

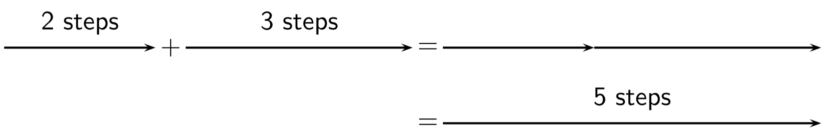
Vectors are mathematical objects and we need to understand the mathematical properties of vectors, like adding and subtracting.

For all the examples in this section, we will use displacement as our vector quantity. Displacement was discussed in Chapter 3. Displacement is defined as the distance together with direction of the straight line joining a final point to an initial point.

Remember that displacement is just one example of a vector. We could just as well have decided to use forces or velocities to illustrate the properties of vectors.

### 11.6.1 Adding Vectors

When vectors are added, we need to add both a magnitude **and** a direction. For example, take 2 steps in the forward direction, stop and then take another 3 steps in the forward direction. The first 2 steps is a displacement vector and the second 3 steps is also a displacement vector. If we did not stop after the first 2 steps, we would have taken 5 steps in the forward direction in total. Therefore, if we add the displacement vectors for 2 steps and 3 steps, we should get a total of 5 steps in the forward direction. Graphically, this can be seen by first following the first vector two steps forward and then following the second one three steps forward:



We add the second vector at the end of the first vector, since this is where we now are after the first vector has acted. The vector from the tail of the first vector (the starting point) to the head of the last (the end point) is then the sum of the vectors. This is the *head-to-tail* method of vector addition.

As you can convince yourself, the order in which you add vectors does not matter. In the example above, if you decided to first go 3 steps forward and then another 2 steps forward, the end result would still be 5 steps forward.

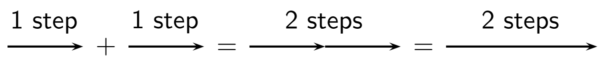
The final answer when adding vectors is called the *resultant*. The resultant displacement in this case will be 5 steps forward.

**Definition: Resultant of Vectors**
The resultant of a number of vectors is the single vector whose effect is the same as the individual vectors acting together.

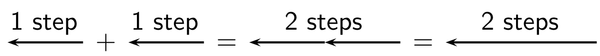
In other words, the individual vectors can be replaced by the resultant – the overall effect is the same. If vectors $\vec{a}$ and $\vec{b}$ have a resultant $\vec{R}$, this can be represented mathematically as,

$$\vec{R} = \vec{a} + \vec{b}.$$

Let us consider some more examples of vector addition using displacements. The arrows tell you how far to move and in what direction. Arrows to the right correspond to steps forward, while arrows to the left correspond to steps backward. Look at all of the examples below and check them.

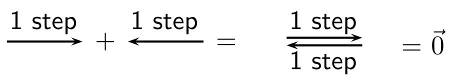


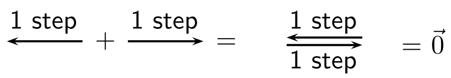

This example says 1 step forward and then another step forward is the same as an arrow twice as long – two steps forward.




This examples says 1 step backward and then another step backward is the same as an arrow twice as long – two steps backward.

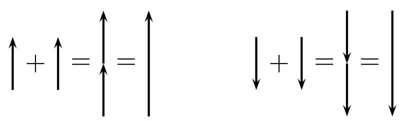
It is sometimes possible that you end up back where you started. In this case the net result of what you have done is that you have gone nowhere (your start and end points are at the same place). In this case, your resultant displacement is a vector with length zero units. We use the symbol $\vec{0}$ to denote such a vector:



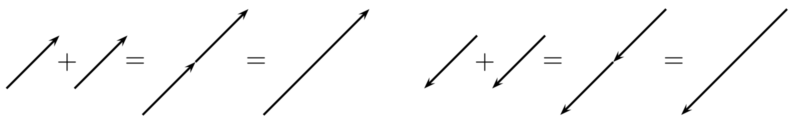




Check the following examples in the same way. Arrows up the page can be seen as steps left and arrows down the page as steps right.

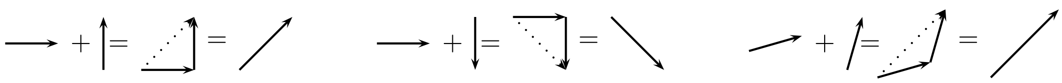
Try a couple to convince yourself!



It is important to realise that the directions are not special– 'forward and backwards' or 'left and right' are treated in the same way. The same is true of any set of parallel directions:



In the above examples the separate displacements were parallel to one another. However the same head-to-tail technique of vector addition can be applied to vectors in any direction.

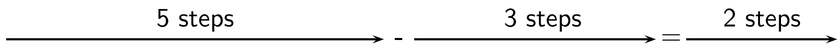


Now you have discovered one use for vectors; describing resultant displacement – how far and in what direction you have travelled after a series of movements.

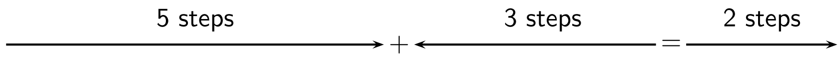
Although vector addition here has been demonstrated with displacements, all vectors behave in exactly the same way. Thus, if given a number of forces acting on a body you can use the same method to determine the resultant force acting on the body. We will return to vector addition in more detail later.

### 11.6.2 Subtracting Vectors

What does it mean to subtract a vector? Well this is really simple; if we have 5 apples and we subtract 3 apples, we have only 2 apples left. Now lets work in steps; if we take 5 steps forward and then subtract 3 steps forward we are left with only two steps forward:




What have we done? You originally took 5 steps forward but then you took 3 steps back. That backward displacement would be represented by an arrow pointing to the left (backwards) with length 3. The net result of adding these two vectors is 2 steps forward:




Thus, subtracting a vector from another is the same as adding a vector in the opposite direction (i.e. subtracting 3 steps forwards is the same as adding 3 steps backwards).

**Important:** Subtracting a vector from another is the same as adding a vector in the opposite direction.

This suggests that in this problem to the right was chosen as the positive direction. Arrows to the right are positive and arrows to the left are negative. More generally, vectors in opposite directions differ in sign (i.e. if we define up as positive, then vectors acting down are negative). Thus, changing the sign of a vector simply reverses its direction:

$$- \longrightarrow = \longleftarrow \qquad - \longleftarrow = \longrightarrow$$

$$- \uparrow = \downarrow \qquad - \downarrow = \uparrow$$

$$- \swarrow = \nearrow \qquad - \nearrow = \swarrow$$

In mathematical form, subtracting $\vec{a}$ from $\vec{b}$ gives a new vector $\vec{c}$:

$$\begin{aligned} \vec{c} &= \vec{b} - \vec{a} \\ &= \vec{b} + (-\vec{a}) \end{aligned}$$

This clearly shows that subtracting vector $\vec{a}$ from $\vec{b}$ is the same as adding $(-\vec{a})$ to $\vec{b}$. Look at the following examples of vector subtraction.

$$\longrightarrow - \longrightarrow = \longrightarrow + \longleftarrow = \vec{0}$$

$$\longrightarrow - \longleftarrow = \longrightarrow + \longrightarrow = \xrightarrow{\hspace{3em}}$$

### 11.6.3 Scalar Multiplication

What happens when you multiply a vector by a scalar (an ordinary number)?

Going back to normal multiplication we know that $2 \times 2$ is just $2$ groups of $2$ added together to give $4$. We can adopt a similar approach to understand how vector multiplication works.

$$2 \times \longrightarrow = \longrightarrow + \longrightarrow = \xrightarrow{\hspace{3em}}$$

## 11.7 Techniques of Vector Addition

Now that you have learned about the mathematical properties of vectors, we return to vector addition in more detail. There are a number of techniques of vector addition. These techniques fall into two main categories - graphical and algebraic techniques.

### 11.7.1 Graphical Techniques

Graphical techniques involve drawing accurate scale diagrams to denote individual vectors and their resultants. We next discuss the two primary graphical techniques, the head-to-tail technique and the parallelogram method.

### The Head-to-Tail Method

In describing the mathematical properties of vectors we used displacements and the head-to-tail graphical method of vector addition as an illustration. The head-to-tail method of graphically adding vectors is a standard method that must be understood.

### Method: Head-to-Tail Method of Vector Addition

1. Choose a scale and include a reference direction.
2. Choose any of the vectors and draw it as an arrow in the correct direction and of the correct length – remember to put an arrowhead on the end to denote its direction.
3. Take the next vector and draw it as an arrow starting from the arrowhead of the first vector in the correct direction and of the correct length.
4. Continue until you have drawn each vector – each time starting from the head of the previous vector. In this way, the vectors to be added are drawn one after the other head-to-tail.
5. The resultant is then the vector drawn from the tail of the first vector to the head of the last. Its magnitude can be determined from the length of its arrow using the scale. Its direction too can be determined from the scale diagram.

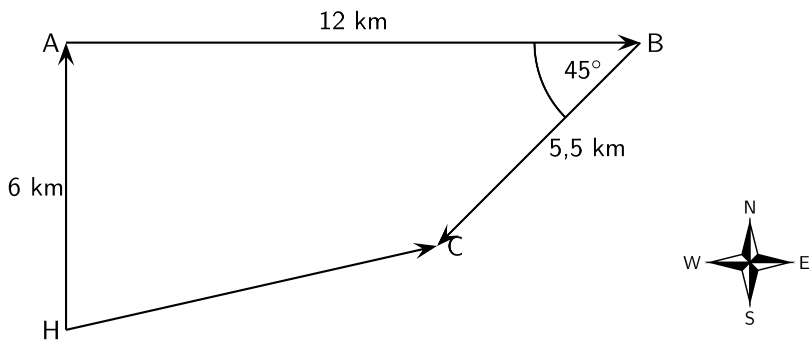
**Worked Example 50: Head-to-Tail Addition I**

**Question:** A ship leaves harbour H and sails 6 km north to port A. From here the ship travels 12 km east to port B, before sailing 5,5 km south-west to port C. Determine the ship's resultant displacement using the head-to-tail technique of vector addition.

**Answer**

**Step 1 : Draw a rough sketch of the situation**

Its easy to understand the problem if we first draw a quick sketch. The rough sketch should include all of the information given in the problem. All of the magnitudes of the displacements are shown and a compass has been included as a reference direction. In a rough sketch one is interested in the approximate shape of the vector diagram.



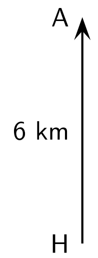

**Step 2 : Choose a scale and include a reference direction.**

The choice of scale depends on the actual question – you should choose a scale such that your vector diagram fits the page.

It is clear from the rough sketch that choosing a scale where 1 cm represents 2 km (scale: 1 cm = 2 km) would be a good choice in this problem. The diagram will then take up a good fraction of an A4 page. We now start the accurate construction.

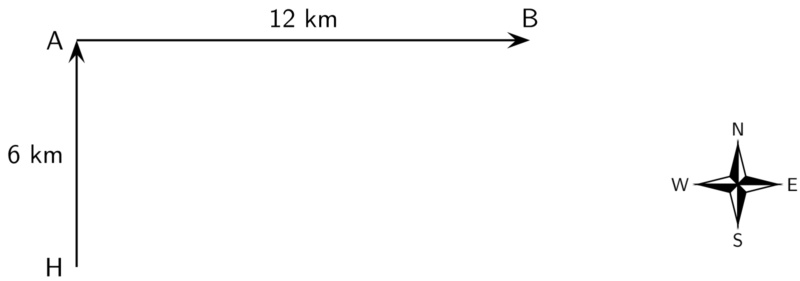
**Step 3 : Choose any of the vectors to be summed and draw it as an arrow in the correct direction and of the correct length – remember to put an arrowhead on the end to denote its direction.**

Starting at the harbour H we draw the first vector 3 cm long in the direction north.



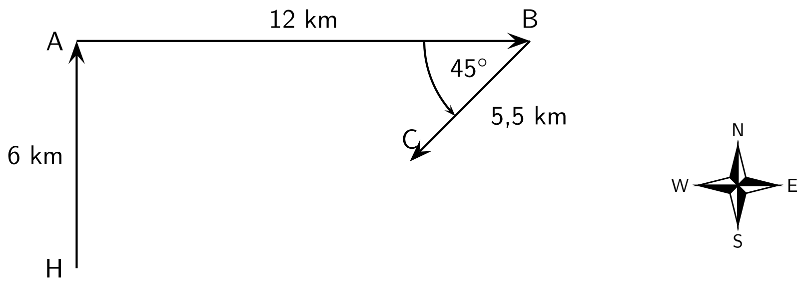

**Step 4 : Take the next vector and draw it as an arrow starting from the head of the first vector in the correct direction and of the correct length.**

Since the ship is now at port A we draw the second vector 6 cm long starting from point A in the direction east.



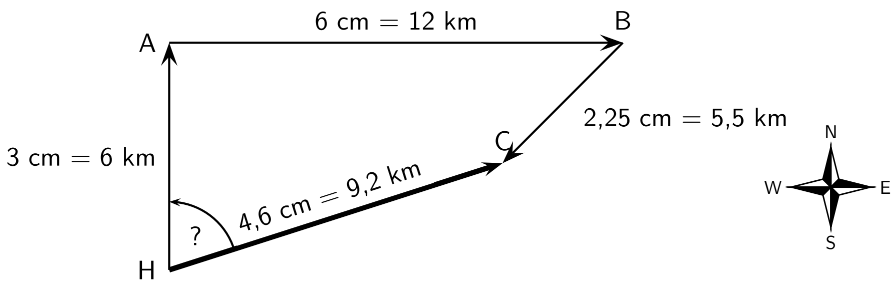

**Step 5 : Take the next vector and draw it as an arrow starting from the head of the second vector in the correct direction and of the correct length.**

Since the ship is now at port B we draw the third vector 2,25 cm long starting from this point in the direction south-west. A protractor is required to measure the angle of 45°.




**Step 6 : The resultant is then the vector drawn from the tail of the first vector to the head of the last. Its magnitude can be determined from the length of its arrow using the scale. Its direction too can be determined from the scale diagram.**

As a final step we draw the resultant displacement from the starting point (the harbour H) to the end point (port C). We use a ruler to measure the length of this arrow and a protractor to determine its direction.




**Step 7 : Apply the scale conversion**

We now use the scale to convert the length of the resultant in the scale diagram to the actual displacement in the problem. Since we have chosen a scale of 1 cm = 2 km in this problem the resultant has a magnitude of 9,2 km. The direction can be specified in terms of the angle measured either as 072,3° east of north or on a bearing of 072,3°.

**Step 8 : Quote the final answer**

The resultant displacement of the ship is 9,2 km on a bearing of 072,3°.

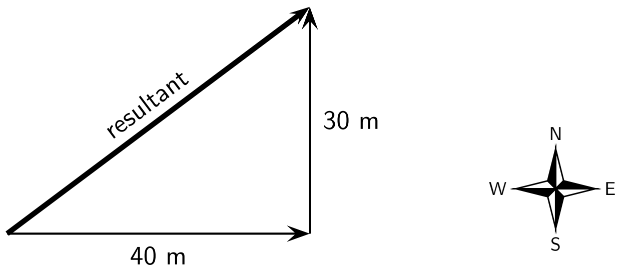
### Worked Example 51: Head-to-Tail Graphical Addition II

**Question:** A man walks 40 m East, then 30 m North.

1. What was the total distance he walked?
2. What is his resultant displacement?

**Answer**

**Step 1 : Draw a rough sketch**




**Step 2 : Determine the distance that the man traveled**

In the first part of his journey he traveled 40 m and in the second part he traveled 30 m. This gives us a total distance traveled of 40 m + 30 m = 70 m.

**Step 3 : Determine his resultant displacement**

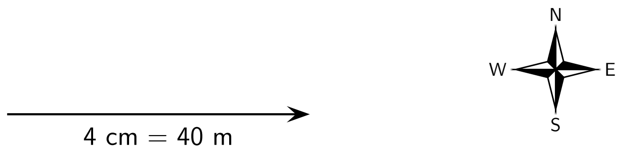
The man's resultant displacement is the **vector** from where he started to where he ended. It is the vector sum of his two separate displacements. We will use the head-to-tail method of accurate construction to find this vector.

**Step 4 : Choose a suitable scale**

A scale of 1 cm represents 10 m (1 cm = 10 m) is a good choice here. Now we can begin the process of construction.

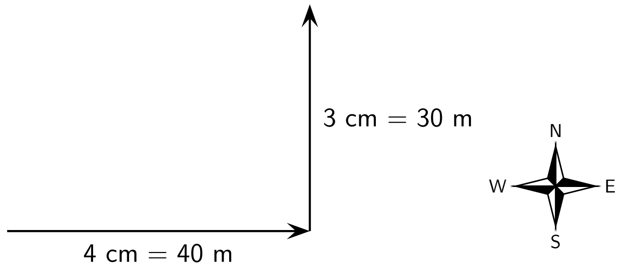
**Step 5 : Draw the first vector to scale**

We draw the first displacement as an arrow 4 cm long in an eastwards direction.



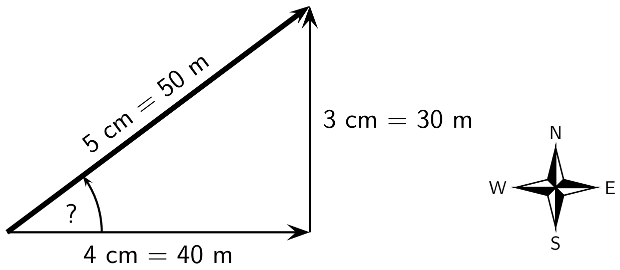

**Step 6 : Draw the second vector to scale**

Starting from the head of the first vector we draw the second vector as an arrow 3 cm long in a northerly direction.




**Step 7 : Determine the resultant vector**

Now we connect the starting point to the end point and measure the length and direction of this arrow (the resultant).




**Step 8 : Find the direction**
To find the direction you measure the angle between the resultant and the 40 m vector. You should get about 37°.

**Step 9 : Apply the scale conversion**
Finally we use the scale to convert the length of the resultant in the scale diagram to the actual magnitude of the resultant displacement. According to the chosen scale 1 cm = 10 m. Therefore 5 cm represents 50 m. The resultant displacement is then 50 m 37° north of east.

### The Parallelogram Method

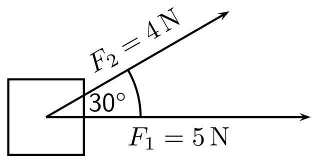
The *parallelogram method* is another graphical technique of finding the resultant of two vectors.

### Method: The Parallelogram Method

1. Choose a scale and a reference direction.
2. Choose either of the vectors to be added and draw it as an arrow of the correct length in the correct direction.
3. Draw the second vector as an arrow of the correct length in the correct direction from the tail of the first vector.
4. Complete the parallelogram formed by these two vectors.
5. The resultant is then the diagonal of the parallelogram. The magnitude can be determined from the length of its arrow using the scale. The direction too can be determined from the scale diagram.

**Worked Example 52: Parallelogram Method of Vector Addition I**

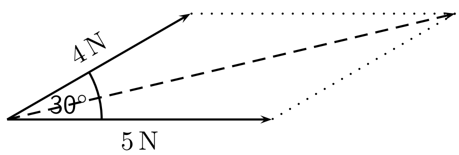
**Question:** A force of $F_1 = 5\,\mathrm{N}$ is applied to a block in a horizontal direction. A second force $F_2 = 4\,\mathrm{N}$ is applied to the object at an angle of 30° above the horizontal.




Determine the resultant force acting on the block using the parallelogram method of accurate construction.

**Answer**

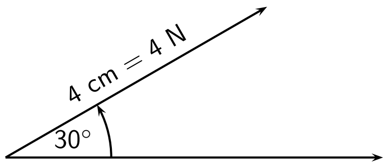
**Step 1 : Firstly make a rough sketch of the vector diagram**



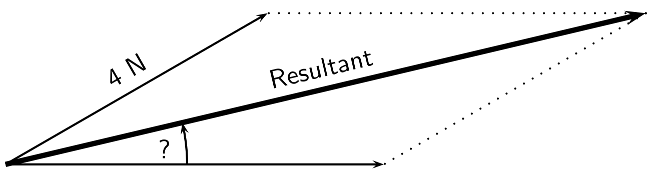
**Step 2 : Choose a suitable scale**
In this problem a scale of 1 cm = 1 N would be appropriate, since then the vector diagram would take up a reasonable fraction of the page. We can now begin the accurate scale diagram.
**Step 3 : Draw the first scaled vector**
Let us draw $F_1$ first. According to the scale it has length 5 cm.




**Step 4 : Draw the second scaled vector**
Next we draw $F_2$. According to the scale it has length 4 cm. We make use of a protractor to draw this vector at 30° to the horizontal.




**Step 5 : Determine the resultant vector**
Next we complete the parallelogram and draw the diagonal.




The resultant has a measured length of 8,7 cm.
**Step 6 : Find the direction**
We use a protractor to measure the angle between the horizontal and the resultant. We get 13,3°.
**Step 7 : Apply the scale conversion**
Finally we use the scale to convert the measured length into the actual magnitude. Since 1 cm = 1 N, 8,7 cm represents 8,7 N. Therefore the resultant force is 8,7 N at 13,3° above the horizontal.

The parallelogram method is restricted to the addition of just two vectors. However, it is arguably the most intuitive way of adding two forces acting at a point.

## 11.7.2 Algebraic Addition and Subtraction of Vectors

### Vectors in a Straight Line

Whenever you are faced with adding vectors acting in a straight line (i.e. some directed left and some right, or some acting up and others down) you can use a very simple algebraic technique:

### Method: Addition/Subtraction of Vectors in a Straight Line

1. Choose a positive direction. As an example, for situations involving displacements in the directions west and east, you might choose west as your positive direction. In that case, displacements east are negative.
2. Next simply add (or subtract) the vectors using the appropriate signs.
3. As a final step the direction of the resultant should be included in words (positive answers are in the positive direction, while negative resultants are in the negative direction).

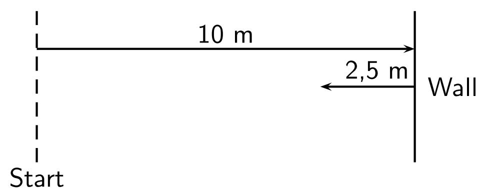
Let us consider a few examples.

**Worked Example 53: Adding vectors algebraically I**

**Question:** A tennis ball is rolled towards a wall which is 10 m away from the wall. If after striking the wall the ball rolls a further 2,5 m along the ground away from the wall, calculate algebraically the ball's resultant displacement.

**Answer**

**Step 1 : Draw a rough sketch of the situation**




**Step 2 : Decide which method to use to calculate the resultant**

We know that the resultant displacement of the ball ($\vec{x}_R$) is equal to the sum of the ball's separate displacements ($\vec{x}_1$ and $\vec{x}_2$):

$$\vec{x}_R = \vec{x}_1 + \vec{x}_2$$

Since the motion of the ball is in a straight line (i.e. the ball moves towards and away from the wall), we can use the method of algebraic addition just explained.

**Step 3 : Choose a positive direction**

Let's make towards the wall the **positive** direction. This means that away from the wall becomes the **negative** direction.

**Step 4 : Now define our vectors algebraically**

With right positive:

$$\begin{aligned}\vec{x}_1 &= +10{,}0\,\mathrm{m}\\ \vec{x}_2 &= -2{,}5\,\mathrm{m}\end{aligned}$$

**Step 5 : Add the vectors**

Next we simply add the two displacements to give the resultant:

$$\begin{aligned}\vec{x}_R &= (+10\,\mathrm{m}) + (-2{,}5\,\mathrm{m})\\ &= (+7{,}5)\,\mathrm{m}\end{aligned}$$

**Step 6 : Quote the resultant**

Finally, in this case towards the wall means positive so: $\vec{x}_R$ = 7,5 m towards the wall.

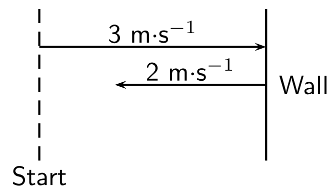
**Worked Example 54: Subtracting vectors algebraically I**

**Question:** Suppose that a tennis ball is thrown horizontally towards a wall at an initial velocity of 3 m·s$^{-1}$to the right. After striking the wall, the ball returns to the thrower at 2 m·s$^{-1}$. Determine the change in velocity of the ball.

**Answer**

**Step 1 : Draw a sketch**

A quick sketch will help us understand the problem.




**Step 2 : Decide which method to use to calculate the resultant**
Remember that velocity is a vector. The change in the velocity of the ball is equal to the difference between the ball's initial and final velocities:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

Since the ball moves along a straight line (i.e. left and right), we can use the algebraic technique of vector subtraction just discussed.
**Step 3 : Choose a positive direction**
Choose towards the wall as the **positive** direction. This means that away from the wall becomes the **negative** direction.
**Step 4 : Now define our vectors algebraically**

$$\begin{aligned} \vec{v}_i &= +3\,\mathrm{m\cdot s^{-1}} \\ \vec{v}_f &= -2\,\mathrm{m\cdot s^{-1}} \end{aligned}$$

**Step 5 : Subtract the vectors**
Thus, the change in velocity of the ball is:

$$\begin{aligned} \Delta\vec{v} &= (-2\,\mathrm{m\cdot s^{-1}}) - (+3\,\mathrm{m\cdot s^{-1}}) \\ &= (-5)\,\mathrm{m\cdot s^{-1}} \end{aligned}$$

**Step 6 : Quote the resultant**
Remember that in this case towards the wall means positive so: $\Delta\vec{v} = 5\,\mathrm{m\cdot s^{-1}}$ to the away from the wall.

?

**Exercise: Resultant Vectors**

1. Harold walks to school by walking 600 m Northeast and then 500 m N 40° W. Determine his resultant displacement by using accurate scale drawings.
2. A dove flies from her nest, looking for food for her chick. She flies at a velocity of 2 m·s$^{-1}$on a bearing of 135° and then at a velocity of 1,2 m·s$^{-1}$on a bearing of 230°. Calculate her resultant velocity by using accurate scale drawings.
3. A squash ball is dropped to the floor with an initial velocity of 2,5 m·s$^{-1}$. I rebounds (comes back up) with a velocity of 0,5 m·s$^{-1}$.
   3.1 What is the change in velocity of the squash ball?
   3.2 What is the resultant velocity of the squash ball?

Remember that the technique of addition and subtraction just discussed can only be applied to vectors acting along a straight line. When vectors are not in a straight line, i.e. at an angle to each other, the following method can be used:

### A More General Algebraic technique

Simple geometric and trigonometric techniques can be used to find resultant vectors.

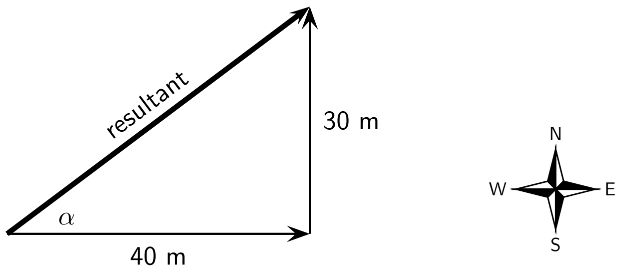
**Worked Example 55: An Algebraic Solution I**

**Question:** A man walks 40 m East, then 30 m North. Calculate the man's resultant displacement.

**Answer**

**Step 1 : Draw a rough sketch**

As before, the rough sketch looks as follows:




**Step 2 : Determine the length of the resultant**

Note that the triangle formed by his separate displacement vectors and his resultant displacement vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let $x$ represent the length of the resultant vector. Then:

$$\begin{aligned} x_R^2 &= (40\,\mathrm{m})^2 + (30\,\mathrm{m})^2 \\ x_R^2 &= 2\ 500\,\mathrm{m}^2 \\ x_R &= 50\,\mathrm{m} \end{aligned}$$

**Step 3 : Determine the direction of the resultant**

Now we have the length of the resultant displacement vector but not yet its direction. To determine its direction we calculate the angle $\alpha$ between the resultant displacement vector and East, by using simple trigonometry:

$$\begin{aligned} \tan\alpha &= \frac{\mathrm{oppositeside}}{\mathrm{adjacentside}} \\ \tan\alpha &= \frac{30}{40} \\ \alpha &= \tan^{-1}(0{,}75) \\ \alpha &= 36{,}9^\circ \end{aligned}$$

**Step 4 : Quote the resultant**

The resultant displacement is then 50 m at 36,9° North of East.
This is exactly the same answer we arrived at after drawing a scale diagram!

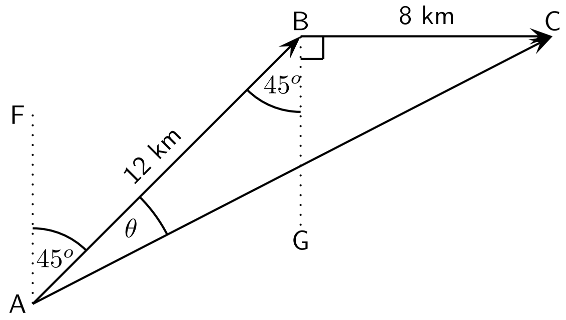
In the previous example we were able to use simple trigonometry to calculate the resultant displacement. This was possible since the directions of motion were perpendicular (north and east). Algebraic techniques, however, are not limited to cases where the vectors to be combined are along the same straight line or at right angles to one another. The following example illustrates this.

**Worked Example 56: An Algebraic Solution II**

**Question:** A man walks from point A to point B which is 12 km away on a bearing of 45°. From point B the man walks a further 8 km east to point C. Calculate the resultant displacement.

**Answer**

**Step 1 : Draw a rough sketch of the situation**




$B\hat{A}F = 45°$ since the man walks initially on a bearing of 45°. Then, $A\hat{B}G = B\hat{A}F = 45°$ (parallel lines, alternate angles). Both of these angles are included in the rough sketch.

**Step 2 : Calculate the length of the resultant**

The resultant is the vector AC. Since we know both the lengths of AB and BC and the included angle $A\hat{B}C$, we can use the cosine rule:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 - 2 \cdot AB \cdot BC \cos(A\hat{B}C) \\ &= (12)^2 + (8)^2 - 2 \cdot (12)(8) \cos(135°) \\ &= 343{,}8 \\ AC &= 18{,}5 \text{ km} \end{aligned}$$

**Step 3 : Determine the direction of the resultant**

Next we use the sine rule to determine the angle $\theta$:

$$\begin{aligned} \frac{\sin\theta}{8} &= \frac{\sin 135°}{18{,}5} \\ \sin\theta &= \frac{8 \times \sin 135°}{18{,}5} \\ \theta &= \sin^{-1}(0{,}3058) \\ \theta &= 17{,}8° \end{aligned}$$

To find $F\hat{A}C$, we add 45°. Thus, $F\hat{A}C = 62{,}8°$.

**Step 4 : Quote the resultant**

The resultant displacement is therefore 18,5 km on a bearing of 062,8°.

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**Exercise: More Resultant Vectors**

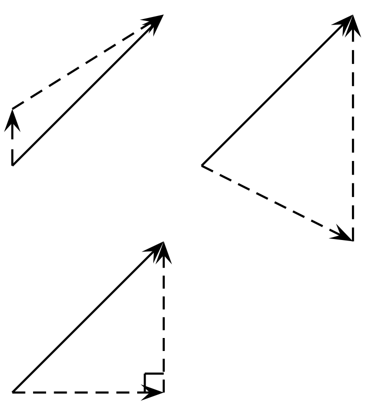
1. Hector, a long distance athlete, runs at a velocity of 3 m·s$^{-1}$in a northerly direction. He turns and runs at a velocity of 5 m·s$^{-1}$in a westerly direction. Find his resultant velocity by using appropriate calculations. Include a rough sketch of the situation in your answer.

2. Sandra walks to the shop by walking 500 m Northwest and then 400 m N 30° E. Determine her resultant displacement by doing appropriate calculations.

## 11.8 Components of Vectors

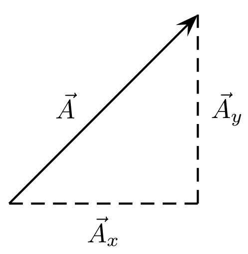
In the discussion of vector addition we saw that a number of vectors acting together can be combined to give a single vector (the resultant). In much the same way a single vector can be broken down into a number of vectors which when added give that original vector. These vectors which sum to the original are called **components** of the original vector. The process of breaking a vector into its components is called **resolving into components**.

While summing a given set of vectors gives just one answer (the resultant), a single vector can be resolved into infinitely many sets of components. In the diagrams below the same black vector is resolved into different pairs of components. These components are shown as dashed lines. When added together the dashed vectors give the original black vector (i.e. the original vector is the resultant of its components).



In practice it is most useful to resolve a vector into components which are at right angles to one another, usually horizontal and vertical.

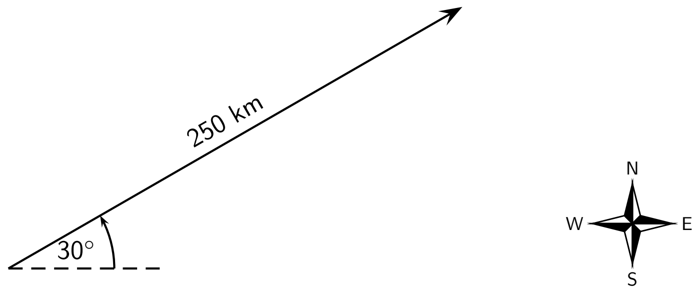
Any vector can be resolved into a horizontal and a vertical component. If $\vec{A}$ is a vector, then the horizontal component of $\vec{A}$ is $\vec{A}_x$ and the vertical component is $\vec{A}_y$.




**Worked Example 57: Resolving a vector into components**

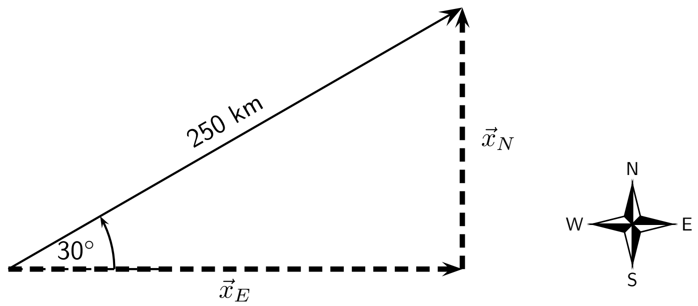
**Question:** A motorist undergoes a displacement of 250 km in a direction 30° north of east. Resolve this displacement into components in the directions north ($\vec{x}_N$) and east ($\vec{x}_E$).

**Answer**
**Step 1 : Draw a rough sketch of the original vector**




**Step 2 : Determine the vector component**

Next we resolve the displacement into its components north and east. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original displacement as its hypotenuse.




Notice how the two components acting together give the original vector as their resultant.

**Step 3 : Determine the lengths of the component vectors**

Now we can use trigonometry to calculate the magnitudes of the components of the original displacement:

$$\begin{aligned} x_N &= (250)(\sin 30°) \\ &= 125 \text{ km} \end{aligned}$$

and

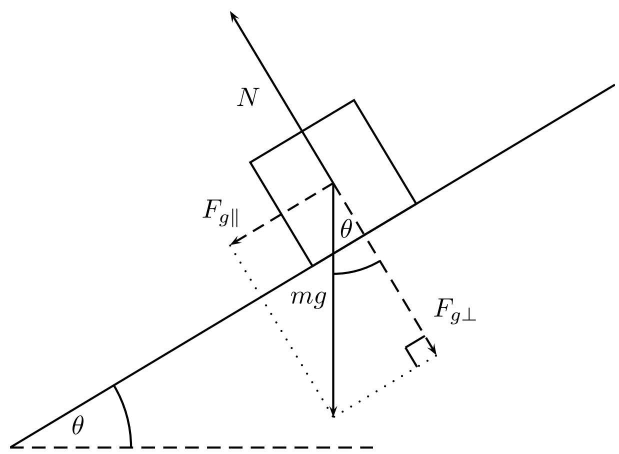
$$\begin{aligned} x_E &= (250)(\cos 30°) \\ &= 216{,}5 \text{ km} \end{aligned}$$

Remember $x_N$ and $x_E$ are the magnitudes of the components – they are in the directions north and east respectively.

*Extension: Block on an incline*

As a further example of components let us consider a block of mass $m$ placed on a frictionless surface inclined at some angle $\theta$ to the horizontal. The block will obviously slide down the incline, but what causes this motion?

The forces acting on the block are its weight $mg$ and the normal force $N$ exerted by the surface on the object. These two forces are shown in the diagram below.




Now the object's weight can be resolved into components parallel and perpendicular to the inclined surface. These components are shown as dashed arrows in the diagram above and are at right angles to each other. The components have been drawn acting from the same point. Applying the parallelogram method, the two components of the block's weight sum to the weight vector.

To find the components in terms of the weight we can use trigonometry:

$$\begin{aligned} F_{g\|} &= mg\sin\theta \\ F_{g\perp} &= mg\cos\theta \end{aligned}$$

The component of the weight perpendicular to the slope $F_{g\perp}$ exactly balances the normal force $N$ exerted by the surface. The parallel component, however, $F_{g\|}$ is unbalanced and causes the block to slide down the slope.

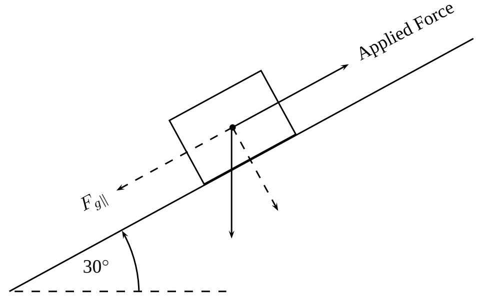
*Extension: Worked example*

**Worked Example 58: Block on an incline plane**

**Question:** Determine the force needed to keep a 10 kg block from sliding down a frictionless slope. The slope makes an angle of 30° with the horizontal.

**Answer**

**Step 1 : Draw a diagram of the situation**




The force that will keep the block from sliding is equal to the parallel component of the weight, but its direction is up the slope.

**Step 2 : Calculate $F_{g\|}$**

$$F_{g\|} = mg\sin\theta = (10)(9{,}8)(\sin 30°) = 49\text{N}$$

**Step 3 : Write final answer**
The force is 49 N up the slope.

### 11.8.1 Vector addition using components

Components can also be used to find the resultant of vectors. This technique can be applied to both graphical and algebraic methods of finding the resultant. The method is simple: make a rough sketch of the problem, find the horizontal and vertical components of each vector, find the sum of all horizontal components and the sum of all the vertical components and then use them to find the resultant.

Consider the two vectors, $\vec{A}$ and $\vec{B}$, in Figure 11.3, together with their resultant, $\vec{R}$.

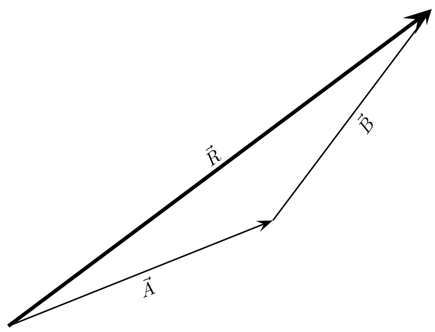



Figure 11.3: An example of two vectors being added to give a resultant

Each vector in Figure 11.3 can be broken down into a component in the $x$-direction and one in the $y$-direction. These components are two vectors which when added give you the original vector as the resultant. This is shown in Figure 11.4 where we can see that:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$
$$\vec{B} = \vec{B}_x + \vec{B}_y$$
$$\vec{R} = \vec{R}_x + \vec{R}_y$$

But, $\vec{R}_x = \vec{A}_x + \vec{B}_x$
and $\vec{R}_y = \vec{A}_y + \vec{B}_y$

In summary, addition of the $x$ components of the two original vectors gives the $x$ component of the resultant. The same applies to the $y$ components. So if we just added all the components together we would get the same answer! This is another important property of vectors.

**Worked Example 59: Adding Vectors Using Components**

**Question:** If in Figure 11.4, $\vec{A} = 5{,}385\,\text{m}$ at an angle of 21.8° to the horizontal and $\vec{B} = 5\,\text{m}$ at an angle of 53,13° to the horizontal, find $\vec{R}$.

**Answer**
**Step 1 : Decide how to tackle the problem**

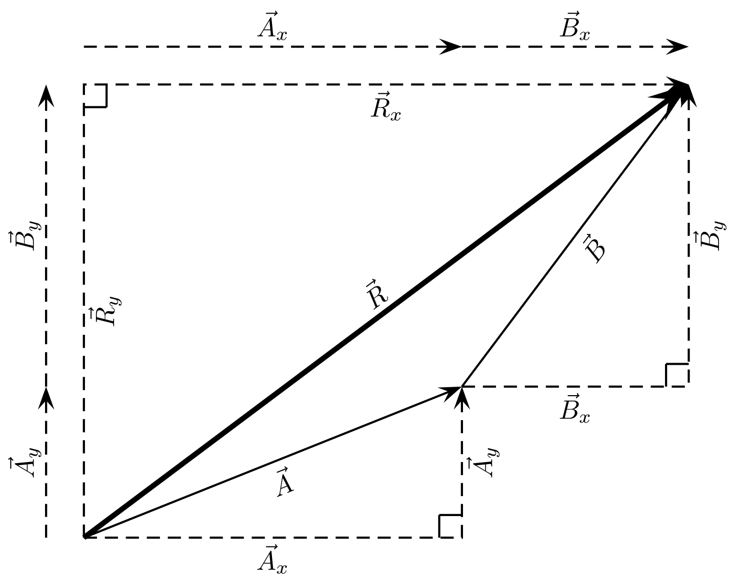



Figure 11.4: Adding vectors using components.

The first thing we must realise is that the order that we add the vectors does not matter. Therefore, we can work through the vectors to be added in any order.

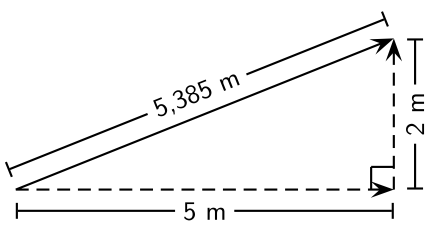
**Step 2 : Resolve $\vec{A}$ into components**

We find the components of $\vec{A}$ by using known trigonometric ratios. First we find the magnitude of the vertical component, $A_y$:

$$\begin{aligned} \sin\theta &= \frac{A_y}{A} \\ \sin 21{,}8^\circ &= \frac{A_y}{5{,}385} \\ A_y &= (5{,}385)(\sin 21{,}8^\circ) \\ &= 2\,\mathrm{m} \end{aligned}$$

Secondly we find the magnitude of the horizontal component, $A_x$:

$$\begin{aligned} \cos\theta &= \frac{A_x}{A} \\ \cos 21.8^\circ &= \frac{A_x}{5{,}385} \\ A_x &= (5{,}385)(\cos 21{,}8^\circ) \\ &= 5\,\mathrm{m} \end{aligned}$$




The components give the sides of the right angle triangle, for which the original vector is the hypotenuse.

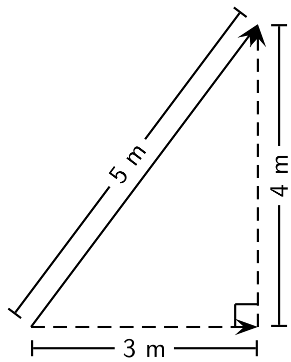
**Step 3 : Resolve $\vec{B}$ into components**

We find the components of $\vec{B}$ by using known trigonometric ratios. First we find the magnitude of the vertical component, $B_y$:

$$\begin{aligned} \sin\theta &= \frac{B_y}{B} \\ \sin 53{,}13^\circ &= \frac{B_y}{5} \\ B_y &= (5)(\sin 53{,}13^\circ) \\ &= 4\,\text{m} \end{aligned}$$

Secondly we find the magnitude of the horizontal component, $B_x$:

$$\begin{aligned} \cos\theta &= \frac{B_x}{B} \\ \cos 21{,}8^\circ &= \frac{B_x}{5{,}385} \\ B_x &= (5{,}385)(\cos 53{,}13^\circ) \\ &= 5\,\text{m} \end{aligned}$$




**Step 4 : Determine the components of the resultant vector**

Now we have all the components. If we add all the horizontal components then we will have the $x$-component of the resultant vector, $\vec{R}_x$. Similarly, we add all the vertical components then we will have the $y$-component of the resultant vector, $\vec{R}_y$.

$$\begin{aligned} R_x &= A_x + B_x \\ &= 5\,\text{m} + 3\,\text{m} \\ &= 8\,\text{m} \end{aligned}$$

Therefore, $\vec{R}_x$ is 8 m to the right.

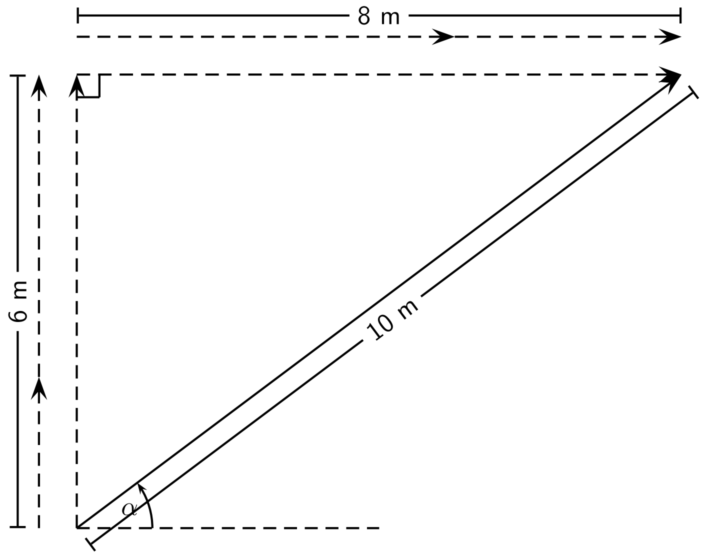
$$\begin{aligned} R_y &= A_y + B_y \\ &= 2\,\text{m} + 4\,\text{m} \\ &= 6\,\text{m} \end{aligned}$$

Therefore, $\vec{R}_y$ is 6 m up.

**Step 5 : Determine the magnitude and direction of the resultant vector**

Now that we have the components of the resultant, we can use the Theorem of Pythagoras to determine the magnitude of the resultant, $R$.

$$\begin{aligned} R^2 &= (R_x)^2 + (R_y)^2 \\ R^2 &= (6)^2 + (8)^2 \\ R^2 &= 100 \\ \therefore R &= 10\,\text{m} \end{aligned}$$




The magnitude of the resultant, $R$ is 10 m. So all we have to do is calculate its direction. We can specify the direction as the angle the vectors makes with a known direction. To do this you only need to visualise the vector as starting at the origin of a coordinate system. We have drawn this explicitly below and the angle we will calculate is labeled $\alpha$.

Using our known trigonometric ratios we can calculate the value of $\alpha$;

$$\begin{aligned} \tan\alpha &= \frac{6\,\mathrm{m}}{8\,\mathrm{m}} \\ \alpha &= \tan^{-1}\frac{6\,\mathrm{m}}{8\,\mathrm{m}} \\ \alpha &= 36{,}8^o. \end{aligned}$$

**Step 6 : Quote the final answer**

$\vec{R}$ is 10 m at an angle of $36{,}8°$ to the positive $x$-axis.

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**Exercise: Adding and Subtracting Components of Vectors**

1. Harold walks to school by walking 600 m Northeast and then 500 m N $40^o$ W. Determine his resultant displacement by means of addition of components of vectors.
2. A dove flies from her nest, looking for food for her chick. She flies at a velocity of 2 m·s$^{-1}$on a bearing of 135$^o$ and then at a velocity of 1,2 m·s$^{-1}$on a bearing of 230$^o$. Calculate her resultant velocity by adding the horizontal and vertical components of vectors.

---

*Extension: Vector Multiplication*

Vectors are special, they are more than just numbers. This means that multiplying vectors is not necessarily the same as just multiplying their magnitudes. There are two different types of multiplication defined for vectors. You can find the dot product of two vectors or the cross product.

The *dot* product is most similar to regular multiplication between scalars. To take the dot product of two vectors, you just multiply their magnitudes to get out a scalar answer. The maths definition of the dot product is:

$$\vec{a} \bullet \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$$

Take two vectors $\vec{a}$ and $\vec{b}$:


You can draw in the component of $\vec{b}$ that is parallel to $\vec{a}$:


In this way we can arrive at the definition of the dot product. You find how much of $\vec{b}$ is lined up with $\vec{a}$ by finding the component of $\vec{b}$ parallel to $\vec{a}$. Then multiply the magnitude of that component, $|\vec{b}| \cos\theta$, with the magnitude of $\vec{a}$ to get a scalar.

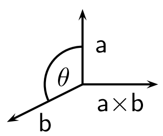
The second type of multiplication is more subtle and uses the directions of the vectors in a more complicated way to get another vector as the answer. The maths definition of the cross product is:

$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta$$

This gives the magnitude of the answer, but we still need to find the direction of the resultant vector. We do this by applying the *right hand rule*.

**Method: Right Hand Rule**

1. Using your right hand:
2. Point your index finger in the direction of $\vec{a}$.
3. Point the middle finger in the direction of $\vec{b}$.
4. Your thumb will show the direction of $\vec{a} \times \vec{b}$.

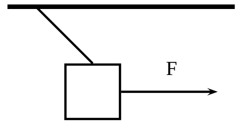


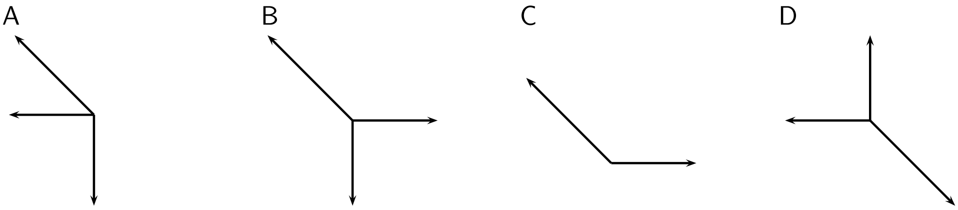

## 11.8.2 Summary

1. A scalar is a physical quantity with magnitude only.
2. A vector is a physical quantity with magnitude and direction.
3. Vectors are drawn as arrows where the length of the arrow indicates the magnitude and the arrowhead indicates the direction of the vector.
4. The direction of a vector can be indicated by referring to another vector or a fixed point (eg. 30° from the river bank); using a compass (eg. N 30° W); or bearing (eg. 053°).
5. Vectors can be added using the head-to-tail method, the parallelogram method or the component method.
6. The resultant of a vector is the single vector whose effect is the same as the individual vectors acting together.

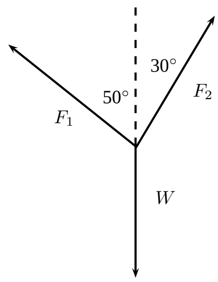
### 11.8.3 End of chapter exercises: Vectors

1. An object is suspended by means of a light string. The sketch shows a horizontal force $F$ which pulls the object from the vertical position until it reaches an equilibrium position as shown. Which one of the following vector diagrams best represents all the forces acting on the object?




2. A load of weight $W$ is suspended from two strings. $F_1$ and $F_2$ are the forces exerted by the strings on the load in the directions show in the figure above. Which one of the following equations is valid for this situation?



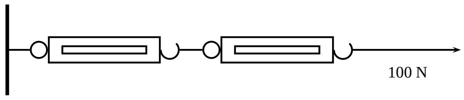

   A $W = F_1^2 + F_2^2$

   B $F_1 \sin 50° = F_2 \sin 30°$

   C $F_1 \cos 50° = F_2 \cos 30°$

   D $W = F_1 + F_2$

3. Two spring balances $P$ and $Q$ are connected by means of a piece of string to a wall as shown. A horizontal force of 100 N is exerted on spring balance Q. What will be the readings on spring balances $P$ and $Q$?




| | P | Q |
|---|---|---|
| A | 100 N | 0 N |
| B | 25 N | 75 N |
| C | 50 N | 50 N |
| D | 100 N | 100 N |

4. A point is acted on by two forces in equilibrium. The forces

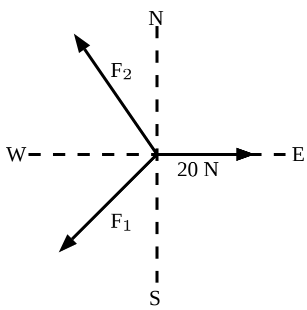
   A have equal magnitudes and directions.

   B have equal magnitudes but opposite directions.

   C act perpendicular to each other.

   D act in the same direction.

5. A point in equilibrium is acted on by three forces. Force $F_1$ has components 15 N due south and 13 N due west. What are the components of force $F_2$?




   A 13 N due north and 20 due west

   B 13 N due north and 13 N due west

   C 15 N due north and 7 N due west

   D 15 N due north and 13 N due east

6. Which of the following contains two vectors and a scalar?

   A distance, acceleration, speed

   B displacement, velocity, acceleration

   C distance, mass, speed

   D displacement, speed, velocity

7. Two vectors act on the same point. What should the angle between them be so that a maximum resultant is obtained?

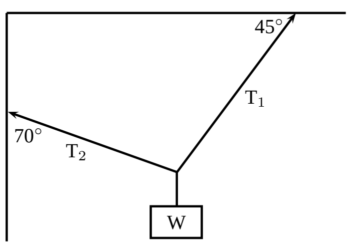
A 0° B 90° C 180° D cannot tell

8. Two forces, 4 N and 11 N, act on a point. Which one of the following cannot be a resultant?

A 4 N B 7 N C 11 N D 15 N

### 11.8.4 End of chapter exercises: Vectors - Long questions

1. A helicopter flies due east with an air speed of 150 km.h$^{-1}$. It flies through an air current which moves at 200 km.h$^{-1}$ north. Given this information, answer the following questions:
   1.1 In which direction does the helicopter fly?
   1.2 What is the ground speed of the helicopter?
   1.3 Calculate the ground distance covered in 40 minutes by the helicopter.
2. A plane must fly 70 km due north. A cross wind is blowing to the west at 30 km.h$^{-1}$. In which direction must the pilot steer if the plane goes at 200 km.h$^{-1}$ in windless conditions?
3. A stream that is 280 m wide flows along its banks with a velocity of 1.80m.s$^{-1}$. A raft can travel at a speed of 2.50 m.s$^{-1}$ across the stream. Answer the following questions:
   3.1 What is the shortest time in which the raft can cross the stream?
   3.2 How far does the raft drift downstream in that time?
   3.3 In what direction must the raft be steered against the current so that it crosses the stream perpendicular to its banks?
   3.4 How long does it take to cross the stream in question 3?
4. A helicopter is flying from place $X$ to place $Y$. $Y$ is $1000$ km away in a direction $50°$ east of north and the pilot wishes to reach it in two hours. There is a wind of speed $150$ km.h$^{-1}$ blowing from the northwest. Find, by accurate construction and measurement (with a scale of $1\ \mathrm{cm} = 50\ \mathrm{km.h^{-1}}$), the
   4.1 the direction in which the helicopter must fly, and
   4.2 the magnitude of the velocity required for it to reach its destination on time.
5. An aeroplane is flying towards a destination $300$ km due south from its present position. There is a wind blowing from the north east at $120$ km.h$^{-1}$. The aeroplane needs to reach its destination in $30$ minutes. Find, by accurate construction and measurement (with a scale of $1\ \mathrm{cm} = 30\ \mathrm{km.s^{-1}}$), or otherwise, the
   5.1 the direction in which the aeroplane must fly and
   5.2 the speed which the aeroplane must maintain in order to reach the destination on time.
   5.3 Confirm your answers in the previous 2 subquestions with calculations.
6. An object of weight $W$ is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are $T_1$ and $T_2$ respectively. Tension $T_1 = 1200$ N. Determine the tension $T_2$ and weight $W$ of the object by accurate construction and measurement or by calculation.



7. In a map-work exercise, hikers are required to walk from a tree marked A on the map to another tree marked B which lies 2,0 km due East of A. The hikers then walk in a straight line to a waterfall in position C which has components measured from B of 1,0 km E and 4,0 km N.

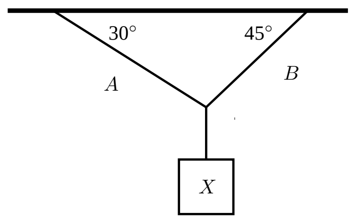
   7.1 Distinguish between quantities that are described as being *vector* and *scalar*.

   7.2 Draw a labelled displacement-vector diagram (not necessarily to scale) of the hikers' complete journey.

   7.3 What is the total distance walked by the hikers from their starting point at A to the waterfall C?

   7.4 What are the magnitude and bearing, to the nearest degree, of the displacement of the hikers from their starting point to the waterfall?

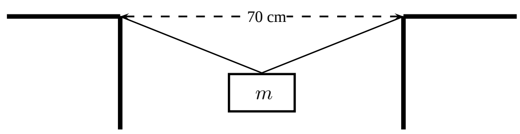
8. An object $X$ is supported by two strings, $A$ and $B$, attached to the ceiling as shown in the sketch. Each of these strings can withstand a maximum force of 700 N. The weight of $X$ is increased gradually.




   8.1 Draw a rough sketch of the triangle of forces, and use it to explain which string will break first.

   8.2 Determine the maximum weight of $X$ which can be supported.

9. A rope is tied at two points which are 70 cm apart from each other, on the same horizontal line. The total length of rope is 1 m, and the maximum tension it can withstand in any part is 1000 N. Find the largest mass ($m$), in kg, that can be carried at the midpoint of the rope, without breaking the rope. Include a vector diagram in your answer.



# Chapter 12

# Force, Momentum and Impulse - Grade 11

## 12.1 Introduction

In Grade 10 we studied motion but not what caused the motion. In this chapter we will learn that a net force is needed to cause motion. We recall what a force is and learn about how force and motion are related. We are introduced to two new concepts, momentum and impulse, and we learn more about turning forces and the force of gravity.

## 12.2 Force

### 12.2.1 What is a force?

A force is anything that can cause a change to objects. Forces can:

- change the shape of an object
- move or stop an object
- change the direction of a moving object.

A force can be classified as either a *contact force* or a *non-contact force*.

A contact force must touch or *be in contact* with an object to cause a change. Examples of contact forces are:

- the force that is used to push or pull things, like on a door to open or close it
- the force that a sculptor uses to turn clay into a pot
- the force of the wind to turn a windmill

A non-contact force does not have to touch an object to cause a change. Examples of non-contact forces are:

- the force due to gravity, like the Earth pulling the Moon towards itself
- the force due to electricity, like a proton and an electron attracting each other
- the force due to magnetism, like a magnet pulling a paper clip towards itself

The unit of force is the *newton* (symbol $N$). This unit is named after Sir Isaac Newton who first defined force. Force is a vector quantity and has a magnitude and a direction. We use the abbreviation $F$ for force.

There is a popular story that while Sir Isaac Newton was sitting under an apple tree, an apple fell on his head, and he suddenly thought of the Universal Law of Gravitation. Coincidently, the weight of a small apple is approximately 1 N.

Force was first described by Archimedes of Syracuse (circa 287 BC - 212 BC). Archimedes was a Greek mathematician, astronomer, philosopher, physicist and engineer. He was killed by a Roman soldier during the sack of the city, despite orders from the Roman general, Marcellus, that he was not to be harmed.

This chapter will often refer to the *resultant force* acting on an object. The resultant force is simply the vector sum of all the forces acting on the object. It is very important to remember that all the forces must be acting on the *same* object. The resultant force is the force that has the same effect as all the other forces added together.

### 12.2.2 Examples of Forces in Physics

Most of Physics revolves around forces. Although there are many different forces, all are handled in the same way. All forces in Physics can be put into one of four groups. These are gravitational forces, electromagnetic forces, strong nuclear force and weak nuclear force. You will mostly come across gravitational or electromagnetic forces at school.

#### Gravitational Forces

Gravity is the attractive force between two objects due to the mass of the objects. When you throw a ball in the air, its mass and the Earth's mass attract each other, which leads to a force between them. The ball falls back towards the Earth, and the Earth accelerates towards the ball. The movement of the Earth towards the ball is, however, so small that you couldn't possibly measure it.

#### Electromagnetic Forces

Almost all of the forces that we experience in everyday life are electromagnetic in origin. They have this unusual name because long ago people thought that electric forces and magnetic forces were different things. After much work and experimentation, it has been realised that they are actually different manifestations of the same underlying theory.

#### Electric or Electrostatic Forces

If we have objects carrying electrical charge, which are not moving, then we are dealing with electrostatic forces (Coulomb's Law). This force is actually much stronger than gravity. This may seem strange, since gravity is obviously very powerful, and holding a balloon to the wall seems to be the most impressive thing electrostatic forces have done, but if we think about it: for gravity to be detectable, we need to have a very large mass nearby. But a balloon rubbed in someone's hair can stick to a wall with a force so strong that it overcomes the force of gravity—with just the charges in the balloon and the wall!

#### Magnetic Forces

The magnetic force is a different manifestation of the electromagnetic force. It stems from the interaction between *moving charges* as opposed to the *fixed* charges involved in Coulomb's Law. Examples of the magnetic force in action include magnets, compasses,car engines and computer data storage. Magnets are also used in the wrecking industry to pick up cars and move them around sites.

#### Friction

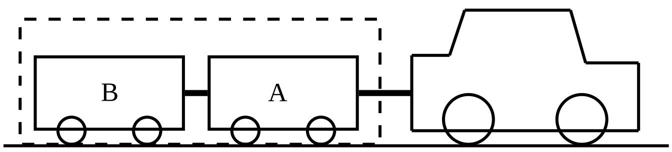
According to Newton's First Law (we will discuss this later in the chapter) an object moving *without a force acting on it* will keep on moving. Then why does a box sliding on a table stop? The answer is friction. Friction arises from the interaction between the molecules on the bottom of a box with the molecules on a table. This interaction is electromagnetic in origin, hence friction is just another view of the electromagnetic force. Later in this chapter we will discuss frictional forces a little more.

#### Drag Forces

This is the force an object experiences while travelling through a medium like an aeroplane flying through air. When something travels through the air it needs to displace air as it travels and because of this the air exerts a force on the object. This becomes an important force when you move fast and a lot of thought is taken to try and reduce the amount of drag force a sports car or an aeroplane experiences. The drag force is very useful for parachutists. They jump from high altitudes and if there was no drag force, then they would continue accelerating all the way to the ground. Parachutes are wide because the more surface area you show, the greater the drag force and hence the slower you hit the ground.

### 12.2.3 Systems and External Forces

The concepts of a system and an external forces are very important in Physics. A system is any collection of objects. If one draws an imaginary box around such a system then an external force is one that is applied by an object or person outside the box. Imagine for example a car pulling two trailers.



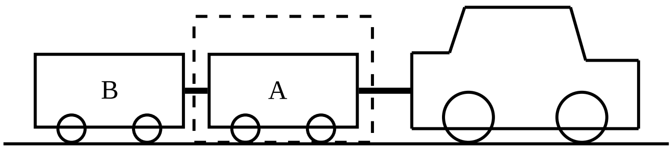

If we draw a box around the two trailers they can be considered a closed system or unit. When we look at the forces on this closed system the following forces will apply:

- The force of the car pulling the unit (trailer A and B)
- The force of friction between the wheels of the trailers and the road (opposite to the direction of motion)
- The force of the Earth pulling downwards on the system (gravity)
- The force of the road pushing upwards on the system

These forces are called external forces to the system.
The following forces will not apply:

- The force of A pulling B
- The force of B pulling A
- The force of friction between the wheels of the car and the road (opposite to the direction of motion)

We can also draw a box around trailer A or B, in which case the forces will be different.

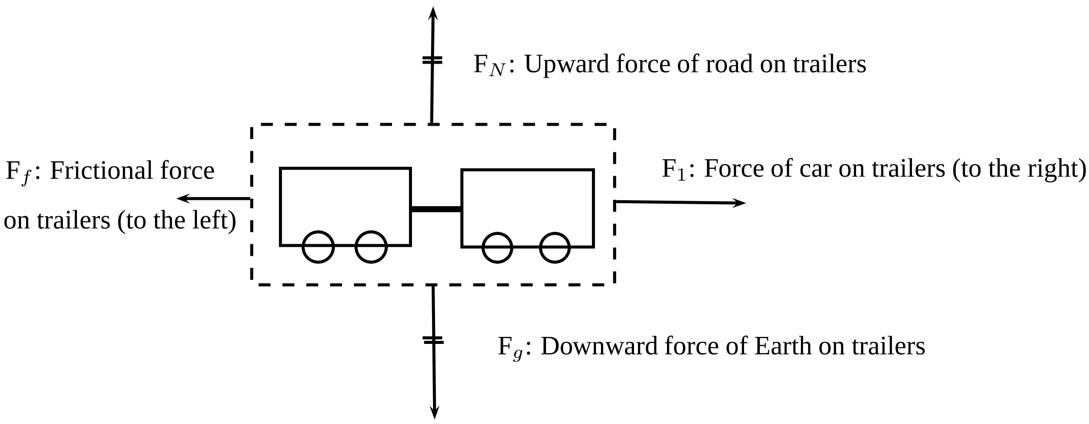



If we consider trailer A as a system, the following external forces will apply:

- The force of the car pulling on A (towards the right)
- The force of B pulling on A (towards the left)
- The force of the Earth pulling downwards on the trailer (gravity)
- The force of the road pushing upwards on the trailer

### 12.2.4 Force Diagrams

If we look at the example above and draw a force diagram of all the forces acting on the two-trailer-unit, the diagram would look like this:




It is important to keep the following in mind when you draw force diagrams:

- Make your drawing large and clear.
- You must use arrows and the direction of the arrow will show the direction of the force.
- The length of the arrow will indicate the size of the force, in other words, the longer arrows in the diagram ($F_1$ for example) indicates a bigger force than a shorter arrow ($F_f$). Arrows of the same length indicate forces of equal size ($F_N$ and $F_g$). Use ?little lines? like in maths to show this.
- Draw neat lines using a ruler. The arrows must touch the system or object.

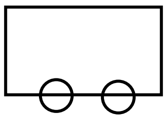
- All arrows must have labels. Use letters with a key on the side if you do not have enough space on your drawing.
- The labels must indicate what is applying the force (the force of the car?) on what the force is applied (?on the trailer?) and in which direction (to the right)
- If the values of the forces are known, these values can be added to the diagram or key.

**Worked Example 60: Force diagrams**

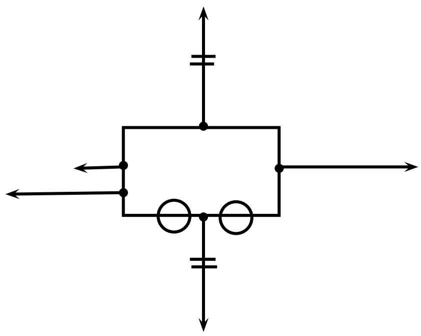
**Question:** Draw a labeled force diagram to indicate all the forces acting on trailer A in the example above.

**Answer**

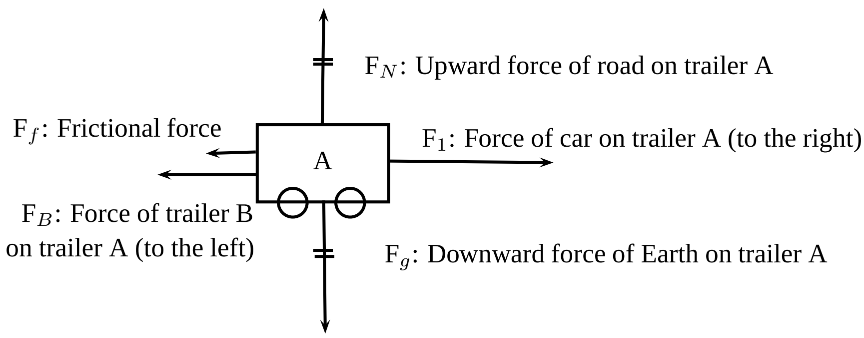
**Step 1 : Draw a large diagram of the ?picture? from your question**



**Step 2 : Add all the forces**

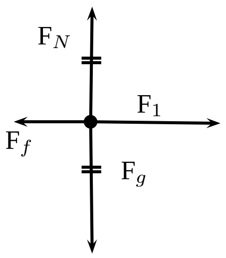


**Step 3 : Add the labels**




## 12.2.5 Free Body Diagrams

In a free-body diagram, the object of interest is drawn as a dot and all the forces acting on it are drawn as arrows pointing away from the dot. A free body diagram for the two-trailer-system will therefore look like this:




$F_1$: Force of car on trailers (to the right)
$F_f$: Frictional force on trailers (to the left)
$F_g$: Downward force of Earth on trailers
$F_N$: Upward force of road on trailers

**Worked Example 61: Free body diagram**

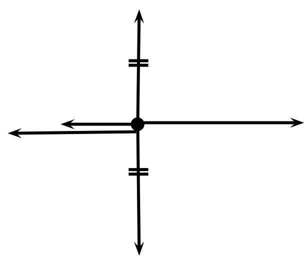
**Question:** Draw a free body diagram of all the forces acting on trailer A in the example above.

**Answer**

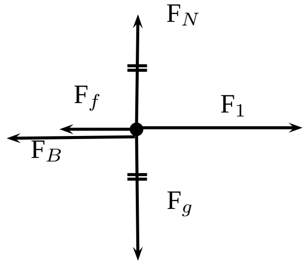
**Step 1 : Draw a dot to indicate the object**

•

**Step 2 : Draw arrows to indicate all the forces acting on the object**



**Step 3 : Label the forces**




$F_1$: Force of car on trailer A (to the right)
$F_B$: Force of trailer B on trailer A (to the left)
$F_f$: Frictional force on trailer A (to the left)
$F_g$: Downward force of Earth on trailer A
$F_N$: Upward force of road on trailer A

### 12.2.6 Finding the Resultant Force

The easiest way to determine a resultant force is to draw a free body diagram. Remember from Chapter **??** that we use the length of the arrow to indicate the vector's magnitude and the direction of the arrow to show which direction it acts in.

After we have done this, we have a diagram of vectors and we simply find the sum of the vectors to get the resultant force.

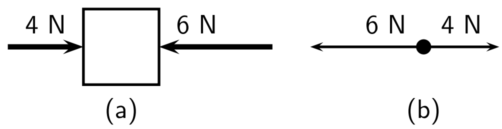



Figure 12.1: (a) Force diagram of 2 forces acting on a box. (b) Free body diagram of the box.

For example, two people push on a box from opposite sides with forces of 4 N and 6 N respectively as shown in Figure 12.1(a). The free body diagram in Figure 12.1(b) shows the object represented by a dot and the two forces are represented by arrows with their tails on the dot.

As you can see, the arrows point in opposite directions and have different lengths. The resultant force is 2 N to the left. This result can be obtained algebraically too, since the two forces act along the same line. First, as in motion in one direction, choose a frame of reference. Secondly, add the two vectors taking their directions into account.

For the example, assume that the positive direction is to the right, then:

$$\begin{aligned} F_R &= (+4\,\mathrm{N}) + (-6\,\mathrm{N}) \\ &= -2\,\mathrm{N} \\ &= 2\,\mathrm{N}\text{ to the left} \end{aligned}$$

Remember that a negative answer means that the force acts in the *opposite* direction to the one that you chose to be positive. You can *choose* the positive direction to be any way you want, but once you have chosen it you *must* keep it.

As you work with more force diagrams in which the forces exactly balance, you may notice that you get a zero answer (e.g. 0 N). This simply means that the forces are balanced and that the object will not move.

Once a force diagram has been drawn the techniques of vector addition introduced in Chapter **??** can be used. Depending on the situation you might choose to use a graphical technique such as the tail-to-head method or the parallelogram method, or else an algebraic approach to determine the resultant. Since force is a vector all of these methods apply.

**Worked Example 62: Finding the resultant force**

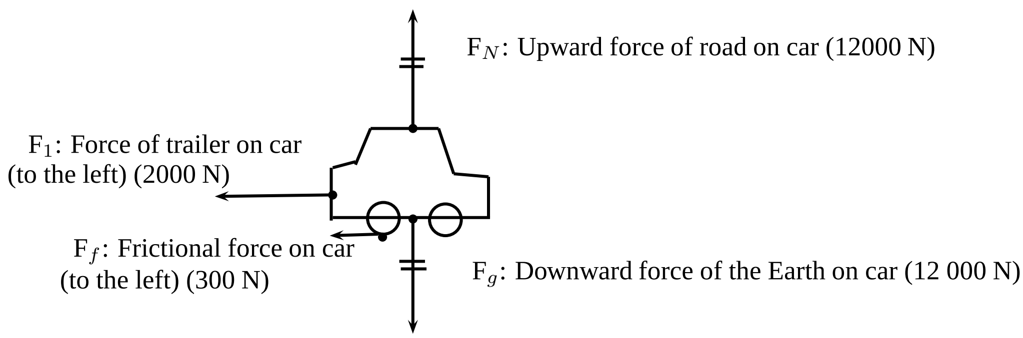
**Question:** A car (mass 1200 kg) applies a force of 2000 N on a trailer (mass 250 kg). A constant frictional force of 200 N is acting on the trailer, and 300 N is acting on the car.

1. Draw a force diagram of all the forces acting on the car.
2. Draw a free body diagram of all the horizontal forces acting on the trailer.
3. Use the force diagram to determine the resultant force on the trailer.

**Answer**

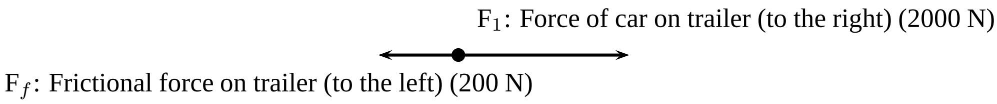
**Step 1 : Draw the force diagram for the car.**

The question asks us to draw all the forces on the car. This means that we must include horizontal and vertical forces.



**Step 2 : Draw the free body diagram for the trailer.**

The question only asks for horizontal forces. We will therefore not include the force of the Earth on the trailer, or the force of the road on the trailer as these forces are in a vertical direction.




**Step 3 : Determine the resultant force on the trailer.**

To find the resultant force we need to add all the horizontal forces together. We do not add vertical forces as the movement of the car and trailer will be in a horizontal direction, and not up and down. $F_R = 2000 + (-200) = 1800$ N to the right.

### 12.2.7 Exercise

1. A force acts on an object. Name three effects that the force can have on the object.

2. Identify each of the following forces as contact or non-contact forces.

   2.1 The force between the north pole of a magnet and a paper clip.

   2.2 The force required to open the door of a taxi.

   2.3 The force required to stop a soccer ball.

   2.4 The force causing a ball, dropped from a height of 2 m, to fall to the floor.

3. A book of mass 2 kg is lying on a table. Draw a labeled force diagram indicating all the forces on the book.

4. A boy pushes a shopping trolley (mass 15 kg) with a constant force of 75 N. A constant frictional force of 20 N is present.

   4.1 Draw a labeled force diagram to identify all the forces acting on the shopping trolley.

   4.2 Draw a free body diagram of all the horizontal forces acting on the trolley.

   4.3 Determine the resultant force on the trolley.

5. A donkey (mass 250 kg) is trying to pull a cart (mass 80 kg) with a force of 400 N. The rope between the donkey and the cart makes an angle of 30° with the cart. The cart does not move.

   5.1 Draw a free body diagram of all the forces acting on the donkey.

   5.2 Draw a force diagram of all the forces acting on the cart.

   5.3 Find the magnitude and direction of the frictional force preventing the cart from moving.

## 12.3 Newton's Laws

In grade 10 you learned about motion, but did not look at how things start to move. You have also learned about forces. In this section we will look at the effect of forces on objects and how we can make things move.

### 12.3.1 Newton's First Law

Sir Isaac Newton was a scientist who lived in England (1642-1727). He was interested in the reason why objects move. He suggested that objects that are stationary will remain stationary, unless a force acts on them and objects that are moving will keep on moving, unless a force slows them down, speeds them up or let them change direction. From this he formulated what is known as Newton's First Law of Motion:

**Definition: Newton's First Law of Motion**
An object will remain in a state of rest or continue traveling at constant velocity, unless acted upon by an unbalanced (net) force.

Let us consider the following situations:

An ice skater pushes herself away from the side of the ice rink and skates across the ice. She will continue to move in a straight line across the ice unless something stops her. Objects are also like that. If we kick a soccer ball across a soccer field, according to Newton's First Law, the soccer ball should keep on moving forever! However, in real life this does not happen. Is Newton's Law wrong? Not really. Newton's First Law applies to situations where there aren't any external forces present. This means that friction is not present. In the case of the ice skater, the friction between the skates and the ice is very little and she will continue moving for quite a distance. In the case of the soccer ball, air resistance (friction between the air and the ball) and friction between the grass and the ball is present and this will slow the ball down.

**Newton's First Law in action**

We experience Newton's First Law in every day life. Let us look at the following examples:

**Rockets**:

A spaceship is launched into space. The force of the exploding gases pushes the rocket through the air into space. Once it is in space, the engines are switched off and it will keep on moving at a constant velocity. If the astronauts want to change the direction of the spaceship they need to fire an engine. This will then apply a force on the rocket and it will change its direction.

**Seat belts:**

We wear seat belts in cars. This is to protect us when the car is involved in an accident. If a car is traveling at 120 km·hr$^{-1}$, the passengers in the car is also traveling at 120 km·hr$^{-1}$.

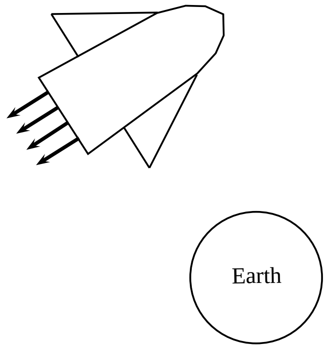
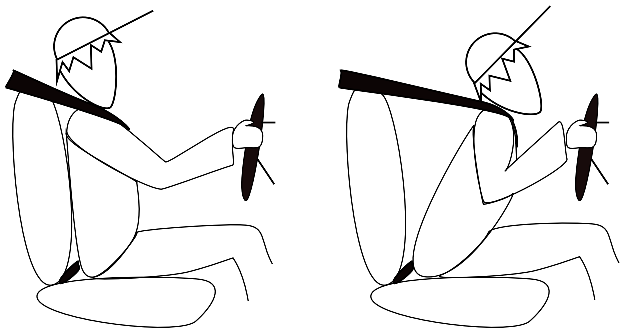



Figure 12.2: Newton's First Law and rockets

When the car suddenly stops a force is exerted on the car (making it slow down), but not on the passengers. The passengers will carry on moving forward at 120 km·hr$^{-1}$according to Newton I. If they are wearing seat belts, the seat belts will stop them and therefore prevent them from getting hurt.



**Worked Example 63: Newton's First Law in action**

**Question:** Why do passengers get thrown to the side when the car they are driving in goes around a corner?

**Answer**

**Step 1 : What happens before the car turns**

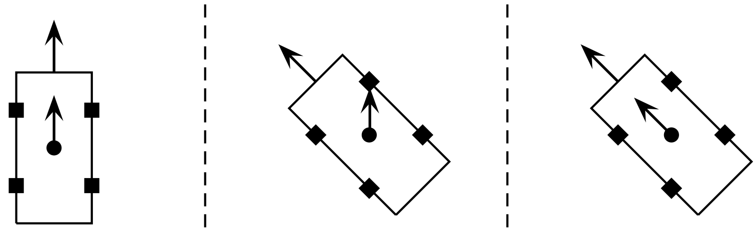
Before the car starts turning both the passengers and the car are traveling at the same velocity. (picture A)

**Step 2 : What happens while the car turns**

The driver turns the wheels of the car, which then exert a force on the car and the car turns. This force acts on the car but not the passengers, hence (by Newton's First Law) the passengers continue moving with the same original velocity. (picture B)

**Step 3 : Why passengers get thrown to the side?**

If the passengers are wearing seat belts they will exert a force on the passengers until the passengers' velocity is the same as that of the car (picture C). Without a seat belt the passenger may hit the side of the car.



A: Both the car and the person travelling at the same velocity

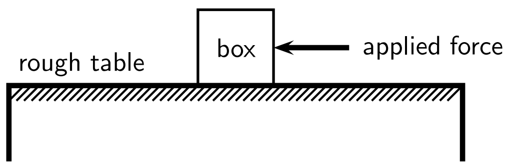
B: The cars turns but not the person

C: Both the car and the person are travelling at the same velocity again

## 12.3.2 Newton's Second Law of Motion

According to Newton I, things 'like to keep on doing what they are doing'. In other words, if an object is moving, it likes to keep on moving and if an object is stationary, it likes to stay stationary. So how do objects start to move then?

Let us look at the example of a 10 kg box on a rough table. If we push lightly on the box as indicated in the diagram, the box won't move. Let's say we applied a force of 100 N, yet the box remains stationary. At this point a frictional force of 100 N is acting on the box, preventing the box from moving. If we increase the force, lets say to 150 N and the box just about starts to move, the frictional force is 150 N. To be able to move the box, we need to push hard enough to overcome the friction and then move the box. If we therefore apply a force of 200 N remembering that a frictional force of 150 N is present, the 'first' 150 N will be used to overcome or 'cancel' the friction and the other 50 N will be used to move (accelerate) the block. In order to accelerate an object we must have a resultant force acting on the block.



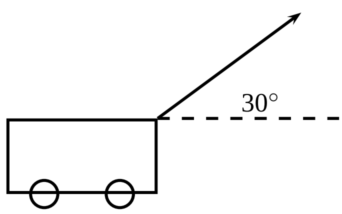

Now, what do you think will happen if we pushed harder, lets say 300 N? Or, what do you think will happen if the mass of the block was more, say 20 kg, or what if it was less? Let us investigate how the motion of an object is affected by mass and force.

**Activity :: Investigation : Newton's Second Law of Motion**

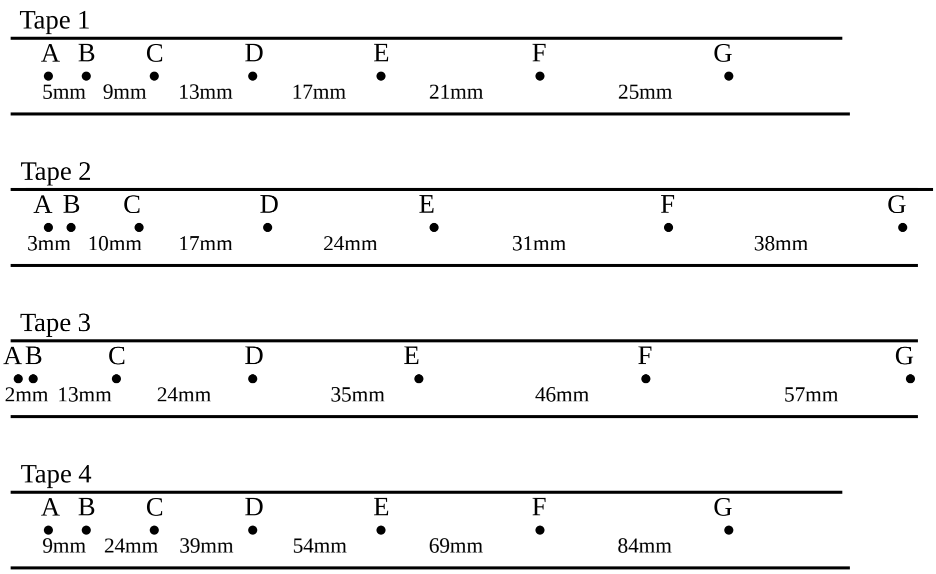
**Aim:**

To investigate the relationship between the acceleration produced on different masses by a constant resultant force.

**Method:**



1. A constant force of 20 N, acting at an angle of 30° to the horizontal, is applied to a dynamics trolley.
2. Ticker tape attached to the trolley runs through a ticker timer of frequency 20 Hz as the trolley is moving on the frictionless surface.
3. The above procedure is repeated 4 times, each time using the same force, but varying the mass of the trolley.
4. Shown below are sections of the four ticker tapes obtained. The tapes are marked with the letters A, B, C, D, etc. A is the first dot, B is the second dot and so on. The distance between each dot is also shown.




Tapes are not drawn to scale

Instructions:

1. Use each tape to calculate the instantaneous velocity (in m·s$^{-1}$) of the trolley at points B and F. Use these velocities to calculate the trolley?s acceleration in each case.
2. Use Newton's second law to calculate the mass of the trolley in each case.
3. Tabulate the mass and corresponding acceleration values as calculated in each case. Ensure that each column and row in your table is appropriately labeled.
4. Draw a graph of acceleration vs. mass, using a scale of 1 cm = 1 m·s$^{-2}$on the y-axis and 1 cm = 1 kg on the x-axis.
5. Use your graph to read off the acceleration of the trolley if its mass is 5 kg.
6. Write down a conclusion for the experiment.

---

You will have noted in the investigation above that the heavier the trolley is, the slower it moved. The acceleration is indirectly proportional to the mass. In mathematical terms: $a \propto \frac{1}{m}$

In a similar investigation where the mass is kept constant, but the applied force is varied, you will find that the bigger the force is, the faster the object will move. The acceleration of the trolley is therefore directly proportional to the resultant force. In mathematical terms: a $\propto$ F.

If we rearrange the above equations, we get $a \propto \frac{F}{m}$ OR $F = ma$

Newton formulated his second law as follows:

**Definition: Newton's Second Law of Motion**
If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and indirectly proportional to the mass of the body. The mathematical representation is $a \propto \frac{F}{m}$.

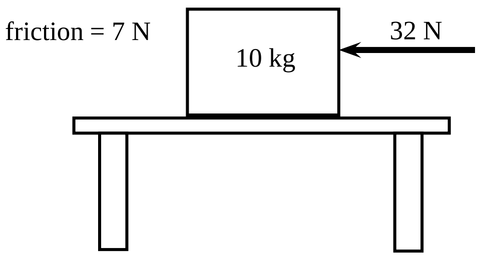
### Applying Newton's Second Law

Newton's Second Law can be applied to a variety of situations. We will look at the main types of examples that you need to study.

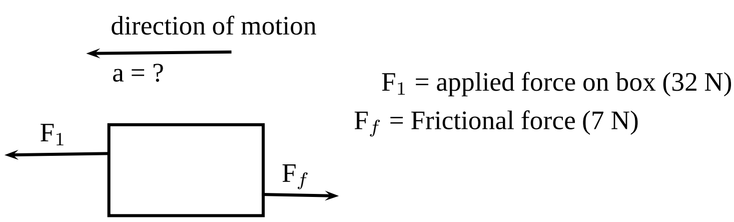
**Worked Example 64: Newton II - Box on a surface 1**

**Question:** A 10 kg box is placed on a table. A horizontal force of 32 N is applied to the box. A frictional force of 7 N is present between the surface and the box.

1. Draw a force diagram indicating all the horizontal forces acting on the box.
2. Calculate the acceleration of the box.




**Answer**
**Step 1 : Identify the horizontal forces and draw a force diagram**
We only look at the forces acting in a horizontal direction (left-right) and not vertical (up-down) forces. The applied force and the force of friction will be included. The force of gravity, which is a vertical force, will not be included.




**Step 2 : Calculate the acceleration of the box**
We have been given:
Applied force $F_1 = 32$ N
Frictional force $F_f =$ - 7 N
Mass m = 10 kg

To calculate the acceleration of the box we will be using the equation $F_R = ma$. Therefore:

$$\begin{aligned} F_R &= ma \\ F_1 + F_f &= (10)(a) \\ 32 - 7 &= 10\ a \\ 25 &= 10\ a \\ a &= 2{,}5\,\mathrm{m \cdot s^{-1}} \text{towards the left} \end{aligned}$$

**Worked Example 65: Newton II - box on surface 2**

**Question:** Two crates, 10 kg and 15 kg respectively, are connected with a thick rope according to the diagram. A force of 500 N is applied. The boxes move with an acceleration of 2 m·s$^{-2}$. One third of the total frictional force is acting on the 10 kg block and two thirds on the 15 kg block. Calculate:

1. the magnitude and direction of the frictional force present.
2. the magnitude of the tension in the rope at T.

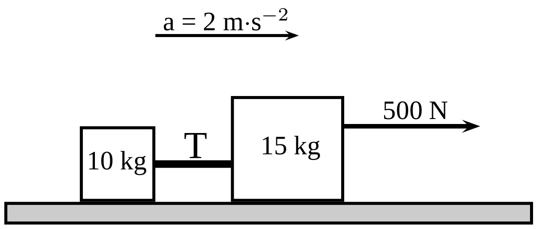



Figure 12.3: Two crates on a surface

**Answer**

**Step 3 : Draw a force diagram**

Always draw a force diagram although the question might not ask for it. The acceleration of the whole system is given, therefore a force diagram of the whole system will be drawn. Because the two crates are seen as a unit, the force diagram will look like this:

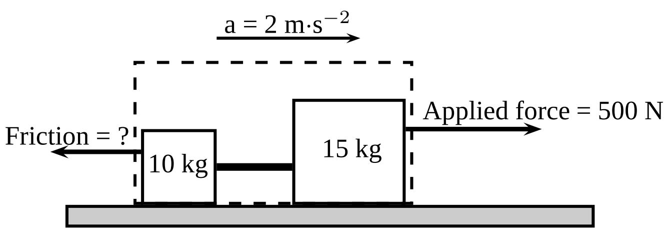



Figure 12.4: Force diagram for two crates on a surface

**Step 4 : Calculate the frictional force**

To find the frictional force we will apply Newton's Second Law. We are given the mass (10 + 15 kg) and the acceleration (2 m·s$^{-2}$). Choose the direction of motion to be the positive direction (to the right is positive).

$$\begin{aligned} F_R &= ma \\ F_{\text{applied}} + F_f &= ma \\ 500 + F_f &= (10+15)(2) \\ F_f &= 50 - 500 \\ F_f &= -450N \end{aligned}$$

The frictional force is 450 N opposite to the direction of motion (to the left).

**Step 5 : Find the tension in the rope**
To find the tension in the rope we need to look at one of the two crates on their own. Let's choose the 10 kg crate. Firstly, we need to draw a force diagram:

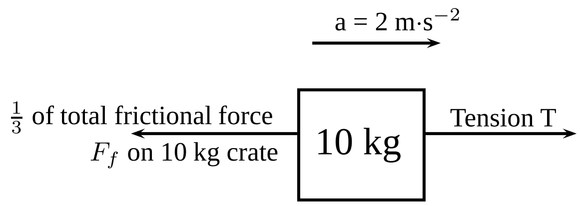



Figure 12.5: Force diagram of 10 kg crate

The frictional force on the 10 kg block is one third of the total, therefore:
$F_f = \frac{1}{3} \times 450$
$F_f = 150$ N

If we apply Newton's Second Law:

$$\begin{aligned} F_R &= ma \\ T + F_f &= (10)(2) \\ T + (-150) &= 20 \\ T &= 170 \text{ N} \end{aligned}$$

Note: If we had used the same principle and applied it to 15 kg crate, our calculations would have been the following:

$$\begin{aligned} F_R &= ma \\ F_{\text{applied}} + T + F_f &= (15)(2) \\ 500 + T + (-300) &= 30 \\ T &= -170 \text{ N} \end{aligned}$$

The negative answer here means that the force is in the direction opposite to the motion, in other words to the left, which is correct. However, the question asks for the magnitude of the force and your answer will be quoted as 170 N.

**Worked Example 66: Newton II - Man pulling a box**

**Question:** A man is pulling a 20 kg box with a rope that makes an angle of 60° with the horizontal. If he applies a force of 150 N and a frictional force of 15 N is present, calculate the acceleration of the box.

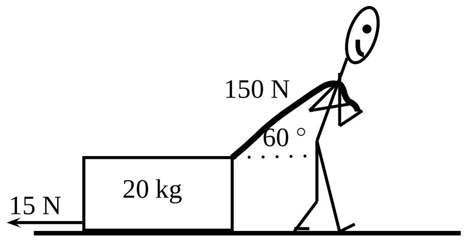



Figure 12.6: Man pulling a box

**Answer**

**Step 1 : Draw a force diagram**

The motion is horizontal and therefore we will only consider the forces in a horizontal direction. Remember that vertical forces do not influence horizontal motion and vice versa.

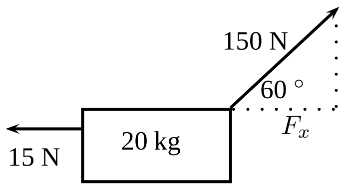



Figure 12.7: Force diagram

**Step 2 : Calculate the horizontal component of the applied force**

The applied force is acting at an angle of 60 ° to the horizontal. We can only consider forces that are parallel to the motion. The horizontal component of the applied force needs to be calculated before we can continue:

$$\begin{aligned} F_x &= 150\cos 60^\circ \\ F_x &= 75\text{N} \end{aligned}$$

**Step 3 : Calculate the acceleration**

To find the acceleration we apply Newton's Second Law:

$$\begin{aligned} F_R &= ma \\ F_x + F_f &= (20)(a) \\ 75 + (-15) &= 20a \\ a &= 3\,\text{m}\cdot\text{s}^{-2}\text{to the right} \end{aligned}$$

**Worked Example 67: Newton II - Truck and trailor**

**Question:** A 2000 kg truck pulls a 500 kg trailer with a constant acceleration. The engine of the truck produces a thrust of 10 000 N. Ignore the effect of friction.

1. Calculate the acceleration of the truck.
2. Calculate the tension in the tow bar T between the truck and the trailer, if the tow bar makes an angle of 25° with the horizontal.

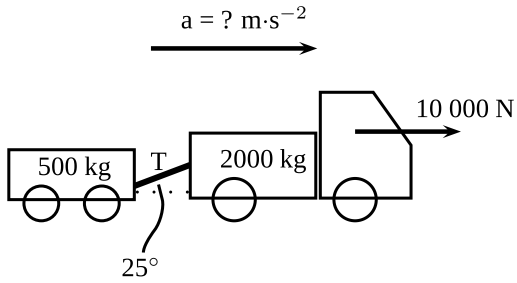



Figure 12.8: Truck pulling a trailer

**Answer**

**Step 1 : Draw a force diagram**

Draw a force diagram indicating all the horizontal forces on the system as a whole:

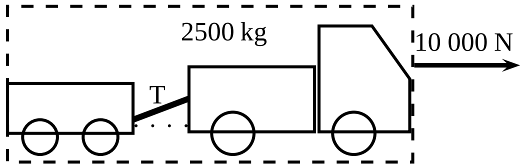



Figure 12.9: Force diagram for truck pulling a trailer

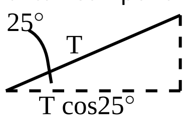
**Step 2 : Find the acceleration of the system**

In the absence of friction, the only force that causes the system to accelerate is the thrust of the engine. If we now apply Newton's Second Law:

$$\begin{aligned} F_R &= ma \\ 10000 &= (500 + 2000)a \\ a &= 4\,\mathrm{m \cdot s^{-2}} \text{ to the right} \end{aligned}$$

**Step 3 : Find the horizontal component of T**

We are asked to find the tension in the tow bar, but because the tow bar is acting at an angle, we need to find the horizontal component first. We will find the horizontal component in terms of T and then use it in the next step to find T.




The horizontal component is T cos 25°.

**Step 4 : Find the tension in the tow bar**

To find T, we will apply Newton's Second Law:

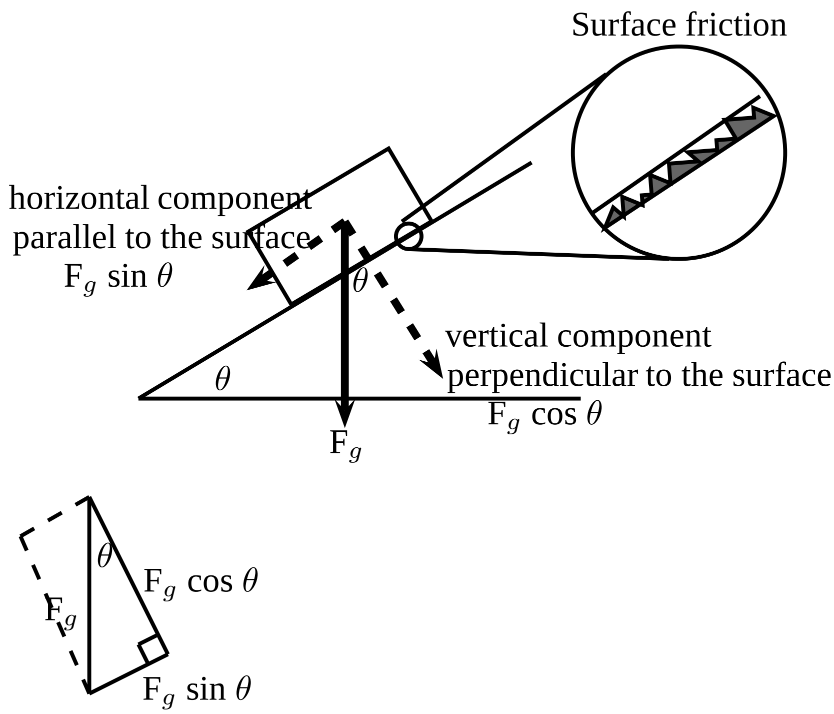
$$\begin{aligned} F_R &= ma \\ F - T\cos 25° &= ma \\ 10000 - T\cos 25° &= (2000)(4) \\ T\cos 25° &= 2000 \\ T &= 2206{,}76\mathrm{N} \end{aligned}$$

### Object on an inclined plane

When we place an object on a slope the force of gravity ($F_g$) acts straight down and not perpendicular to the slope. Due to gravity pulling straight down, the object will tend to slide down the slope with a force equal to the horizontal component of the force of gravity ($F_g$ sin $\theta$). The object will 'stick' to the slope due to the frictional force between the object and the surface. As you increase the angle of the slope, the horizontal component will also increase until the frictional force is overcome and the object starts to slide down the slope.

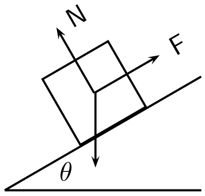
The force of gravity will also tend to push an object 'into' the slope. This force is equal to the vertical component of the force of gravity ($F_g$ cos $\theta$). There is no movement in this direction as this force is balanced by the slope pushing up against the object. This ?pushing force? is called the normal force (N) and is equal to the resultant force in the vertical direction, $F_g$ sin $\theta$ in this case, but opposite in direction.

**Important:** Do not use the abbreviation W for weight as it is used to abbreviate 'work'. Rather use the force of gravity $F_g$ for weight.




**Worked Example 68: Newton II - Box on inclined plane**

**Question:** A body of mass M is at rest on an inclined plane.




What is the magnitude of the frictional force acting on the body?

A Mg

B Mg cos $\theta$

C Mg sin $\theta$

D Mg tan $\theta$

**Answer**

**Step 1 : Analyse the situation**

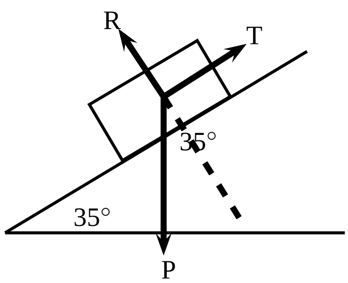
The question asks us to identify the frictional force. The body is said to be at rest on the plane, which means that it is not moving and therefore there is no resultant force. The frictional force must therefore be balanced by the force F up the inclined plane.

**Step 2 : Choose the correct answer**

The frictional force is equal to the horizontal component of the weight (Mg) which is equal to Mg sin $\theta$.

**Worked Example 69: Newton II - Object on a slope**

**Question:** A force T = 312 N is required to keep a body at rest on a frictionless inclined plane which makes an angle of 35° with the horizontal. The forces acting on the body are shown. Calculate the magnitudes of forces P and R, giving your answers to three significant figures.




**Answer**

**Step 1 : Find the magnitude of P**

We are usually asked to find the magnitude of T, but in this case T is given and we are asked to find P. We can use the same equation. T is the force that balances the horizontal component of P ($P_x$) and therefore it has the same magnitude.

$$\begin{aligned} T &= P \sin\theta \\ 312 &= P \sin 35° \\ P &= 544 \text{ N} \end{aligned}$$

**Step 2 : Find the magnitude of R**

R can also be determined with the use of trigonometric ratios. The tan or cos ratio can be used. We recommend that you use the tan ratio because it does not involve using the value for P (for in case you made a mistake in calculating P).

$$\begin{aligned} \tan 55° &= \frac{R}{T} \\ \tan 55° &= \frac{R}{312} \\ R &= \tan 55° \times 312 \\ R &= 445{,}6 \text{ N} \\ R &= 446 \text{ N} \end{aligned}$$

Note that the question asks that the answers be given to 3 significant figures. We therefore round 445,6 N up to 446 N.

**Lifts and rockets**

So far we have looked at objects being pulled or pushed across a surface, in other words horizontal motion. Here we only considered horizontal forces, but we can also lift objects up or let them fall. This is vertical motion where only vertical forces are being considered.

Let us consider a 500 kg lift, with no passengers, hanging on a cable. The purpose of the cable is to pull the lift upwards so that it can reach the next floor or to let go a little so that it can move downwards to the floor below. We will look at five possible stages during the motion of the lift.

**Stage 1:**
The 500 kg lift is stationary at the second floor of a tall building.
Because the lift is stationary (not moving) there is no resultant force acting on the lift. This means that the upward forces must be balanced by the downward forces. The only force acting down is the force of gravity which is equal to (500 x 9,8 = 4900 N) in this case. The cable must therefore pull upwards with a force of 4900 N to keep the lift stationary at this point.

**Stage 2:**
The lift moves upwards at an acceleration of 1 m·s$^{-2}$.
If the lift is accelerating, it means that there is a resultant force in the direction of the motion. This means that the force acting upwards is now bigger than the force of gravity $F_g$ (down). To find the magnitude of the force applied by the cable ($F_c$) we can do the following calculation: (Remember to choose a direction as positive. We have chosen upwards as positive.)

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ F_c + (-4900) &= (500)(1) \\ F_c &= 5400 \text{ N upwards} \end{aligned}$$

The answer makes sense as we need a bigger force upwards to cancel the effect of gravity as well as make the lift go faster.

**Stage 3:**
The lift moves at a constant velocity.
When the lift moves at a constant velocity, it means that all the forces are balanced and that there is no resultant force. The acceleration is zero, therefore $F_R = 0$. The force acting upwards is equal to the force acting downwards, therefore $F_c = 4900$ N.

**Stage 4:**
The lift slow down at a rate of 2m·s$^{-2}$.
As the lift is now slowing down there is a resultant force downwards. This means that the force acting downwards is bigger than the force acting upwards. To find the magnitude of the force applied by the cable ($F_c$) we can do the following calculation: Again we have chosen upwards as positive, which means that the acceleration will be a negative number.

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ F_c + (-4900) &= (500)(-2) \\ F_c &= 3900 \text{ N upwards} \end{aligned}$$

This makes sense as we need a smaller force upwards to ensure a resultant force down. The force of gravity is now bigger than the upward pull of the cable and the lift will slow down.

**Stage 5:**
The cable snaps.
When the cable snaps, the force that used to be acting upwards is no longer present. The only force that is present would be the force of gravity. The lift will freefall and its acceleration can be calculated as follows:

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ 0 + (-4900) &= (500)(a) \\ a &= -9{,}8\,\mathrm{m \cdot s^{-2}} \\ a &= 9{,}8\,\mathrm{m \cdot s^{-2}}\text{downwards} \end{aligned}$$

### Rockets

Like with lifts, rockets are also examples of objects in vertical motion. The force of gravity pulls the rocket down while the thrust of the engine pushes the rocket upwards. The force that the engine exerts must overcome the force of gravity so that the rocket can accelerate upwards. The worked example below looks at the application of Newton's Second Law in launching a rocket.

**Worked Example 70: Newton II - rocket**

**Question:** A rocket is launched vertically upwards into the sky at an acceleration of 20 m·s$^{-2}$. If the mass of the rocket is 5000 kg, calculate the magnitude and direction of the thrust of the rocket?s engines.

**Answer**

**Step 1 : Analyse what is given and what is asked**

We have the following:
m = 5000 kg
a = 20 m·s$^{-2}$
$F_g$ = 5000 x 9,8 = 49000 N
We are asked to find the thrust of the rocket engine $F_1$.

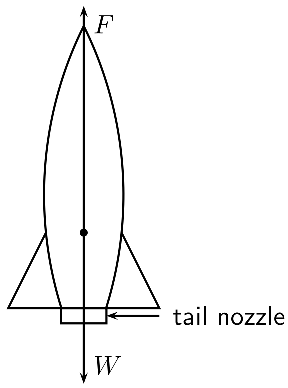
**Step 2 : Find the thrust of the engine**

We will apply Newton's Second Law:

$$\begin{aligned} F_R &= ma \\ F_1 + F_g &= ma \\ F_1 + (-49000) &= (5000)(20) \\ F_1 &= 149\ 000\ \text{N upwards} \end{aligned}$$

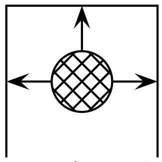
**Worked Example 71: Rockets**

**Question:** How do rockets accelerate in space?




**Answer**

- Gas explodes inside the rocket.
- This exploding gas exerts a force on each side of the rocket (as shown in the picture below of the explosion chamber inside the rocket).



Note that the forces shown in this picture are representative. With an explosion there will be forces in all directions.

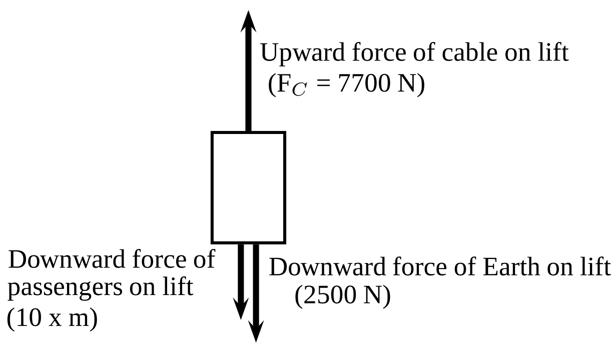
- Due to the symmetry of the situation, all the forces exerted on the rocket are balanced by forces on the opposite side, except for the force opposite the open side. This force on the upper surface is unbalanced.
- This is therefore the resultant force acting on the rocket and it makes the rocket accelerate forwards.

**Worked Example 72: Newton II - lifts**

**Question:** A lift, mass 250 kg, is initially at rest on the ground floor of a tall building. Passengers with an unknown total mass, m, climb into the lift. The lift accelerates upwards at 1,6 m·s$^{-2}$. The cable supporting the lift exerts a constant upward force of 7700 N. Use g = 10 m·s$^{-2}$.

1. Draw a labeled force diagram indicating all the forces acting on the lift while it accelerates upwards.
2. What is the maximum mass, m, of the passengers the lift can carry in order to achieve a constant upward acceleration of 1,6 m·s$^{-2}$.

**Answer**
**Step 1 : Draw a force diagram.**



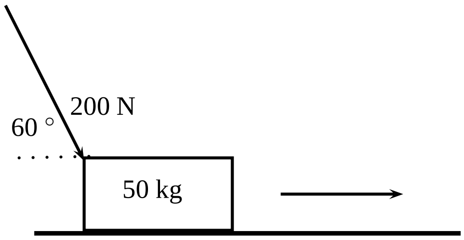
**Step 2 : Find the mass, m.**

Let us look at the lift with its passengers as a unit. The mass of this unit will be (250 + m) kg and the force of the Earth pulling downwards ($F_g$) will be (250 + m) x 10. If we apply Newton's Second Law to the situation we get:

$$\begin{aligned} F_{net} &= ma \\ F_C - F_g &= ma \\ 7700 - (250+m)(10) &= (250+m)(1{,}6) \\ 7700 - 2500 - 10\ m &= 400 + 1{,}6\ m \\ 4800 &= 11{,}6\ m \\ m &= 413{,}79\,\text{kg} \end{aligned}$$

### 12.3.3 Exercise

1. A tug is capable of pulling a ship with a force of 100 kN. If two such tugs are pulling on one ship, they can produce any force ranging from a minimum of 0 N to a maximum of 200 kN. Give a detailed explanation of how this is possible. Use diagrams to support your result.
2. A car of mass 850 kg accelerates at 2 m·s$^{-2}$. Calculate the magnitude of the resultant force that is causing the acceleration.
3. Find the force needed to accelerate a 3 kg object at 4 m·s$^{-2}$.
4. Calculate the acceleration of an object of mass 1000 kg accelerated by a force of 100 N.
5. An object of mass 7 kg is accelerating at 2,5 m·s$^{-2}$. What resultant force acts on it?
6. Find the mass of an object if a force of 40 N gives it an acceleration of 2 m·s$^{-2}$.
7. Find the acceleration of a body of mass 1 000 kg that has a 150 N force acting on it.
8. Find the mass of an object which is accelerated at 2 m·s$^{-2}$ by a force of 40 N.
9. Determine the acceleration of a mass of 24 kg when a force of 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved?
10. A mass of 8 kg is accelerating at 5 m·s$^{-2}$.
    10.1 Determine the resultant force that is causing the acceleration.
    10.2 What acceleration would be produced if we doubled the force and reduced the mass by half?
11. A motorcycle of mass 100 kg is accelerated by a resultant force of 500 N. If the motorcycle starts from rest:
    11.1 What is its acceleration?
    11.2 How fast will it be travelling after 20 s?
    11.3 How long will it take to reach a speed of 35 m·s$^{-1}$?
    11.4 How far will it travel from its starting point in 15 s?
12. A force acting on a trolley on a frictionless horizontal plane causes an acceleration of magnitude 6 m·s$^{-2}$. Determine the mass of the trolley.
13. A force of 200 N, acting at 60° to the horizontal, accelerates a block of mass 50 kg along a horizontal plane as shown.



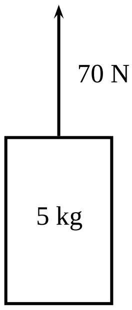

13.1 Calculate the component of the 200 N force that accelerates the block horizontally.

13.2 If the acceleration of the block is 1,5 m·s$^{-2}$, calculate the magnitude of the frictional force on the block.

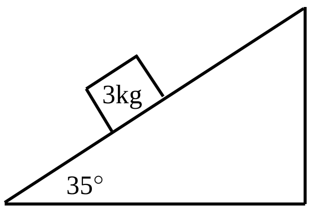
13.3 Calculate the vertical force exerted by the block on the plane.

14. A toy rocket of mass 0,5 kg is supported vertically by placing it in a bottle. The rocket is then ignited. Calculate the force that is required to accelerate the rocket vertically upwards at 8 m·s$^{-2}$.

15. A constant force of 70 N is applied vertically to a block of mass 5 kg as shown. Calculate the acceleration of the block.




16. A stationary block of mass 3kg is on top of a plane inclined at 35° to the horizontal.




16.1 Draw a force diagram (not to scale). Include the weight ($F_g$) of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.

16.2 Determine the values of the weight's perpendicular and parallel components ($F_{gx}$ and $F_{gy}$).

16.3 Determine the magnitude and direction of the frictional force between the block and plane.

17. A student of mass 70 kg investigates the motion of a lift. While he stands in the lift on a bathroom scale (calibrated in newton), he notes three stages of his journey.

17.1 For 2 s immediately after the lift starts, the scale reads 574 N.

17.2 For a further 6 s it reads 700 N.

17.3 For the final 2 s it reads 854 N.

Answer the following questions:

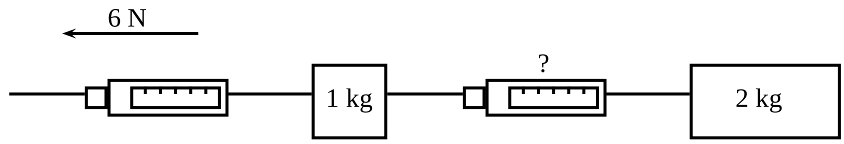
17.1 Is the motion of the lift upward or downward? Give a reason for your answer.

17.2 Write down the magnitude and the direction of the resultant force acting on the student for each of the stages I, II and III.

17.3 Calculate the magnitude of the acceleration of the lift during the first 2s.

18. A car of mass 800 kg accelerates along a level road at 4 $m{\cdot}s^{-2}$. A frictional force of 700 N opposes its motion. What force is produced by the car's engine?

19. Two objects, with masses of 1 kg and 2 kg respectively, are placed on a smooth surface and connected with a piece of string. A horizontal force of 6 N is applied with the help of a spring balance to the 1 kg object. Ignoring friction, what will the force acting on the 2 kg mass, as measured by a second spring balance, be?




20. A rocket of mass 200 kg has a resultant force of 4000 N upwards on it.

20.1 What is its acceleration in space, where it has no weight?

20.2 What is its acceleration on the Earth, where it has weight?

20.3 What driving force does the rocket engine need to exert on the back of the rocket in space?

20.4 What driving force does the rocket engine need to exert on the back of the rocket on the Earth?

21. A car going at 20 $m{\cdot}s^{-1}$stops in a distance of 20 m.

21.1 What is its acceleration?

21.2 If the car is 1000 kg how much force do the brakes exert?

### 12.3.4 Newton's Third Law of Motion

Newton's Third Law of Motion deals with the interaction between pairs of objects. For example, if you hold a book up against a wall you are exerting a force on the book (to keep it there) and the book is exerting a force back at you (to keep you from falling through the book). This may sound strange, but if the book was not pushing back at you, your hand will push through the book! These two forces (the force of the hand on the book ($F_1$) and the force of the book on the hand ($F_2$)) are called an action-reaction pair of forces. They have the same magnitude, but act in opposite directions and act on different objects (the one force is onto the book and the other is onto your hand).

There is another action-reaction pair of forces present in this situation. The book is pushing against the wall (action force) and the wall is pushing back at the book (reaction). The force of the book on the wall ($F_3$) and the force of the wall on the book ($F_4$) are shown in the diagram.

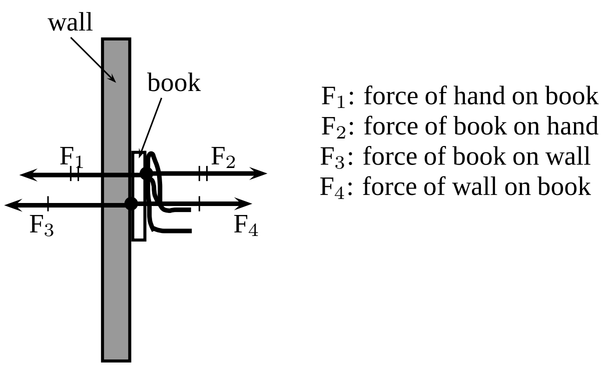



Figure 12.10: Newton's action-reaction pairs

**Definition: Newton's Third Law of Motion**
If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.

Newton's action-reaction pairs can be found everywhere in life where two objects interact with one another. The following worked examples will illustrate this:

**Worked Example 73: Newton III - seat belt**

**Question:** Dineo is seated in the passenger seat of a car with the seat belt on. The car suddenly stops and he moves forwards until the seat belt stops him. Draw a labeled force diagram identifying two action-reaction pairs in this situation.

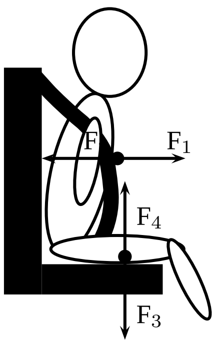
**Answer**

**Step 1 : Draw a force diagram**

Start by drawing the picture. You will be using arrows to indicate the forces so make your picture large enough so that detailed labels can also be added. The picture needs to be accurate, but not artistic! Use stickmen if you have to.

**Step 2 : Label the diagram**

Take one pair at a time and label them carefully. If there is not enough space on the drawing, then use a key on the side.




$F_1$: The force of Dineo on the seat belt
$F_2$: The force of the seat belt on Dineo
$F_3$: The force of Dineo on the seat (downwards)
$F_4$: The force of the seat on Dineo (upwards)

**Worked Example 74: Newton III - forces in a lift**

**Question:** Tammy travels from the ground floor to the fifth floor of a hotel in a lift. Which ONE of the following statements is TRUE about the force exerted by the floor of the lift on Tammy's feet?

A It is greater than the magnitude of Tammy's weight.

B It is equal in magnitude to the force Tammy's feet exert on the floor.

C It is equal to what it would be in a stationary lift.

D It is greater than what it would be in a stationary lift.

**Answer**

**Step 1 : Analyse the situation**

This is a Newton's Third Law question and not Newton II. We need to focus on the action-reaction pairs of forces and not the motion of the lift. The following diagram will show the action-reaction pairs that are present when a person is standing on a scale in a lift.

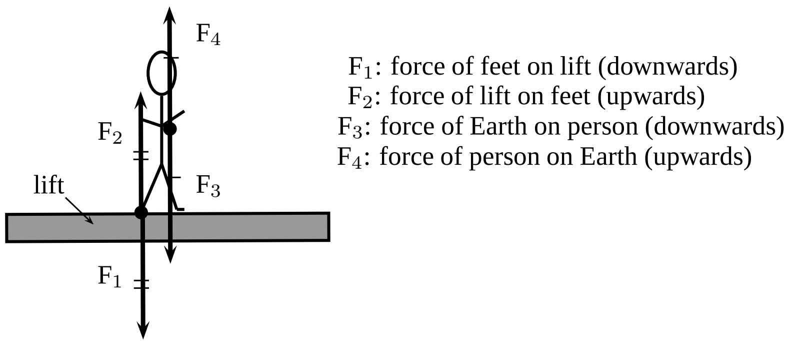



Figure 12.11: Newton's action-reaction pairs in a lift

In this question statements are made about the force of the floor (lift) on Tammy's feet. This force corresponds to $F_2$ in our diagram. The reaction force that pairs up with this one is $F_1$, which is the force that Tammy's feet exerts on the floor of the lift. The magnitude of these two forces are the same, but they act in opposite directions.

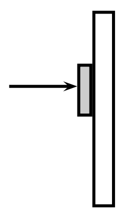
**Step 2 : Choose the correct answer**

It is important to analyse the question first, before looking at the answers as the answers might confuse you. Make sure that you understand the situation and know what is asked before you look at the options.
The correct answer is B.

**Worked Example 75: Newton III - book and wall**

**Question:** Tumi presses a book against a vertical wall as shown in the sketch.

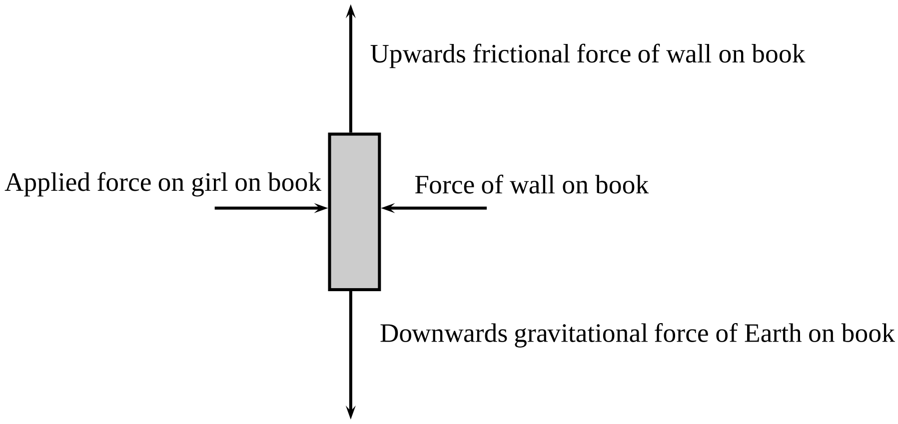
1. Draw a labelled force diagram indicating all the forces acting on the book.
2. State, in words, Newton's Third Law of Motion.
3. Name the action-reaction pairs of forces acting in the horizontal plane.



**Answer**

**Step 1 : Draw a force diagram**

A force diagram will look like this:




Note that we had to draw all the force acting on the book and not the action-reaction pairs. None of the forces drawn are action-reaction pairs, because they all act on the same object (the book). When you label forces, be as specific as possible, including the direction of the force and both objects involved, for example, do not say gravity (which is an incomplete answer) but rather say 'Downward (direction) gravitational force of the Earth (object) on the book (object)'.

**Step 2 : State Newton's Third Law**
If body A exerts a force onto body B, then body B will exert a force equal in magnitude, but opposite in direction, onto body A.

**Step 3 : Name the action-reaction pairs**
The question only asks for action-reaction forces in the horizontal plane. Therefore:
Pair 1: Action: Applied force of the girl on the book; Reaction: The force of the book on the girl.
Pair 2: Action: Force of the book on the wall; Reaction: Force of the wall on the book.
Note that a Newton III pair will always involve the same combination of words, like 'book on wall' and wall on book'. The objects are 'swopped around' in naming the pairs.

**Activity :: Experiment : Balloon Rocket**
**Aim:**
In this experiment for the entire class, you will use a balloon rocket to investigate Newton's Third Law. A fishing line will be used as a track and a plastic straw taped to the balloon will help attach the balloon to the track.
**Apparatus:**
You will need the following items for this experiment:

1. balloons (one for each team)
2. plastic straws (one for each team)
3. tape (cellophane or masking)
4. fishing line, 10 meters in length
5. a stopwatch - optional (a cell phone can also be used)
6. a measuring tape - optional

**Method:**

1. Divide into groups of at least five.
2. Attach one end of the fishing line to the blackboard with tape. Have one teammate hold the other end of the fishing line so that it is taut and roughly horizontal. The line must be held steady and **must not** be moved up or down during the experiment.
3. Have one teammate blow up a balloon and hold it shut with his or her fingers. Have another teammate tape the straw along the side of the balloon. Thread the fishing line through the straw and hold the balloon at the far end of the line.
4. Let go of the rocket and observe how the rocket moves forward.
5. Optionally, the rockets of each group can be timed to determine a winner of the fastest rocket.
   5.1 Assign one teammate to time the event. The balloon should be let go when the time keeper yells "Go!" Observe how your rocket moves toward the blackboard.
   5.2 Have another teammate stand right next to the blackboard and yell "Stop!" when the rocket hits its target. If the balloon does not make it all the way to the blackboard, "Stop!" should be called when the balloon stops moving. The timekeeper should record the flight time.
   5.3 Measure the exact distance the rocket traveled. Calculate the average speed at which the balloon traveled. To do this, divide the distance traveled by the time the balloon was "in flight." Fill in your results for Trial 1 in the Table below.
   5.4 Each team should conduct two more races and complete the sections in the Table for Trials 2 and 3. Then calculate the average speed for the three trials to determine your team's race entry time.

**Results:**

| | **Distance (m)** | **Time (s)** | **Speed (m·s$^{-1}$)** |
|---|---|---|---|
| Trial 1 | | | |
| Trial 2 | | | |
| Trial 3 | | | |
| | | Average: | |

**Conclusions:**

The winner of this race is the team with the fastest average balloon speed.

While doing the experiment, you should think about,

1. What made your rocket move?
2. How is Newton's Third Law of Motion demonstrated by this activity?
3. Draw pictures using labeled arrows to show the forces acting on the inside of the balloon before it was released and after it was released.

---

### 12.3.5 Exercise

1. A fly hits the front windscreen of a moving car. Compared to the magnitude of the force the fly exerts on the windscreen, the magnitude of the force the windscreen exerts on the fly during the collision, is ...

   A zero.

   B smaller, but not zero.

   C bigger.

   D the same.

2. A log of wood is attached to a cart by means of a light, inelastic rope. A horse pulls the cart along a rough, horizontal road with an applied force F. The total system accelerates initally with an acceleration of magnitude a (figure 1). The forces acting on the cart during the acceleration, are indicated in Figure 2.

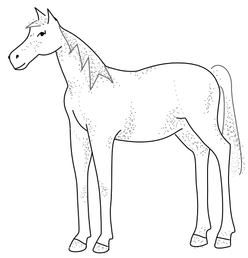


Figure 1

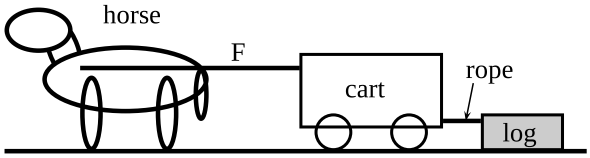
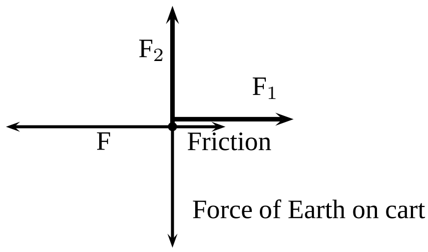



Figure 2



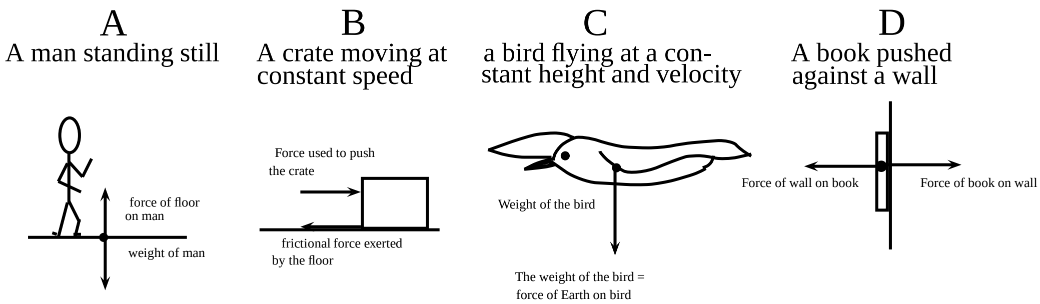

A $F_1$: Force of log on cart; $F_2$: Reaction force of Earth on cart

B $F_1$: Force of log on cart; $F_2$: Force of road on cart

C $F_1$: Force of rope on cart; $F_2$: Reaction force of Earth on cart

D $F_1$: Force of rope on cart; $F_2$: Force of road on cart

3. Which of the following pairs of forces correctly illustrates Newton's Third Law?




## 12.3.6 Different types of forces

### Tension

Tension is the magnitude of the force that exists in objects like ropes, chains and struts that are providing support. For example, there are tension forces in the ropes supporting a child's swing hanging from a tree.

### Contact and non-contact forces

In this chapter we have come across a number of different types of forces, for example a push or a pull, tension in a string, frictional forces and the normal. These are all examples of contact forces where there is a physical point of contact between applying the force and the object. Non-contact forces are forces that act over a distance, for example magnetic forces, electrostatic forces and gravitational forces.

When an object is placed on a surface, two types of surface forces can be identified. Friction is a force that acts between the surface and the object and parallel to the surface. The normal force is a force that acts between the object and the surface and parallel to the surface.

### The normal force

A 5 kg box is placed on a rough surface and a 10 N force is applied at an angle of 36,9° to the horizontal. The box does not move. The normal force (N or $F_N$) is the force between the box and the surface acting in the vertical direction. If this force is not present the box would fall through the surface because the force of gravity pulls it downwards. The normal force therefore acts upwards. We can calculate the normal force by considering all the forces in the vertical direction. All the forces in the vertical direction must add up to zero because there is no movement in the vertical direction.

$$\begin{aligned} N + F_y + F_g &= 0 \\ N + 6 + (-49) &= 0 \\ N &= 43\,\mathrm{N} \text{ upwards} \end{aligned}$$

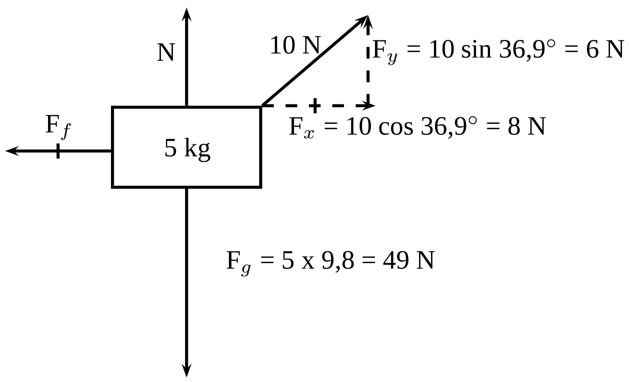



Figure 12.12: Friction and the normal force

The most interesting and illustrative normal force question, that is often asked, has to do with a scale in a lift. Using Newton's third law we can solve these problems quite easily.

When you stand on a scale to measure your weight you are pulled down by gravity. There is no acceleration downwards because there is a reaction force we call the normal force acting upwards on you. This is the force that the scale would measure. If the gravitational force were less then the reading on the scale would be less.

#### Worked Example 76: Normal Forces 1

**Question:** A man with a mass of 100 kg stands on a scale (measuring newtons). What is the reading on the scale?

**Answer**

**Step 1 : Identify what information is given and what is asked for**

We are given the mass of the man. We know the gravitational acceleration that acts on him is 9,8 = m·s$^{-2}$.

**Step 2 : Decide what equation to use to solve the problem**

The scale measures the normal force on the man. This is the force that balances gravity. We can use Newton's laws to solve the problem:

$$F_r = F_g + F_N$$

where $F_r$ is the resultant force on the man.

**Step 3 : Firstly we determine the force on the man due to gravity**

$$\begin{aligned} F_g &= mg \\ &= (100\,\text{kg})(9{,}8\,\text{m}\cdot\text{s}^{-2}) \\ &= 980\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2} \\ &= 980\,\text{N downwards} \end{aligned}$$

**Step 4 : Now determine the normal force acting upwards on the man**

We now know the gravitational force downwards. We know that the sum of all the forces must equal the resultant acceleration times the mass. The overall resultant acceleration of the man on the scale is 0 - so $F_r = 0$.

$$\begin{aligned} F_r &= F_g + F_N \\ 0 &= -980\,\text{N} + \text{F}_\text{N} \\ F_N &= 980\,\text{N upwards} \end{aligned}$$

**Step 5 : Quote the final answer**

The normal force is 980 N upwards. It exactly balances the gravitational force downwards so there is no net force and no acceleration on the man. The reading on the scale is 980 N.

Now we are going to add things to exactly the same problem to show how things change slightly. We will now move to a lift moving at constant velocity. Remember if velocity is constant then acceleration is zero.

**Worked Example 77: Normal Forces 2**

**Question:** A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that moving downwards at a constant velocity of 2 m·s$^{-1}$. What is the reading on the scale?

**Answer**

**Step 6 : Identify what information is given and what is asked for**

We are given the mass of the man and the acceleration of the lift. We know the gravitational acceleration that acts on him.

**Step 7 : Decide which equation to use to solve the problem**

Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, $F_r$.

$$F_r = F_g + F_N$$

**Step 8 : Determine the force due to gravity**

$$\begin{aligned} F_g &= mg \\ &= (100\,\text{kg})(9{,}8\,\text{m}\cdot\text{s}^{-2}) \\ &= 980\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2} \\ &= 980\,\text{N downwards} \end{aligned}$$

**Step 9 : Now determine the normal force acting upwards on the man**

The scale measures this normal force, so once we have determined it we will know the reading on the scale. Because the lift is moving at constant velocity the overall resultant acceleration of the man on the scale is 0. If we write out the equation:

$$\begin{aligned} F_r &= F_g + F_N \\ ma &= F_g + F_N \\ (100)(0) &= -980\,\mathrm{N} + \mathrm{F_N} \\ F_N = 980\,\mathrm{N}\ \text{upwards} \end{aligned}$$

**Step 10 : Quote the final answer**
The normal force is 980 N upwards. It exactly balances the gravitational force downwards so there is no net force and no acceleration on the man. The reading on the scale is 980 N.

In the previous two examples we got exactly the same result because the net acceleration on the man was zero! If the lift is accelerating downwards things are slightly different and now we will get a more interesting answer!

**Worked Example 78: Normal Forces 3**

**Question:** A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that is accelerating downwards at 2 m·s$^{-2}$. What is the reading on the scale?
**Answer**
**Step 1 : Identify what information is given and what is asked for**
We are given the mass of the man and his resultant acceleration - this is just the acceleration of the lift. We know the gravitational acceleration also acts on him.
**Step 2 : Decide which equation to use to solve the problem**
Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, $F_r$.

$$F_r = F_g + F_N$$

**Step 3 : Determine the force due to gravity, $F_g$**

$$\begin{aligned} F_g &= mg \\ &= (100\,\mathrm{kg})(9{,}8\,\mathrm{m\cdot s^{-2}}) \\ &= 980\ \mathrm{kg\cdot\ m\cdot s^{-2}} \\ &= 980\,\mathrm{N}\ \text{downwards} \end{aligned}$$

**Step 4 : Determine the resultant force, $F_r$**
The resultant force can be calculated by applying Newton's Second Law:

$$\begin{aligned} F_r &= ma \\ F_r &= (100)(-2) \\ &= -200\ N \\ &= 200\ N\ \text{down} \end{aligned}$$

**Step 5 : Determine the normal force, $F_N$**
The sum of all the vertical forces is equal to the resultant force, therefore

$$\begin{aligned} F_r &= F_g + F_N \\ -200 &= -980 + F_N \\ F_N &= 780\,\mathrm{N}\ \text{upwards} \end{aligned}$$

**Step 6 : Quote the final answer**

The normal force is 780 N upwards. It balances the gravitational force downwards just enough so that the man only accelerates downwards at 2 m·s$^{-2}$. The reading on the scale is 780 N.

**Worked Example 79: Normal Forces 4**

**Question:** A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that is accelerating upwards at 4 m·s$^{-2}$. What is the reading on the scale?

**Answer**

**Step 1 : Identify what information is given and what is asked for**

We are given the mass of the man and his resultant acceleration - this is just the acceleration of the lift. We know the gravitational acceleration also acts on him.

**Step 2 : Decide which equation to use to solve the problem**

Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, $F_r$.

$$F_r = F_g + F_N$$

**Step 3 : Determine the force due to gravity, $F_g$**

$$\begin{aligned} F_g &= mg \\ &= (100\,\mathrm{kg})(9{,}8\,\mathrm{m\cdot s^{-2}}) \\ &= 980\ \mathrm{kg\cdot\ m\cdot s^{-2}} \\ &= 980\,\mathrm{N\ downwards} \end{aligned}$$

**Step 4 : Determine the resultant force, $F_r$**

The resultant force can be calculated by applying Newton's Second Law:

$$\begin{aligned} F_r &= ma \\ F_r &= (100)(4) \\ &= 400\ N\ \text{upwards} \end{aligned}$$

**Step 5 : Determine the normal force, $F_N$**

The sum of all the vertical forces is equal to the resultant force, therefore

$$\begin{aligned} F_r &= F_g + F_N \\ 400 &= -980 + F_N \\ F_N &= 1380\,\mathrm{N\ upwards} \end{aligned}$$

**Step 6 : Quote the final answer**

The normal force is 1380 N upwards. It balances the gravitational force downwards and then in addition applies sufficient force to accelerate the man upwards at 4m·s$^{-2}$. The reading on the scale is 1380 N.

### Friction forces

When the surface of one object slides over the surface of another, each body exerts a frictional force on the other. For example if a book slides across a table, the table exerts a frictional force onto the book and the book exerts a frictional force onto the table (Newton's Third Law). Frictional forces act parallel to the surfaces.

A force is not always big enough to make an object move, for example a small applied force might not be able to move a heavy crate. The frictional force opposing the motion of the crate is equal to the applied force but acting in the opposite direction. This frictional force is called *static friction*. When we increase the applied force (push harder), the frictional force will also increase until the applied force overcomes it. This frictional force can vary from zero (when no other forces are present and the object is stationary) to a maximum that depends on the surfaces. When the applied force is greater than the frictional force and the crate will move. The frictional force will now decrease to a new constant value which is also dependent on the surfaces. This is called *kinetic friction*. In both cases the maximum frictional force is related to the normal force and can be calculated as follows:

For static friction: $F_f \leq \mu_s$ N

Where $\mu_s$ = the coefficient of static friction
and N = normal force

For kinetic friction: $F_f = \mu_k$ N

Where $\mu_k$ = the coefficient of kinetic friction
and N = normal force

Remember that static friction is present when the object is not moving and kinetic friction while the object is moving. For example when you drive at constant velocity in a car on a tar road you have to keep the accelerator pushed in slightly to overcome the kinetic friction between the tar road and the wheels of the car. The higher the value for the coefficient of friction, the more 'sticky' the surface is and the lower the value, the more 'slippery' the surface is.

The frictional force ($F_f$) acts in the horizontal direction and can be calculated in a similar way to the normal for as long as there is no movement. If we use the same example as in figure 12.12 and we choose to the right as positive,

$$\begin{aligned} F_f + F_x &= 0 \\ F_f + (+8) &= 0 \\ F_f &= -8 \\ F_f &= 8\,\mathrm{N} \text{ to the left} \end{aligned}$$

**Worked Example 80: Forces on a slope**

**Question:** A 50 kg crate is placed on a slope that makes an angle of 30° with the horizontal. The box does not slide down the slope. Calculate the magnitude and direction of the frictional force and the normal force present in this situation.

**Answer**

**Step 1 : Draw a force diagram**

Draw a force diagram and fill in all the details on the diagram. This makes it easier to understand the problem.

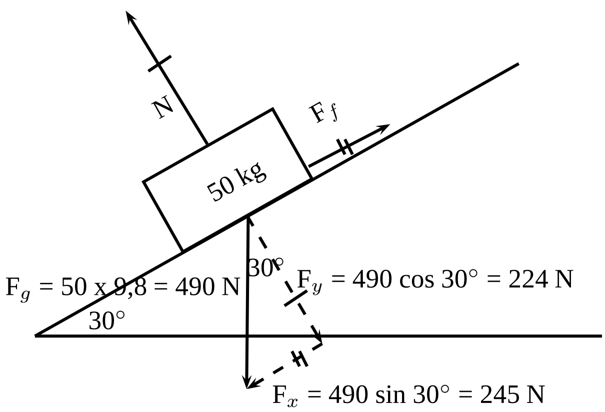



Figure 12.13: Friction and the normal forces on a slope

**Step 2 : Calculate the normal force**

The normal force acts perpendicular to the surface (and not vertically upwards). It's magnitude is equal to the component of the weight perpendicular to the slope. Therefore:

$$\begin{aligned} N &= F_g \; cos \; 30° \\ N &= 490 \; cos \; 30° \\ N &= 224\,\text{N perpendicular to the surface} \end{aligned}$$

**Step 3 : Calculate the frictional force**

The frictional force acts parallel to the surface and up the slope. It's magnitude is equal to the component of the weight parallel to the slope. Therefore:

$$\begin{aligned} F_f &= F_g \; sin \; 30° \\ F_f &= 490 \; sin \; 30° \\ F_f &= 245\,\text{N up the slope} \end{aligned}$$

We often think about friction in a negative way but very often friction is useful without us realizing it. If there was no friction and you tried to prop a ladder up against a wall, it would simply slide to the ground. Rock climbers use friction to maintain their grip on cliffs. The brakes of cars would be useless if it wasn't for friction!

**Worked Example 81: Coefficients of friction**

**Question:** A block of wood weighing 32 N is placed on a rough slope and a rope is tied to it. The tension in the rope can be increased to 8 N before the block starts to slide. A force of 4 N will keep the block moving at constant speed once it has been set in motion. Determine the coefficients of static and kinetic friction.

**Answer**

**Step 1 : Analyse the question and determine what is asked**

The weight of the block is given (32 N) and two situations are identified: One where the block is not moving (applied force is 8 N), and one where the block is moving (applied force is 4 N).

We are asked to find the coefficient for static friction $\mu_s$ and kinetic friction $\mu_k$.

**Step 2 : Find the coefficient of static friction**

$$\begin{aligned} F_f &= \mu_s N \\ 8 &= \mu_s(32) \\ \mu_s &= 0{,}25 \end{aligned}$$

Note that the coefficient of friction does not have a unit as it shows a ratio. The value for the coefficient of friction friction can have any value up to a maximum of 0,25. When a force less than 8 N is applied, the coefficient of friction will be less than 0,25.

**Step 3 : Find the coefficient of kinetic friction**
The coefficient of kinetic friction is sometimes also called the coefficient of dynamic friction. Here we look at when the block is moving:

$$\begin{aligned} F_f &= \mu_k N \\ 4 &= \mu_k(32) \\ \mu_k &= 0{,}125 \end{aligned}$$

### 12.3.7 Exercise

1. A 12 kg box is placed on a rough surface. A force of 20 N applied at an angle of 30° to the horizontal cannot move the box. Calculate the magnitude and direction of the normal and friction forces.

2. A 100 kg crate is placed on a slope that makes an angle of 45° with the horizontal. The box does not slide down the slope. Calculate the magnitude and acceleration of the frictional force and the normal force present in this situation.

3. What force T at an angle of 30° above the horizontal, is required to drag a block weighing 20 N to the right at constant speed, if the coefficient of kinetic friction between the block and the surface is 0,20?

4. A block weighing 20 N rests on a horizontal surface. The coefficient of static friction between the block and the surface is 0,40 and the coefficient of dynamic friction is 0,20.

   4.1 What is the magnitude of the frictional force exerted on the block while the block is at rest?

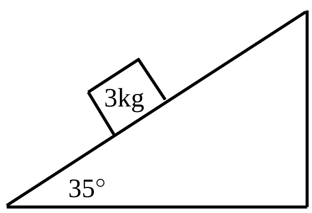
   4.2 What will the magnitude of the frictional force be if a horizontal force of 5 N is exerted on the block?

   4.3 What is the minimum force required to start the block moving?

   4.4 What is the minimum force required to keep the block in motion once it has been started?

   4.5 If the horizontal force is 10 N, determine the frictional force.

5. A stationary block of mass 3kg is on top of a plane inclined at $35°$ to the horizontal.




   5.1 Draw a force diagram (not to scale). Include the weight of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.

   5.2 Determine the values of the weight's perpendicular and parallel components.

5.3 There exists a frictional force between the block and plane. Determine this force (magnitude and direction).

6. A lady injured her back when she slipped and fell in a supermarket. She holds the owner of the supermarket accountable for her medical expenses. The owner claims that the floor covering was not wet and meets the accepted standards. He therefore cannot accept responsibility. The matter eventually ends up in court. Before passing judgement, the judge approaches you, a science student, to determine whether the coefficient of static friction of the floor is a minimum of 0,5 as required. He provides you with a tile from the floor, as well as one of the shoes the lady was wearing on the day of the incident.

   6.1 Write down an expression for the coefficient of static friction.

   6.2 Plan an investigation that you will perform to assist the judge in his judgement. Follow the steps outlined below to ensure that your plan meets the requirements.

      i. Formulate an investigation question.
      ii. Apparatus: List *all* the other apparatus, except the tile and the shoe, that you will need.
      iii. A stepwise method: How will you perform the investigation? Include a relevant, labelled free body-diagram.
      iv. Results: What will you record?
      v. Conclusion: How will you interpret the results to draw a conclusion?

### 12.3.8 Forces in equilibrium

At the beginning of this chapter it was mentioned that resultant forces cause objects to accelerate in a straight line. If an object is stationary or moving at constant velocity then either,

- no forces are acting on the object, or
- the forces acting on that object are exactly balanced.

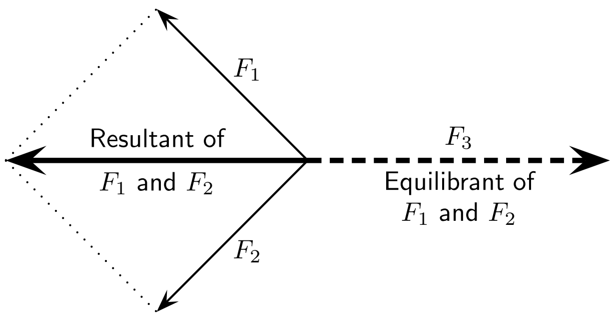
In other words, for stationary objects or objects moving with constant velocity, the resultant force acting on the object is zero. Additionally, if there is a perpendicular moment of force, then the object will rotate. You will learn more about moments of force later in this chapter.

Therefore, in order for an object not to move or to be in *equilibrium*, the sum of the forces (resultant force) must be zero and the sum of the moments of force must be zero.

**Definition: Equilibrium**
An object in equilibrium has both the sum of the forces acting on it and the sum of the moments of the forces equal to zero.

If a resultant force acts on an object then that object can be brought into equilibrium by applying an additional force that exactly balances this resultant. Such a force is called the *equilibrant* and is equal in magnitude but opposite in direction to the original resultant force acting on the object.

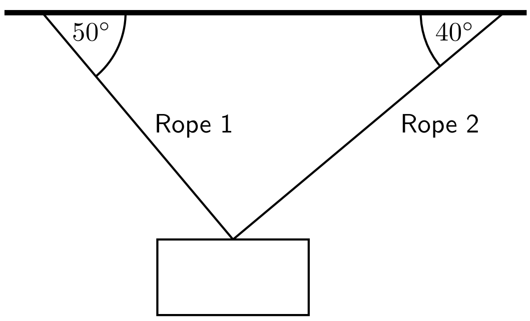
**Definition: Equilibrant**
The equilibrant of any number of forces is the single force required to produce equilibrium, and is equal in magnitude but opposite in direction to the resultant force.



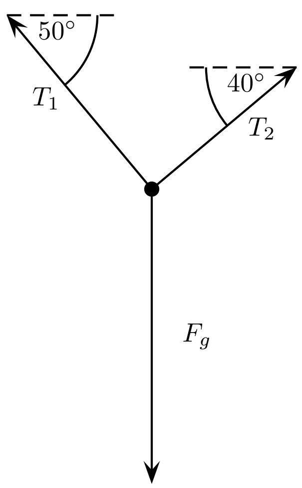

In the figure the resultant of $F_1$ and $F_2$ is shown. The equilibrant of $F_1$ and $F_2$ is then the vector opposite in direction to this resultant with the same magnitude (i.e. $F_3$).

- $F_1$, $F_2$ and $F_3$ are in equilibrium
- $F_3$ is the equilibrant of $F_1$ and $F_2$
- $F_1$ and $F_2$ are kept in equilibrium by $F_3$

As an example of an object in equilibrium, consider an object held stationary by two ropes in the arrangement below:

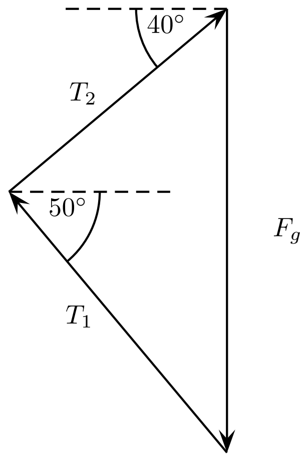


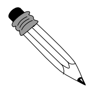

Let us draw a free body diagram for the object. In the free body diagram the object is drawn as a dot and all forces acting on the object are drawn in the correct directions starting from that dot. In this case, three forces are acting on the object.




Each rope exerts a force on the object in the direction of the rope away from the object. These tension forces are represented by $T_1$ and $T_2$. Since the object has mass, it is attracted towards the centre of the Earth. This weight is represented in the force diagram as $F_g$.

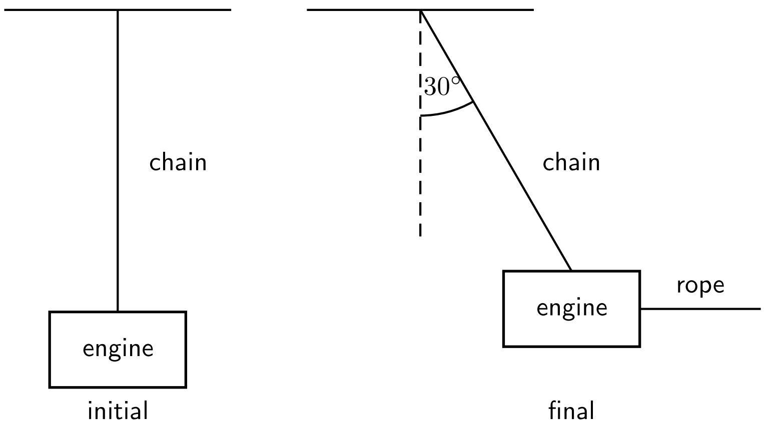
Since the object is stationary, the resultant force acting on the object is zero. In other words the three force vectors drawn tail-to-head form a closed triangle:



**Worked Example 82: Equilibrium**

**Question:** A car engine of weight 2000 N is lifted by means of a chain and pulley system. The engine is initially suspended by the chain, hanging stationary. Then, the engine is pulled sideways by a mechanic, using a rope. The engine is held in such a position that the chain makes an angle of $30°$ with the vertical. In the questions that follow, the masses of the chain and the rope can be ignored.




1. Draw a free body representing the forces acting on the engine in the initial situation.

2. Determine the tension in the chain initially.

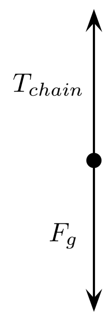
3. Draw a free body diagram representing the forces acting on the engine in the final situation.

4. Determine the magnitude of the applied force and the tension in the chain in the final situations.

**Answer**

**Step 1 : Initial free body diagram for the engine**

There are only two forces acting on the engine initially: the tension in the chain, $T_{chain}$ and the weight of the engine, $F_g$.



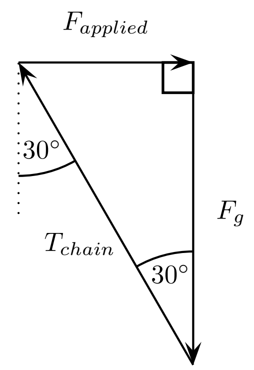

**Step 2 : Determine the tension in the chain**

The engine is initially stationary, which means that the resultant force on the engine is zero. There are also no moments of force. Thus the tension in the chain exactly balances the weight of the engine. The tension in the chain is:

$$\begin{aligned} T_{chain} &= F_g \\ &= 2000\,\mathrm{N} \end{aligned}$$

**Step 3 : Final free body diagram for the engine**

There are three forces acting on the engine finally: The tension in the chain, the applied force and the weight of the engine.




**Step 4 : Calculate the magnitude of the applied force and the tension in the chain in the final situation**

Since no method was specified let us calculate the magnitudes algebraically. Since the triangle formed by the three forces is a right-angle triangle this is easily done:

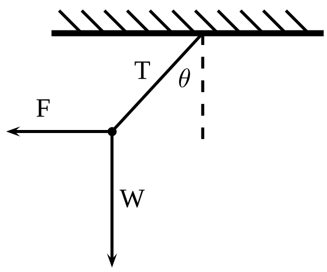
$$\begin{aligned} \frac{F_{applied}}{F_g} &= \tan 30° \\ F_{applied} &= (2000)\tan 30° \\ &= 1\,155\,\mathrm{N} \end{aligned}$$

and

$$\begin{aligned} \frac{T_{chain}}{F_g} &= \frac{1}{\cos 30°} \\ T_{chain} &= \frac{2000}{\cos 30°} \\ &= 2\,309\,\mathrm{N} \end{aligned}$$

## 12.3.9 Exercise

1. The diagram shows an object of weight W, attached to a string. A horizontal force F is applied to the object so that the string makes an angle of $\theta$ with the vertical when the object is at rest. The force exerted by the string is T. Which one of the following expressions is incorrect?




A F + T + W = 0

B W = T cos $\theta$

C tan $\theta = \frac{F}{W}$

D W = T sin $\theta$

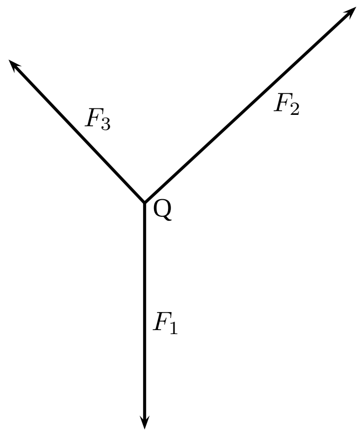
2. The point Q is in equilibrium due to three forces $F_1$, $F_2$ and $F_3$ acting on it. Which of the statements about these forces is INCORRECT?

   A The sum of the forces $F_1$, $F_2$ and $F_3$ is zero.

   B The three forces all lie in the same plane.

   C The resultant force of $F_1$ and $F_3$ is $F_2$.

   D The sum of the components of the forces in any direction is zero.




3. A point is acted on by two forces in equilibrium. The forces

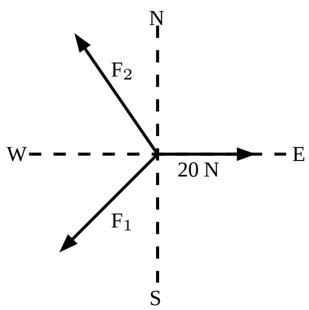
   A have equal magnitudes and directions.

   B have equal magnitudes but opposite directions.

   C act perpendicular to each other.

   D act in the same direction.

4. A point in equilibrium is acted on by three forces. Force $F_1$ has components 15 N due south and 13 N due west. What are the components of force $F_2$?



A 13 N due north and 20 due west

B 13 N due north and 13 N due west

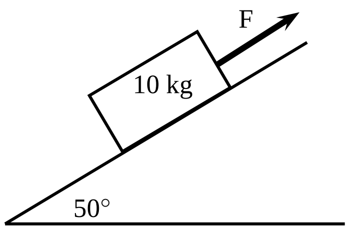
C 15 N due north and 7 N due west

D 15 N due north and 13 N due east

5. 5.1 Define the term 'equilibrant'.

   5.2 Two tugs, one with a pull of 2500 N and the other with a pull of 3 000 N are used to tow an oil drilling platform. The angle between the two cables is 30 °. Determine, either by scale diagram or by calculation (a clearly labelled rough sketch must be given), the equilibrant of the two forces.

6. A 10 kg block is held motionless by a force F on a frictionless plane, which is inclined at an angle of 50° to the horizontal, as shown below:




   6.1 Draw a force diagram (not a triangle) indicating all the forces acting on the block.

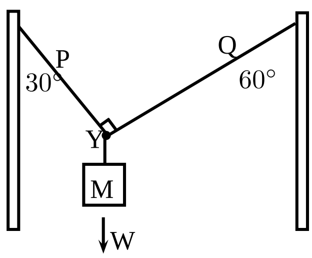
   6.2 Calculate the magnitude of force F. Include a labelled diagram showing a triangle of forces in your answer.

7. A rope of negligible mass is strung between two vertical struts. A mass M of weight W hangs from the rope through a hook fixed at point Y

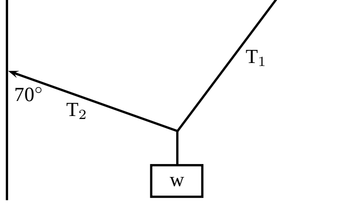
   7.1 Draw a vector diagram, plotted head to tail, of the forces acting at point X. Label each force and show the size of each angle.

   7.2 Where will the force be greatest? Part P or Q? Motivate your answer.

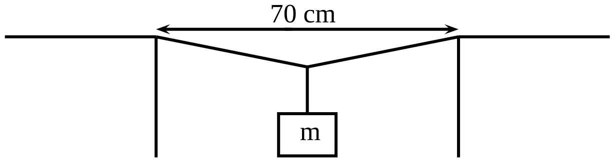
   7.3 When the force in the rope is greater than 600N it will break. What is the maximum mass that the above set up can support?




8. An object of weight $w$ is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are $T_1$ and $T_2$ respectively. Tension $T_1 = 1200$ N. Determine the tension $T_2$ and weight $w$ of the object by accurate construction and measurement or calculation.



9. A rope is tied at two points which are 70 cm apart from each other, on the same horizontal line. The total length of rope is 1 m, and the maximum tension it can withstand in any part is 1000 N. Find the largest mass (m), in kg, that can be carried at the midpoint of the rope, without breaking the rope. Include a labelled diagram showing the triangle of forces in your answer.




## 12.4 Forces between Masses

In Chapter **??**, you saw that gravitational fields exert forces on masses in the field. A field is a region of space in which an object experiences a force. The strength of a field is defined by a field strength. For example, the gravitational field strength, $g$, on or near the surface of the Earth has a value that is approximately 9,8 m·s$^{-2}$.

The force exerted by a field of strength $g$ on an object of mass $m$ is given by:

$$F = m \cdot g \tag{12.1}$$

This can be re-written in terms of $g$ as:

$$g = \frac{F}{m}$$

This means that $g$ can be understood to be a measure of force exerted per unit mass.

The force defined in Equation 12.1 is known as weight.

Objects in a gravitational field exert forces on each other without touching. The gravitational force is an example of a non-contact force.

Gravity is a force and therefore must be described by a vector - so remember magnitude and direction.

### 12.4.1 Newton's Law of Universal Gravitation

**Definition: Newton's Law of Universal Gravitation**
Every point mass attracts every other point mass by a force directed along the line connecting the two. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

The magnitude of the attractive gravitational force between the two point masses, $F$ is given by:

$$F = G\frac{m_1 m_2}{r^2} \tag{12.2}$$

where: $G$ is the gravitational constant, $m_1$ is the mass of the first point mass, $m_2$ is the mass of the second point mass and $r$ is the distance between the two point masses.

Assuming SI units, $F$ is measured in newtons (N), $m_1$ and $m_2$ in kilograms (kg), $r$ in meters (m), and the constant $G$ is approximately equal to $6{,}67 \times 10^{-11} N \cdot m^2 \cdot\ kg^{-2}$. Remember that this is a force of attraction.

For example, consider a man of mass 80 kg standing 10 m from a woman with a mass of 65 kg.

The attractive gravitational force between them would be:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= (6{,}67\times 10^{-11})(\frac{(80)(65)}{(10)^2}) \\ &= 3{,}47\times 10^{-9}\,\mathrm{N} \end{aligned}$$

If the man and woman move to 1 m apart, then the force is:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= (6{,}67\times 10^{-11})(\frac{(80)(65)}{(1)^2}) \\ &= 3{,}47\times 10^{-7}\,\mathrm{N} \end{aligned}$$

As you can see, these forces are very small.

Now consider the gravitational force between the Earth and the Moon. The mass of the Earth is $5{,}98\times 10^{24}$ kg, the mass of the Moon is $7{,}35\times 10^{22}$ kg and the Earth and Moon are $0{,}38\times 10^{9}$ m apart. The gravitational force between the Earth and Moon is:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= (6{,}67\times 10^{-11})(\frac{(5{,}98\times 10^{24})(7{,}35\times 10^{22})}{(0{,}38\times 10^{9})^2}) \\ &= 2{,}03\times 10^{20}\,\mathrm{N} \end{aligned}$$

From this example you can see that the force is very large.

These two examples demonstrate that the bigger the masses, the greater the force between them. The $1/r^2$ factor tells us that the distance between the two bodies plays a role as well. The closer two bodies are, the stronger the gravitational force between them is. We feel the gravitational attraction of the Earth most at the surface since that is the closest we can get to it, but if we were in outer-space, we would barely even know the Earth's gravity existed!

Remember that

$$F = m\cdot a \tag{12.3}$$

which means that every object on Earth feels the same gravitational acceleration! That means whether you drop a pen or a book (from the same height), they will both take the same length of time to hit the ground... in fact they will be head to head for the entire fall if you drop them at the same time. We can show this easily by using the two equations above (Equations 12.2 and 12.3). The force between the Earth (which has the mass $m_e$) and an object of mass $m_o$ is

$$F = \frac{G m_o m_e}{r^2} \tag{12.4}$$

and the acceleration of an object of mass $m_o$ (in terms of the force acting on it) is

$$a_o = \frac{F}{m_o} \tag{12.5}$$

So we substitute equation (12.4) into Equation (12.5), and we find that

$$a_o = \frac{G m_e}{r^2} \tag{12.6}$$

Since it doesn't matter what $m_o$ is, this tells us that the acceleration on a body (due to the Earth's gravity) does not depend on the mass of the body. Thus all objects experience the same gravitational acceleration. The force on different bodies will be different but the acceleration will be the same. Due to the fact that this acceleration caused by gravity is the same on all objects we label it differently, instead of using $a$ we use $g$ which we call the gravitational acceleration.

### 12.4.2 Comparative Problems

Comparative problems involve calculation of something in terms of something else that we know. For example, if you weigh 490 N on Earth and the gravitational acceleration on Venus is 0,903 that of the gravitational acceleration on the Earth, then you would weigh 0,903 x 490 N = 442,5 N on Venus.

#### Principles for answering comparative problems

- Write out equations and calculate all quantities for the given situation
- Write out all relationships between variable from first and second case
- Write out second case
- Substitute all first case variables into second case
- Write second case in terms of first case

**Worked Example 83: Comparative Problem 1**

**Question:** On Earth a man has a mass of 70 kg. The planet Zirgon is the same size as the Earth but has twice the mass of the Earth. What would the man weigh on Zirgon, if the gravitational acceleration on Earth is 9,8 m·s$^{-2}$?

**Answer**

**Step 1 : Determine what information has been given**

The following has been provided:

- the mass of the man on Earth, $m$
- the mass of the planet Zirgon ($m_Z$) in terms of the mass of the Earth ($m_E$), $m_Z = 2m_E$
- the radius of the planet Zirgon ($r_Z$) in terms of the radius of the Earth ($r_E$), $r_Z = r_E$

**Step 2 : Determine how to approach the problem**

We are required to determine the man's weight on Zirgon ($w_Z$). We can do this by using:

$$w = mg = G\frac{m_1 \cdot m_2}{r^2}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Zirgon.

**Step 3 : Situation on Earth**

$$\begin{aligned} w_E &= mg_E = G\frac{m_E \cdot m}{r_E^2} \\ &= (70\,\text{kg})(9{,}8\,\text{m}\cdot\text{s}^{-2}) \\ &= 686\,\text{N} \end{aligned}$$

**Step 4 : Situation on Zirgon in terms of situation on Earth**

Write the equation for the gravitational force on Zirgon and then substitute the values for $m_Z$ and $r_Z$, in terms of the values for the Earth.

$$\begin{aligned} w_Z = mg_Z &= G\frac{m_Z \cdot m}{r_Z^2} \\ &= G\frac{2m_E \cdot m}{r_E^2} \\ &= 2(G\frac{m_E \cdot m}{r_E^2}) \\ &= 2w_E \\ &= 2(686\,\mathrm{N}) \\ &= 1\,372\,\mathrm{N} \end{aligned}$$

**Step 5 : Quote the final answer**

The man weighs 1 372 N on Zirgon.

**Worked Example 84: Comparative Problem 2**

**Question:** On Earth a man weighs 70 kg. On the planet Beeble how much will he weigh if Beeble has mass half of that of the Earth and a radius one quarter that of the Earth. Gravitational acceleration on Earth is 9,8 m·s$^{-2}$.

**Answer**

**Step 1 : Determine what information has been given**

The following has been provided:

- the mass of the man on Earth, $m$
- the mass of the planet Beeble ($m_B$) in terms of the mass of the Earth ($m_E$), $m_B = \frac{1}{2}m_E$
- the radius of the planet Beeble ($r_B$) in terms of the radius of the Earth ($r_E$), $r_B = \frac{1}{4}r_E$

**Step 2 : Determine how to approach the problem**

We are required to determine the man's weight on Beeble ($w_B$). We can do this by using:

$$w = mg = G\frac{m_1 \cdot m_2}{r^2} \tag{12.7}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Beeble.

**Step 3 : Situation on Earth**

$$\begin{aligned} w_E &= mg_E = G\frac{m_E \cdot m}{r_E^2} \\ &= (70\,\mathrm{kg})(9{,}8\,\mathrm{m \cdot s^{-2}}) \\ &= 686\,\mathrm{N} \end{aligned}$$

**Step 4 : Situation on Beeble in terms of situation on Earth**

Write the equation for the gravitational force on Beeble and then substitute the values for $m_B$ and $r_B$, in terms of the values for the Earth.

$$\begin{aligned} w_B = mg_B &= G\frac{m_B \cdot m}{r_B^2} \\ &= G\frac{\frac{1}{2}m_E \cdot m}{(\frac{1}{4}r_E)^2} \\ &= 8(G\frac{m_E \cdot m}{r_E^2}) \\ &= 8w_E \\ &= 8(686\,\mathrm{N}) \\ &= 5\,488\,\mathrm{N} \end{aligned}$$

**Step 5 : Quote the final answer**
The man weighs 5 488 N on Beeble.

### 12.4.3 Exercise

1. Two objects of mass 2m and 3m respectively exert a force F on each other when they are a certain distance apart. What will be the force between two objects situated the same distance apart but having a mass of 5m and 6m respectively?

   A 0,2 F

   B 1,2 F

   C 2,2 F

   D 5 F

2. As the distance of an object above the surface of the Earth is greatly increased, the weight of the object would

   A increase

   B decrease

   C increase and then suddenly decrease

   D remain the same

3. A satellite circles around the Earth at a height where the gravitational force is a factor 4 less than at the surface of the Earth. If the Earth's radius is R, then the height of the satellite above the surface is:

   A R

   B 2 R

   C 4 R

   D 16 R

4. A satellite experiences a force F when at the surface of the Earth. What will be the force on the satellite if it orbits at a height equal to the diameter of the Earth:

   A $\frac{1}{F}$

   B $\frac{1}{2}\ F$

   C $\frac{1}{3}\ F$

   D $\frac{1}{9}\ F$

5. The weight of a rock lying on surface of the Moon is W. The radius of the Moon is R. On planet Alpha, the same rock has weight 8W. If the radius of planet Alpha is half that of the Moon, and the mass of the Moon is M, then the mass, in kg, of planet Alpha is:

   A $\frac{M}{2}$

B $\frac{M}{4}$

C 2 M

D 4 M

6. Consider the symbols of the two physical quantities $g$ and $G$ used in Physics.
   6.1 Name the physical quantities represented by $g$ and $G$.
   6.2 Derive a formula for calculating $g$ near the Earth's surface using Newton's Law of Universal Gravitation. M and R represent the mass and radius of the Earth respectively.
7. Two spheres of mass 800g and 500g respectively are situated so that their centers are 200 cm apart. Calculate the gravitational force between them.
8. Two spheres of mass 2 kg and 3 kg respectively are situated so that the gravitational force between them is 2,5 x $10^{-8}$ N. Calculate the distance between them.
9. Two identical spheres are placed 10 cm apart. A force of 1,6675 x $10^{-9}$ N exists between them. Find the masses of the spheres.
10. Halley's comet, of approximate mass 1 x $10^{15}$ kg was 1,3 x $10^{8}$ km from the Earth, at its point of closest approach during its last sighting in 1986.
    10.1 Name the force through which the Earth and the comet interact.
    10.2 Is the magnitude of the force experienced by the comet the same, greater than or less than the force experienced by the Earth? Explain.
    10.3 Does the acceleration of the comet increase, decrease or remain the same as it moves closer to the Earth? Explain.
    10.4 If the mass of the Earth is 6 x $10^{24}$ kg, calculate the magnitude of the force exerted by the Earth on Halley's comet at its point of closest approach.

## 12.5 Momentum and Impulse

Momentum is a physical quantity which is closely related to forces. Momentum is a property which applies to moving objects.

**Definition: Momentum**
Momentum is the tendency of an object to continue to move in its direction of travel. Momentum is calculated from the product of the mass and velocity of an object.

The momentum (symbol $p$) of an object of mass $m$ moving at velocity $v$ is:

$$p = m \cdot v$$

According to this equation, momentum is related to both the mass and velocity of an object. A small car travelling at the same velocity as a big truck will have a smaller momentum than the truck. The smaller the mass, the smaller the velocity.
A car travelling at 120 km·hr$^{-1}$will have a bigger momentum than the same car travelling at 60 km·hr$^{-1}$. Momentum is also related to velocity; the smaller the velocity, the smaller the momentum.
Different objects can also have the same momentum, for example a car travelling slowly can have the same momentum as a motor cycle travelling relatively fast. We can easily demonstrate this. Consider a car of mass 1 000 kg with a velocity of 8 m·s$^{-1}$(about 30 km·hr$^{-1}$). The momentum of the car is therefore

$$\begin{aligned} p &= m \cdot v \\ &= (1000)(8) \\ &= 8000\,\mathrm{kg \cdot m \cdot s^{-1}} \end{aligned}$$

Now consider a motor cycle of mass 250 kg travelling at 32 m·s$^{-1}$ (about 115 km·hr$^{-1}$). The momentum of the motor cycle is:

$$\begin{aligned} p &= m \cdot v \\ &= (250)(32) \\ &= 8000\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \end{aligned}$$

Even though the motor cycle is considerably lighter than the car, the fact that the motor cycle is travelling much faster than the car means that the momentum of both vehicles is the same.

From the calculations above, you are able to derive the unit for momentum as kg·m·s$^{-1}$. Momentum is also vector quantity, because it is the product of a scalar ($m$) with a vector ($v$). This means that whenever we calculate the momentum of an object, we need to include the direction of the momentum.

**Worked Example 85: Momentum of a Soccer Ball**

**Question:** A soccer ball of mass 420 g is kicked at 20 m·s$^{-1}$ towards the goal post. Calculate the momentum of the ball.

**Answer**

**Step 1 : Identify what information is given and what is asked for**

The question explicitly gives

- the mass of the ball, and
- the velocity of the ball

The mass of the ball must be converted to SI units.

$$420\text{ g} = 0{,}42\,\text{kg}$$

We are asked to calculate the momentum of the ball. From the definition of momentum,

$$p = m \cdot v$$

we see that we need the mass and velocity of the ball, which we are given.

**Step 2 : Do the calculation**

We calculate the magnitude of the momentum of the ball,

$$\begin{aligned} p &= m \cdot v \\ &= (0{,}42)(20) \\ &= 8{,}4\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \end{aligned}$$

**Step 3 : Quote the final answer**

We quote the answer with the direction of motion included, $p =$ 8,4 kg·m·s$^{-1}$ in the direction of the goal post.

**Worked Example 86: Momentum of a cricket ball**

**Question:** A cricket ball of mass 160 g is bowled at 40 m·s$^{-1}$ towards a batsman. Calculate the momentum of the cricket ball.

**Answer**

**Step 1 : Identify what information is given and what is asked for**

The question explicitly gives

- the mass of the ball ($m =$ 160 g = 0,16 kg), and

- the velocity of the ball ($v$ = 40 m·s$^{-1}$)

To calculate the momentum we will use

$$p = m \cdot v$$

.

**Step 2 : Do the calculation**

$$\begin{aligned} p &= m \cdot v \\ &= (0{,}16)(40) \\ &= 6{,}4\,\mathrm{kg \cdot m \cdot s^{-1}} \\ &= 6{,}4\,\mathrm{kg \cdot m \cdot s^{-1}} \text{in the direction of the batsman} \end{aligned}$$

**Worked Example 87: Momentum of the Moon**

**Question:** The Moon is 384 400 km away from the Earth and orbits the Earth in 27,3 days. If the Moon has a mass of 7,35 x 10$^{22}$kg, what is the magnitude of its momentum if we assume a circular orbit?

**Answer**

**Step 1 : Identify what information is given and what is asked for**

The question explicitly gives

- the mass of the Moon (m = 7,35 x 10$^{22}$ kg)
- the distance to the Moon (384 400 km = 384 400 000 m = 3,844 x 10$^{8}$ m)
- the time for one orbit of the Moon (27,3 days = 27,3 x 24 x 60 x 60 = 2,36 x 10$^{6}$ s)

We are asked to calculate only the magnitude of the momentum of the Moon (i.e. we do not need to specify a direction). In order to do this we require the mass and the magnitude of the velocity of the Moon, since

$$p = m \cdot v$$

**Step 2 : Find the magnitude of the velocity of the Moon**

The magnitude of the average velocity is the same as the speed. Therefore:

$$s = \frac{d}{\Delta t}$$

We are given the time the Moon takes for one orbit but not how far it travels in that time. However, we can work this out from the distance to the Moon and the fact that the Moon has a circular orbit. Using the equation for the circumference, $C$, of a circle in terms of its radius, we can determine the distance travelled by the Moon in one orbit:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi(3{,}844 \times 10^{8}) \\ &= 2{,}42 \times 10^{9}\ \mathrm{m} \end{aligned}$$

Combining the distance travelled by the Moon in an orbit and the time taken by the Moon to complete one orbit, we can determine the magnitude of the Moon's velocity or speed,

$$\begin{aligned} s &= \frac{d}{\Delta t} \\ &= \frac{C}{T} \\ &= \frac{2{,}42 \times 10^9}{2{,}36 \times 10^6} \\ &= 1{,}02 \times 10^3\,\mathrm{m \cdot s^{-1}}. \end{aligned}$$

**Step 3 : Finally calculate the momentum and quote the answer**
The magnitude of the Moon's momentum is:

$$\begin{aligned} p &= m \cdot v \\ &= (7{,}35 \times 10^{22})(1{,}02 \times 10^3) \\ &= 7{,}50 \times 10^{25}\,\mathrm{kg \cdot m \cdot s^{-1}} \end{aligned}$$

## 12.5.1 Vector Nature of Momentum

As we have said, momentum is a vector quantity. Since momentum is a vector, the techniques of vector addition discussed in Chapter **??** must be used to calculate the total momentum of a system.

**Worked Example 88: Calculating the Total Momentum of a System**

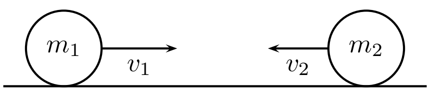
**Question:** Two billiard balls roll towards each other. They each have a mass of 0,3 kg. Ball 1 is moving at $v_1 = 1\,\mathrm{m \cdot s^{-1}}$ to the right, while ball 2 is moving at $v_2 = 0{,}8\,\mathrm{m \cdot s^{-1}}$ to the left. Calculate the total momentum of the system.
**Answer**
**Step 1 : Identify what information is given and what is asked for**
The question explicitly gives

- the mass of each ball,
- the velocity of ball 1, $v_1$, and
- the velocity of ball 2, $v_2$,

all in the correct units!
We are asked to calculate the **total momentum of the system**. In this example our system consists of two balls. To find the total momentum we must determine the momentum of each ball and add them.

$$p_{total} = p_1 + p_2$$

Since ball 1 is moving to the right, its momentum is in this direction, while the second ball's momentum is directed towards the left.




Thus, we are required to find the sum of two vectors acting along the same straight line. The algebraic method of vector addition introduced in Chapter **??** can thus be used.

**Step 2 : Choose a frame of reference**
Let us choose right as the positive direction, then obviously left is negative.

**Step 3 : Calculate the momentum**
The total momentum of the system is then the sum of the two momenta taking the directions of the velocities into account. Ball 1 is travelling at 1 m·s$^{-1}$to the right or $+1$ m·s$^{-1}$. Ball 2 is travelling at 0,8 m·s$^{-1}$to the left or -0,8 m·s$^{-1}$. Thus,

$$\begin{aligned} p_{total} &= m_1v_1 + m_2v_2 \\ &= (0{,}3)(+1) + (0{,}3)(-0{,}8) \\ &= (+0{,}3) + (-0{,}24) \\ &= +0{,}06\,\mathrm{kg \cdot m \cdot s^{-1}} \\ &= 0{,}06\,\mathrm{kg \cdot m \cdot s^{-1}}\text{to the right} \end{aligned}$$

In the last step the direction was added in words. Since the result in the second last line is positive, the total momentum of the system is in the positive direction (i.e. to the right).

### 12.5.2 Exercise

1. 1.1 The fastest recorded delivery for a cricket ball is 161,3 km·hr$^{-1}$, bowled by Shoaib Akhtar of Pakistan during a match against England in the 2003 Cricket World Cup, held in South Africa. Calculate the ball's momentum if it has a mass of 160 g.

   1.2 The fastest tennis service by a man is 246,2 km·hr$^{-1}$by Andy Roddick of the United States of America during a match in London in 2004. Calculate the ball's momentum if it has a mass of 58 g.

   1.3 The fastest server in the women's game is Venus Williams of the United States of America, who recorded a serve of 205 km·hr$^{-1}$during a match in Switzerland in 1998. Calculate the ball's momentum if it has a mass of 58 g.

   1.4 If you had a choice of facing Shoaib, Andy or Venus and didn't want to get hurt, who would you choose based on the momentum of each ball.

2. Two golf balls roll towards each other. They each have a mass of 100 g. Ball 1 is moving at $v_1 = 2{,}4$ m·s$^{-1}$to the right, while ball 2 is moving at $v_2 = 3$ m·s$^{-1}$to the left. Calculate the total momentum of the system.

3. Two motor cycles are involved in a head on collision. Motorcycle A has a mass of 200 kg and was travelling at 120 km·hr$^{-1}$south. Motor cycle B has a mass of 250 kg and was travelling north at 100 km·hr$^{-1}$. A and B is about to collide. Calculate the momentum of the system before the collision takes place.

### 12.5.3 Change in Momentum

Let us consider a tennis ball (mass = 0,1 g) that is dropped at an initial velocity of 5 m·s$^{-1}$and bounces back at a final velocity of 3 m·s$^{-1}$. As the ball approaches the floor it has a momentum that we call the momentum before the collision. When it moves away from the floor it has a different momentum called the momentum after the collision. The bounce on the floor can be thought of as a collision taking place where the floor exerts a force on the tennis ball to change its momentum.

The momentum before the bounce can be calculated as follows:
Because momentum and velocity are vectors, we have to choose a direction as positive. For this example we choose the initial direction of motion as positive, in other words, downwards is

positive.

$$\begin{aligned} p_{before} &= m \cdot v_i \\ &= (0{,}1)(+5) \\ &= 0{,}5\ \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \text{downwards} \end{aligned}$$

When the tennis ball bounces back it changes direction. The final velocity will thus have a negative value. The momentum after the bounce can be calculated as follows:

$$\begin{aligned} p_{after} &= m \cdot v_f \\ &= (0{,}1)(-3) \\ &= -0{,}3\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \\ &= 0{,}3\ \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \text{upwards} \end{aligned}$$

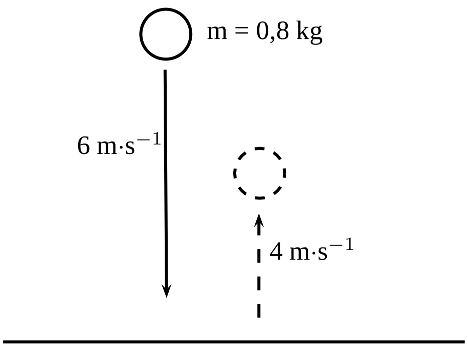
Now let us look at what happens to the momentum of the tennis ball. The momentum changes during this bounce. We can calculate the change in momentum as follows:
Again we have to choose a direction as positive and we will stick to our initial choice as downwards is positive. This means that the final momentum will have a negative number.

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= m \cdot v_f - m \cdot v_i \\ &= (-0{,}3) - (0{,}5) \\ &= -0{,}8\,\text{kg} \cdot \text{m} \cdot \text{s}^{-1} \\ &= 0{,}8\ \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \text{upwards} \end{aligned}$$

You will notice that this number is bigger than the previous momenta calculated. This is should be the case as the ball needed to be stopped and then given momentum to bounce back.

**Worked Example 89: Change in Momentum**

**Question:** A rubber ball of mass 0,8 kg is dropped and strikes the floor with an initial velocity of 6 m·s$^{-1}$. It bounces back with a final velocity of 4 m·s$^{-1}$. Calculate the change in the momentum of the rubber ball caused by the floor.




**Answer**

**Step 1 : Identify the information given and what is asked**

The question explicitly gives

- the ball's mass (m = 0,8 kg),
- the ball's initial velocity ($v_i$ = 6 m·s$^{-1}$), and

- the ball's final velocity ($v_f$ = 4 m·s$^{-1}$)

all in the correct units.

We are asked to calculate the change in momentum of the ball,

$$\Delta p = mv_f - mv_i$$

We have everything we need to find $\Delta p$. Since the initial momentum is directed downwards and the final momentum is in the upward direction, we can use the algebraic method of subtraction discussed in the vectors chapter.

**Step 2 : Choose a frame of reference**
Let us choose down as the positive direction.

**Step 3 : Do the calculation and quote the answer**

$$\begin{aligned}\Delta p &= mv_f - mv_i \\ &= (0{,}8)(-4) - (0{,}8)(+6) \\ &= (-3{,}2) - (4{,}8) \\ &= -8 \\ &= 8\ \mathrm{kg\cdot m\cdot s^{-1}}\ \text{upwards}\end{aligned}$$

### 12.5.4 Exercise

1. Which expression accurately describes the change of momentum of an object?

   A $\frac{F}{m}$
   B $\frac{F}{t}$
   C $F \cdot m$
   D $F \cdot t$

2. A child drops a ball of mass 100 g. The ball strikes the ground with a velocity of 5 m·s$^{-1}$and rebounds with a velocity of 4 m·s$^{-1}$. Calculate the change of momentum of the ball.

3. A 700 kg truck is travelling north at a velocity of 40 km·hr$^{-1}$when it is approached by a 500 kg car travelling south at a velocity of 100 km·hr$^{-1}$. Calculate the total momentum of the system.

### 12.5.5 Newton's Second Law revisited

You have learned about Newton's Second Law of motion earlier in this chapter. Newton's Second Law describes the relationship between the motion of an object and the net force on the object. We said that the motion of an object, and therefore its momentum, can only change when a resultant force is acting on it. We can therefore say that because a net force causes an object to move, it also causes its momentum to change. We can now define Newton's Second Law of motion in terms of momentum.

**Definition: Newton's Second Law of Motion (N2)**
The net or resultant force acting on an object is equal to the rate of change of momentum.

Mathematically, Newton's Second Law can be stated as:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

### 12.5.6 Impulse

Impulse is the product of the net force and the time interval for which the force acts. Impulse is defined as:

$$\text{Impulse} = F \cdot \Delta t \qquad (12.8)$$

However, from Newton's Second Law, we know that

$$\begin{aligned} F &= \frac{\Delta p}{\Delta t} \\ \therefore \quad F \cdot \Delta t &= \Delta p \\ &= \text{Impulse} \end{aligned}$$

Therefore,

$$\text{Impulse} = \Delta p$$

Impulse is equal to the change in momentum of an object. From this equation we see, that for a given change in momentum, $F_{net}\Delta t$ is fixed. Thus, if $F_{net}$ is reduced, $\Delta t$ must be increased (i.e. a smaller resultant force must be applied for longer to bring about the same change in momentum). Alternatively if $\Delta t$ is reduced (i.e. the resultant force is applied for a shorter period) then the resultant force must be increased to bring about the same change in momentum.

**Worked Example 90: Impulse and Change in momentum**

**Question:** A 150 N resultant force acts on a 300 kg trailer. Calculate how long it takes this force to change the trailer's velocity from 2 m·s$^{-1}$to 6 m·s$^{-1}$in the same direction. Assume that the forces acts to the right.

**Answer**

**Step 1 : Identify what information is given and what is asked for**

The question explicitly gives

- the trailer's mass as 300 kg,
- the trailer's initial velocity as 2 m·s$^{-1}$to the right,
- the trailer's final velocity as 6 m·s$^{-1}$to the right, and
- the resultant force acting on the object

all in the correct units!

We are asked to calculate the time taken $\Delta t$ to accelerate the trailer from the 2 to 6 m·s$^{-1}$. From the Law of Momentum,

$$\begin{aligned} F_{net}\Delta t &= \Delta p \\ &= mv_f - mv_i \\ &= m(v_f - v_i). \end{aligned}$$

Thus we have everything we need to find $\Delta t$!

**Step 2 : Choose a frame of reference**

Choose right as the positive direction.

**Step 3 : Do the calculation and quote the final answer**

$$\begin{aligned} F_{net}\Delta t &= m(v_f - v_i) \\ (+150)\Delta t &= (300)((+6) - (+2)) \\ (+150)\Delta t &= (300)(+4) \\ \Delta t &= \frac{(300)(+4)}{+150} \\ \Delta t &= 8\,\text{s} \end{aligned}$$

It takes 8 s for the force to change the object's velocity from 2 m·s$^{-1}$to the right to 6 m·s$^{-1}$to the right.

**Worked Example 91: Impulsive cricketers!**

**Question:** A cricket ball weighing 156 g is moving at 54 km·hr$^{-1}$towards a batsman. It is hit by the batsman back towards the bowler at 36 km·hr$^{-1}$. Calculate

1. the ball's impulse, and
2. the average force exerted by the bat if the ball is in contact with the bat for 0,13 s.

**Answer**

**Step 1 : Identify what information is given and what is asked for**

The question explicitly gives

- the ball's mass,
- the ball's initial velocity,
- the ball's final velocity, and
- the time of contact between bat and ball

We are asked to calculate the impulse

$$\text{Impulse} = \Delta p = F_{net}\Delta t$$

Since we do not have the force exerted by the bat on the ball ($F_{net}$), we have to calculate the impulse from the change in momentum of the ball. Now, since

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= mv_f - mv_i, \end{aligned}$$

we need the ball's mass, initial velocity and final velocity, which we are given.

**Step 2 : Convert to S.I. units**

Firstly let us change units for the mass

$$\begin{aligned} 1000 \text{ g} &= 1\,\text{kg} \\ \text{So, } 1 \text{ g} &= \frac{1}{1000}\,\text{kg} \\ \therefore 156 \times 1 \text{ g} &= 156 \times \frac{1}{1000}\,\text{kg} \\ &= 0{,}156\ \text{kg} \end{aligned}$$

Next we change units for the velocity

$$\begin{aligned} 1 \text{ km}\cdot\text{h}^{-1} &= \frac{1000\,\text{m}}{3\ 600\,\text{s}} \\ \therefore 54 \times 1 \text{ km}\cdot\text{h}^{-1} &= 54 \times \frac{1\,000\,\text{m}}{3\,600\,\text{s}} \\ &= 15\,\text{m}\cdot\text{s}^{-1} \end{aligned}$$

Similarly, 36 km·hr$^{-1}$= 10 m·s$^{-1}$.

**Step 3 : Choose a frame of reference**

Let us choose the direction from the batsman to the bowler as the positive direction. Then the initial velocity of the ball is $v_i$ = -15 m·s$^{-1}$, while the final velocity of the ball is $v_f$ = 10 m·s$^{-1}$.

**Step 4 : Calculate the momentum**

Now we calculate the change in momentum,

$$\begin{aligned} p &= p_f - p_i \\ &= mv_f - mv_i \\ &= m(v_f - v_i) \\ &= (0{,}156)((+10) - (-15)) \\ &= +3{,}9 \\ &= 3{,}9\,\mathrm{kg \cdot m \cdot s^{-1}} \text{in the direction from batsman to bowler} \end{aligned}$$

**Step 5 : Determine the impulse**
Finally since impulse is just the change in momentum of the ball,

$$\begin{aligned} \text{Impulse} &= \Delta p \\ &= 3{,}9\,\mathrm{kg \cdot m \cdot s^{-1}} \text{in the direction from batsman to bowler} \end{aligned}$$

**Step 6 : Determine the average force exerted by the bat**

$$\text{Impulse} = F_{net}\Delta t = \Delta p$$

We are given $\Delta t$ and we have calculated the impulse of the ball.

$$\begin{aligned} F_{net}\Delta t &= \text{Impulse} \\ F_{net}(0{,}13) &= +3{,}9 \\ F_{net} &= \frac{+3{,}9}{0{,}13} \\ &= +30 \\ &= 30\,\mathrm{N} \quad \text{in the direction from batsman to bowler} \end{aligned}$$

### 12.5.7 Exercise

1. Which one of the following is NOT a unit of impulse?

   A $N \cdot s$

   B $kg \cdot \mathrm{m \cdot s^{-1}}$

   C $J \cdot \mathrm{m \cdot s^{-1}}$

   D $J \cdot m^{-1} \cdot s$

2. A toy car of mass 1 kg moves eastwards with a speed of 2 m·s$^{-1}$. It collides head-on with a toy train. The train has a mass of 2 kg and is moving at a speed of 1,5 m·s$^{-1}$westwards. The car rebounds (bounces back) at 3,4 m·s$^{-1}$and the train rebounds at 1,2 m·s$^{-1}$.

   2.1 Calculate the change in momentum for each toy.

   2.2 Determine the impulse for each toy.

   2.3 Determine the duration of the collision if the magnitude of the force exerted by each toy is 8 N.

3. A bullet of mass 20 g strikes a target at 300 m·s$^{-1}$and exits at 200 m·s$^{-1}$. The tip of the bullet takes 0,0001s to pass through the target. Determine:

   3.1 the change of momentum of the bullet.

   3.2 the impulse of the bullet.

   3.3 the magnitude of the force experienced by the bullet.

4. A bullet of mass 20 g strikes a target at 300 m·s$^{-1}$. Determine under which circumstances the bullet experiences the greatest change in momentum, and hence impulse:

   4.1 When the bullet exits the target at 200 m·s$^{-1}$.

4.2 When the bullet stops in the target.

4.3 When the bullet rebounds at 200 m·s$^{-1}$.

5. A ball with a mass of 200 g strikes a wall at right angles at a velocity of 12 m·s$^{-1}$and rebounds at a velocity of 9 m·s$^{-1}$.

   5.1 Calculate the change in the momentum of the ball.

   5.2 What is the impulse of the wall on the ball?

   5.3 Calculate the magnitude of the force exerted by the wall on the ball if the collision takes 0,02s.

6. If the ball in the previous problem is replaced with a piece of clay of 200 g which is thrown against the wall with the same velocity, but then sticks to the wall, calculate:

   6.1 The impulse of the clay on the wall.

   6.2 The force exerted by the clay on the wall if it is in contact with the wall for 0,5 s before it comes to rest.

### 12.5.8 Conservation of Momentum

In the absence of an external force acting on a system, momentum is conserved.

**Definition: Conservation of Linear Momentum**
The total linear momentum of an isolated system is constant. An isolated system has no forces acting on it from the outside.

This means that in an isolated system the total momentum before a collision or explosion is equal to the total momentum after the collision or explosion.

Consider a simple collision of two billiard balls. The balls are rolling on a frictionless surface and the system is isolated. So, we can apply conservation of momentum. The first ball has a mass $m_1$ and an initial velocity $v_{i1}$. The second ball has a mass $m_2$ and moves towards the first ball with an initial velocity $v_{i2}$. This situation is shown in Figure 12.14.


Figure 12.14: Before the collision.

The total momentum of the system before the collision, $p_i$ is:

$$p_i = m_1 v_{i1} + m_2 v_{i2}$$

After the two balls collide and move away they each have a different momentum. If the first ball has a final velocity of $v_{f1}$ and the second ball has a final velocity of $v_{f2}$ then we have the situation shown in Figure 12.15.


Figure 12.15: After the collision.

The total momentum of the system after the collision, $p_f$ is:

$$p_f = m_1 v_{f1} + m_2 v_{f2}$$

This system of two balls is isolated since there are no external forces acting on the balls. Therefore, by the principle of conservation of linear momentum, the total momentum before the collision is equal to the total momentum after the collision. This gives the equation for the conservation of momentum in a collision of two objects,

$$p_i = p_f$$
$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

| | |
|---|---|
| $m_1$ | : mass of object 1 (kg) |
| $m_2$ | : mass of object 2 (kg) |
| $v_{i1}$ | : initial velocity of object 1 (m·s$^{-1}$+ direction) |
| $v_{i2}$ | : initial velocity of object 2 (m·s$^{-1}$+ direction) |
| $v_{f1}$ | : final velocity of object 1 (m·s$^{-1}$+ direction) |
| $v_{f2}$ | : final velocity of object 2 (m·s$^{-1}$+ direction) |

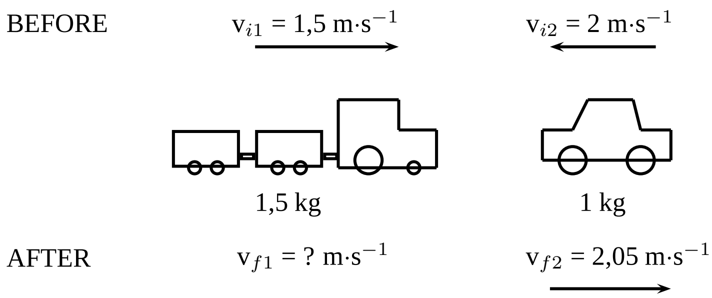
This equation is always true - momentum is always conserved in collisions.

**Worked Example 92: Conservation of Momentum 1**

**Question:** A toy car of mass 1 kg moves westwards with a speed of 2 m·s$^{-1}$. It collides head-on with a toy train. The train has a mass of 1,5 kg and is moving at a speed of 1,5 m·s$^{-1}$eastwards. If the car rebounds at 2,05 m·s$^{-1}$, calculate the velocity of the train.

**Answer**

**Step 1 : Draw rough sketch of the situation**




**Step 2 : Choose a frame of reference**

We will choose to the east as positive.

**Step 3 : Apply the Law of Conservation of momentum**

$$\begin{aligned}
p_i &= p_f \\
m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\
(1{,}5)(+1{,}5) + (2)(-2) &= (1{,}5)(v_{f1}) + (2)(2{,}05) \\
2{,}25 - 4 - 4{,}1 &= 1{,}5\ v_{f1} \\
5{,}85 &= 1{,}5\ v_{f1} \\
v_{f1} &= 3{,}9\,\mathrm{m\cdot s^{-1}}\text{eastwards}
\end{aligned}$$

**Worked Example 93: Conservation of Momentum 2**

**Question:** A helicopter flies at a speed of 275 m·s$^{-1}$. The pilot fires a missile forward out of a gun barrel at a speed of 700 m·s$^{-1}$. The respective masses of the helicopter and the missile are 5000 kg and 50 kg. Calculate the new speed of the helicopter immmediately after the missile had been fired.

**Answer**

**Step 1 : Draw rough sketch of the situation**

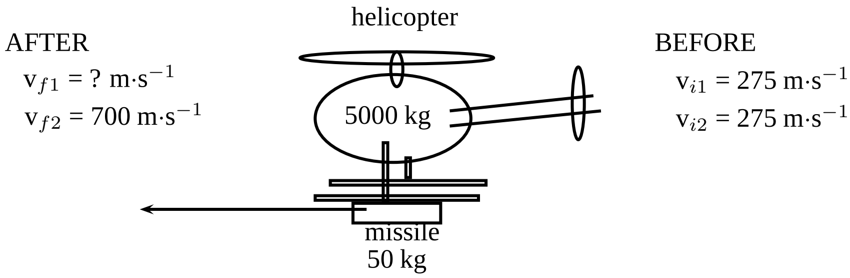



Figure 12.16: helicopter and missile

**Step 2 : Analyse the question and list what is given**

$m_1 = 5000$ kg
$m_2 = 50$ kg
$v_{i1} = v_{i2} = 275$ m·s$^{-1}$
$v_{f1} = ?$
$v_{f2} = 700$ m·s$^{-1}$

**Step 3 : Apply the Law of Conservation of momentum**

The helicopter and missile are connected initially and move at the same velocity. We will therefore combine their masses and change the momentum equation as follows:

$$\begin{aligned} p_i &= p_f \\ (m_1 + m_2)v_i &= m_1 v_{f1} + m_2 v_{f2} \\ (5000 + 50)(275) &= (5000)(v_{f1}) + (50)(700) \\ 1388750 - 35000 &= (5000)(v_{f1}) \\ v_{f1} &= 270{,}75\,\mathrm{m\cdot s^{-1}} \end{aligned}$$

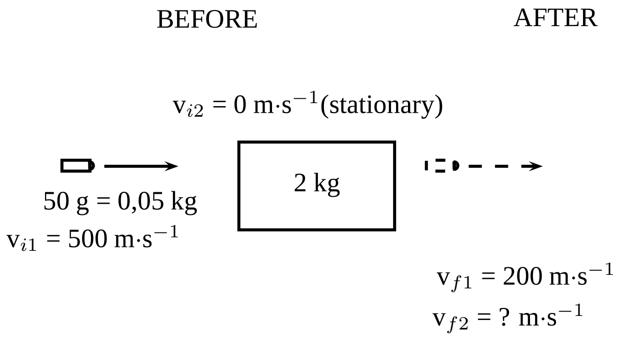
Note that speed is asked and not velocity, therefore no direction is included in the answer.

**Worked Example 94: Conservation of Momentum 3**

**Question:** A bullet of mass 50 g travelling horizontally at 500 m·s$^{-1}$strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet goes through the block and comes out on the other side at 200 m·s$^{-1}$. Calculate the speed of the block after the bullet has come out the other side.

**Answer**

**Step 1 : Draw rough sketch of the situation**




**Step 2 : Choose a frame of reference**
We will choose to the right as positive.

**Step 3 : Apply the Law of Conservation of momentum**

$$\begin{aligned}
p_i &= p_f \\
m_1v_{i1} + m_2v_{i2} &= m_1v_{f1} + m_2v_{f2} \\
(0{,}05)(+500) + (2)(0) &= (0{,}05)(+200) + (2)(v_{f2}) \\
25 + 0 - 10 &= 2\ v_{f2} \\
v_{f2} &= 7{,}5\,\mathrm{m\cdot s^{-1}} \text{in the same direction as the bullet}
\end{aligned}$$

### 12.5.9 Physics in Action: Impulse

A very important application of impulse is improving safety and reducing injuries. In many cases, an object needs to be brought to rest from a certain initial velocity. This means there is a certain specified change in momentum. If the time during which the momentum changes can be increased then the force that must be applied will be less and so it will cause less damage. This is the principle behind arrestor beds for trucks, airbags, and bending your knees when you jump off a chair and land on the ground.

#### Air-Bags in Motor Vehicles

Air bags are used in motor vehicles because they are able to reduce the effect of the force experienced by a person during an accident. Air bags extend the time required to stop the momentum of the driver and passenger. During a collision, the motion of the driver and passenger carries them towards the windshield which results in a large force exerted over a short time in order to stop their momentum. If instead of hitting the windshield, the driver and passenger hit an air bag, then the time of the impact is increased. Increasing the time of the impact results in a decrease in the force.

#### Padding as Protection During Sports

The same principle explains why wicket keepers in cricket use padded gloves or why there are padded mats in gymnastics. In cricket, when the wicket keeper catches the ball, the padding is slightly compressible, thus reducing the effect of the force on the wicket keepers hands. Similarly, if a gymnast falls, the padding compresses and reduces the effect of the force on the gymnast's body.

#### Arrestor Beds for Trucks

An arrestor bed is a patch of ground that is softer than the road. Trucks use these when they have to make an emergency stop. When a trucks reaches an arrestor bed the time interval over which the momentum is changed is increased. This decreases the force and causes the truck to slow down.

#### Follow-Through in Sports

In sports where rackets and bats are used, like tennis, cricket, squash, badminton and baseball, the hitter is often encouraged to follow-through when striking the ball. High speed films of the collisions between bats/rackets and balls have shown that following through increases the time over which the collision between the racket/bat and ball occurs. This increase in the time of the collision causes an increase in the velocity change of the ball. This means that a hitter can cause the ball to leave the racket/bat faster by following through. In these sports, returning the ball with a higher velocity often increases the chances of success.

#### Crumple Zones in Cars

Another safety application of trying to reduce the force experienced is in crumple zones in cars. When two cars have a collision, two things can happen:

1. the cars bounce off each other, or
2. the cars crumple together.

Which situation is more dangerous for the occupants of the cars? When cars bounce off each other, or rebound, there is a larger change in momentum and therefore a larger impulse. A larger impulse means that a greater force is experienced by the occupants of the cars. When cars crumple together, there is a smaller change in momentum and therefore a smaller impulse. The smaller impulse means that the occupants of the cars experience a smaller force. Car manufacturers use this idea and design crumple zones into cars, such that the car has a greater chance of crumpling than rebounding in a collision. Also, when the car crumples, the change in the car's momentum happens over a longer time. Both these effects result in a smaller force on the occupants of the car, thereby increasing their chances of survival.

---

**Activity :: Egg Throw : This activity demonstrates the effect of impulse and how it is used to improve safety. Have two learners hold up a bed sheet or large piece of fabric. Then toss an egg at the sheet. The egg should not break, because the collision between the egg and the bed sheet lasts over an extended period of time since the bed sheet has some give in it. By increasing the time of the collision, the force of the impact is minimized. Take care to aim at the sheet, because if you miss the sheet, you will definitely break the egg and have to clean up the mess!**

---

### 12.5.10 Exercise

1. A canon, mass 500 kg, fires a shell, mass 1 kg, horizontally to the right at 500 m·s$^{-1}$. What is the magnitude and direction of the initial recoil velocity of the canon?
2. The velocity of a moving trolley of mass 1 kg is 3 m·s$^{-1}$. A block of wood, mass 0,5 kg, is dropped vertically on to the trolley. Immediately after the collision, the speed of the trolley and block is 2 m·s$^{-1}$. By way of calculation, show whether momentum is conserved in the collision.
3. A 7200 kg empty railway truck is stationary. A fertilizer firm loads 10800 kg fertilizer into the truck. A second, identical, empty truck is moving at 10 m·s$^{-1}$when it collides with the loaded truck.

3.1 If the empty truck stops completely immediately after the collision, use a conservation law to calculate the velocity of the loaded truck immediately after the collision.

3.2 Calculate the distance that the loaded truck moves after collision, if a constant frictional force of 24 kN acts on the truck.

4. A child drops a squash ball of mass 0,05 kg. The ball strikes the ground with a velocity of 4 m·s$^{-1}$and rebounds with a velocity of 3 m·s$^{-1}$. Does the law of conservation of momentum apply to this situation? Explain.

5. A bullet of mass 50 g travelling horizontally at 600 m·s$^{-1}$strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet gets stuck in the block.

5.1 Name and state the principle which can be applied to find the speed of the block-and-bullet system after the bullet entered the block.

5.2 Calculate the speed of the bullet-and-block system immediately after impact.

5.3 If the time of impact was 5 x 10$^{-4}$ seconds, calculate the force that the bullet exerts on the block during impact.

## 12.6 Torque and Levers

### 12.6.1 Torque

This chapter has dealt with forces and how they lead to motion in a straight line. In this section, we examine how forces lead to rotational motion.

When an object is fixed or supported at one point and a force acts on it a distance away from the support, it tends to make the object turn. The moment of force or *torque* (symbol, $\tau$ read *tau*) is defined as the product of the distance from the support or pivot ($r$) and the component of force perpendicular to the object, $F_\perp$.

$$\tau = F_\perp \cdot r \tag{12.9}$$

Torque can be seen as a rotational force. The unit of torque is N·m and torque is a vector quantity. Some examples of where torque arises are shown in Figures 12.17, 12.18 and 12.19.

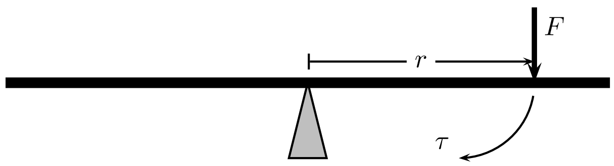



Figure 12.17: The force exerted on one side of a see-saw causes it to swing.

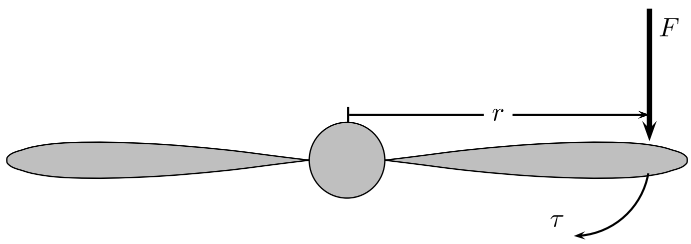



Figure 12.18: The force exerted on the edge of a propellor causes the propellor to spin.

For example in Figure 12.19, if a force $F$ of 10 N is applied perpendicularly to the spanner at a distance $r$ of 0,3 m from the center of the bolt, then the torque applied to the bolt is:

$$\begin{aligned} \tau &= F_\perp \cdot r \\ &= (10\ \mathrm{N})(0{,}3\,\mathrm{m}) \\ &= 3\,\mathrm{N}\cdot\mathrm{m} \end{aligned}$$

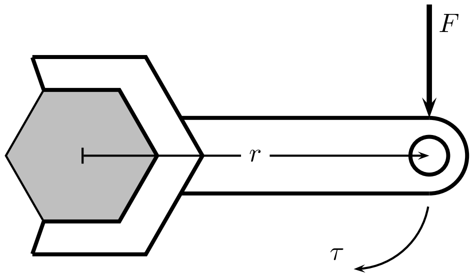



Figure 12.19: The force exerted on a spanner helps to loosen the bolt.

If the force of 10 N is now applied at a distance of 0,15 m from the centre of the bolt, then the torque is:

$$\begin{aligned}\tau &= F_{\perp} \cdot r \\ &= (10\ \mathrm{N})(0{,}15\,\mathrm{m}) \\ &= 1{,}5\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

This shows that there is less torque when the force is applied closer to the bolt than further away.

**Important:** Loosening a bolt

If you are trying to loosen (or tighten) a bolt, apply the force on the spanner further away from the bolt, as this results in a greater torque to the bolt making it easier to loosen.

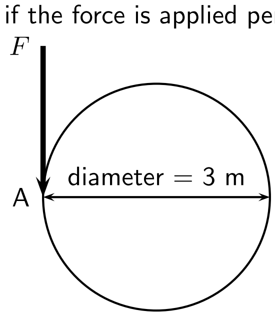
**Important:** Any component of a force exerted parallel to an object will not cause the object to turn. Only perpendicular components cause turning.

**Important:** Torques

The direction of a torque is either clockwise or anticlockwise. When torques are added, choose one direction as positive and the opposite direction as negative. If equal clockwise and anti-clockwise torques are applied to an object, they will cancel out and there will be no net turning effect.

**Worked Example 95: Merry-go-round**

**Question:** Several children are playing in the park. One child pushes the merry-go-round with a force of 50 N. The diameter of the merry-go-round is 3,0 m. What torque does the child apply if the force is applied perpendicularly at point A?



**Answer**
**Step 1 : Identify what has been given**
The following has been given:

- the force applied, $F = 50$ N
- the diameter of the merry-go-round, $2r = 3$ m, therefore $r = 1{,}5$ m.

The quantities are in SI units.
**Step 2 : Decide how to approach the problem**
We are required to determine the torque applied to the merry-go-round. We can do this by using:

$$\tau = F_{\perp} \cdot r$$

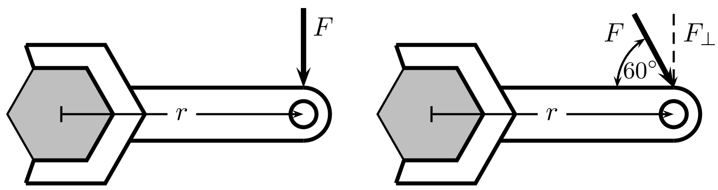
We are given $F_{\perp}$ and we are given the diameter of the merry-go-round. Therefore, $r = 1{,}5$ m.
**Step 3 : Solve the problem**

$$\begin{aligned} \tau &= F_{\perp} \cdot r \\ &= (50\,\mathrm{N})(1{,}5\,\mathrm{m}) \\ &= 75\,\mathrm{N} \cdot \mathrm{m} \end{aligned}$$

**Step 4 : Write the final answer**
75 $\mathrm{N} \cdot \mathrm{m}$ of torque is applied to the merry-go-round.

**Worked Example 96: Flat tyre**

**Question:** Kevin is helping his dad replace the flat tyre on the car. Kevin has been asked to undo all the wheel nuts. Kevin holds the spanner at the same distance for all nuts, but applies the force at two angles (90° and 60°). If Kevin applies a force of 60 N, at a distance of 0,3 m away from the nut, which angle is the best to use? Prove your answer by means of calculations.




**Answer**
**Step 1 : Identify what has been given**
The following has been given:

- the force applied, $F = 60$ N
- the angles at which the force is applied, $\theta = 90°$ and $\theta = 60°$
- the distance from the centre of the nut at which the force is applied, $r = 0{,}3$ m

The quantities are in SI units.

**Step 2 : Decide how to approach the problem**
We are required to determine which angle is more better to use. This means that we must find which angle gives the higher torque. We can use

$$\tau = F_{\perp} \cdot r$$

to determine the torque. We are given $F$ for each situation. $F_{\perp} = F \sin\theta$ and we are given $\theta$. We are also given the distance away from the nut, at which the force is applied.

**Step 3 : Solve the problem for $\theta = 90°$**
$F_\perp = F$

$$\begin{aligned}\tau &= F_\perp \cdot r \\ &= (60\,\mathrm{N})(0{,}3\,\mathrm{m}) \\ &= 18\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

**Step 4 : Solve the problem for $\theta = 60°$**

$$\begin{aligned}\tau &= F_\perp \cdot r \\ &= F\sin\theta\cdot r \\ &= (60\,\mathrm{N})\sin(\theta)(0{,}3\,\mathrm{m}) \\ &= 15{,}6\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

**Step 5 : Write the final answer**
The torque from the perpendicular force is greater than the torque from the force applied at $60°$. Therefore, the best angle is $90°$.

## 12.6.2 Mechanical Advantage and Levers

We can use our knowlegde about the moments of forces (torque) to determine whether situations are balanced. For example two mass pieces are placed on a seesaw as shown in Figure 12.20. The one mass is 3 kg and the other is 6 kg. The masses are placed a distance 2 m and 1 m, respectively from the pivot. By looking at the clockwise and anti-clockwise moments, we can determine whether the seesaw will pivot (move) or not. If the sum of the clockwise and anti-clockwise moments is zero, the seesaw is in equilibrium (i.e. balanced).

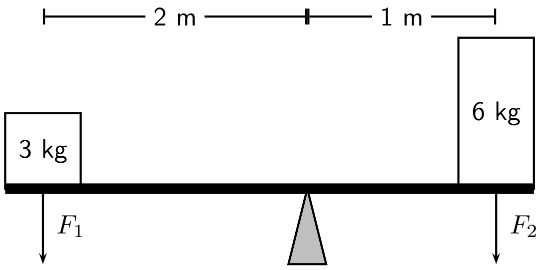



Figure 12.20: The moments of force are balanced.

The clockwise moment can be calculated as follows:

$$\begin{aligned}\tau &= F_\perp \cdot r \\ \tau &= (6)(9{,}8)(1) \\ \tau &= 58{,}8N\cdot m \text{ clockwise}\end{aligned}$$

The anti-clockwise moment can be calculated as follows:

$$\begin{aligned}\tau &= F_\perp \cdot r \\ \tau &= (3)(9{,}8)(2) \\ \tau &= 58{,}8N\cdot m \text{ anti-clockwise}\end{aligned}$$

The sum of the moments of force will be zero:

The resultant moment is zero as the clockwise and anti-clockwise moments of force are in opposite directions and therefore cancel each other.

As we see in Figure 12.20, we can use different distances away from a pivot to balance two different forces. This principle is applied to a lever to make lifting a heavy object much easier.

**Definition: Lever**
A lever is a rigid object that is used with an appropriate fulcrum or pivot point to multiply the mechanical force that can be applied to another object.

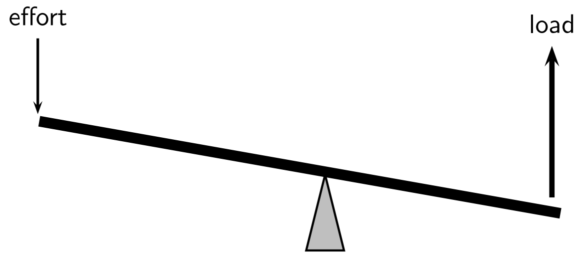



Figure 12.21: A lever is used to put in a small effort to get out a large load.

Archimedes reputedly said: *Give me a lever long enough and a fulcrum on which to place it, and I shall move the world.*

The concept of getting out more than the effort is termed mechanical advantage, and is one example of the principle of moments. The lever allows to do less effort but for a greater distance. For instance to lift a certain unit of weight with a lever with an effort of half a unit we need a distance from the fulcrum in the effort's side to be twice the distance of the weight's side. It also means that to lift the weight 1 meter we need to push the lever for 2 meter. The amount of work done is always the same and independent of the dimensions of the lever (in an ideal lever). The lever only allows to trade effort for distance.

Ideally, this means that the mechanical advantage of a system is the ratio of the force that performs the work (output or load) to the applied force (input or effort), assuming there is no friction in the system. In reality, the mechanical advantage will be less than the ideal value by an amount determined by the amount of friction.

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

For example, you want to raise an object of mass 100 kg. If the pivot is placed as shown in Figure 12.22, what is the mechanical advantage of the lever?

In order to calculate mechanical advantage, we need to determine the load and effort.

**Important:** Effort is the input force and load is the output force.

The load is easy, it is simply the weight of the 100 kg object.

$$F_{load} = m \cdot g = 100\,\text{kg} \cdot 9{,}8\,\text{m} \cdot \text{s}^{-2} = 980\,\text{N}$$

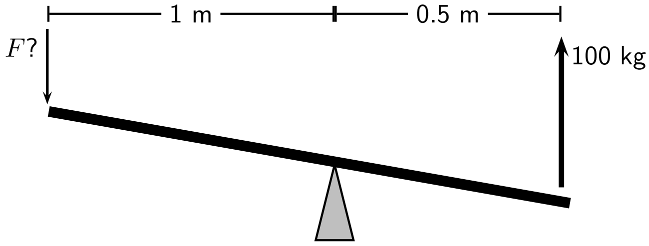



Figure 12.22: A lever is used to put in a small effort to get out a large load.

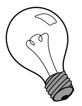
The effort is found by balancing torques.

$$
\begin{aligned}
F_{load} \cdot r_{load} &= F_{effort} \cdot r_{effort} \\
980\,\mathrm{N} \cdot 0.5\,\mathrm{m} &= F_{effort} \cdot 1\,\mathrm{m} \\
F_{effort} &= \frac{980\,\mathrm{N} \cdot 0.5\,\mathrm{m}}{1\,\mathrm{m}} \\
&= 490\,\mathrm{N}
\end{aligned}
$$

The mechanical advantage is:

$$
\begin{aligned}
\text{mechanical advantage} &= \frac{\text{load}}{\text{effort}} \\
&= \frac{980\,\mathrm{N}}{490\,\mathrm{N}} \\
&= 2
\end{aligned}
$$

Since mechanical advantage is a ratio, it does not have any units.



*Extension: Pulleys*

Pulleys change the direction of a tension force on a flexible material, e.g. a rope or cable. In addition, pulleys can be "added together" to create mechanical advantage, by having the flexible material looped over several pulleys in turn. More loops and pulleys increases the mechanical advantage.

### 12.6.3 Classes of levers

**Class 1 levers**

In a class 1 lever the fulcrum is between the effort and the load. Examples of class 1 levers are the seesaw, crowbar and equal-arm balance. The mechanical advantage of a class 1 lever can be increased by moving the fulcrum closer to the load.

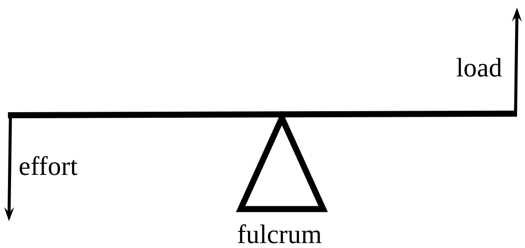



Figure 12.23: Class 1 levers

**Class 2 levers**

In class 2 levers the fulcrum is at the one end of the bar, with the load closer to the fulcrum and the effort on the other end of bar. The mechanical advantage of this type of lever can be increased by increasing the length of the bar. A bottle opener or wheel barrow are examples of class 2 levers.

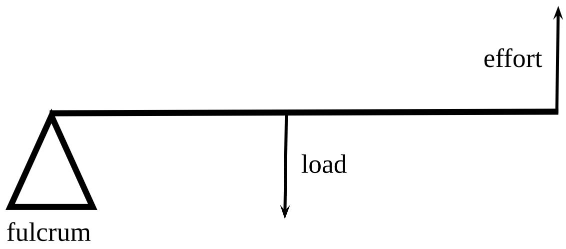



Figure 12.24: Class 2 levers

**Class 3 levers**

In class 3 levers the fulcrum is also at the end of the bar, but the effort is between the fulcrum and the load. An example of this type of lever is the human arm.

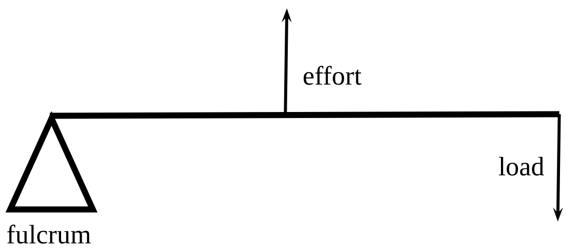



Figure 12.25: Class 3 levers

### 12.6.4 Exercise

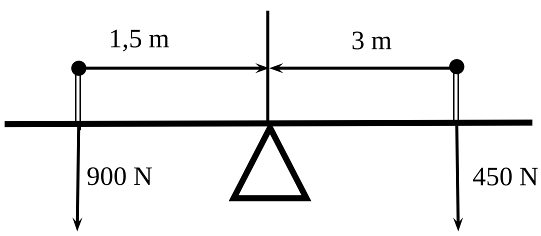
1. Riyaad applies a force of 120 N on a spanner to undo a nut.

    1.1 Calculate the moment of the force if he applies the force 0,15 m from the bolt.

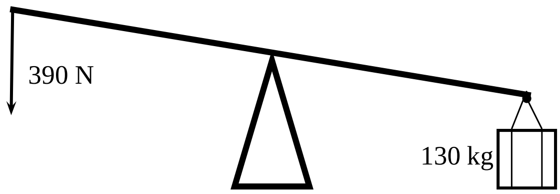
    1.2 The nut does not turn, so Riyaad moves his hand to the end of the spanner and applies the same force 0,2 m away from the bolt. Now the nut begins to move. Calculate the new moment of force. Is it bigger or smaller than before?

    1.3 Once the nuts starts to turn, the moment needed to turn it is less than it was to start it turning. It is now 20 N·m. Calculate the new moment of force that Riyaad now needs to apply 0,2 m away from the nut.

2. Calculate the clockwise and anticlockwise moments in the figure below to see if the see-saw is balanced.




3. Jeffrey uses a force of 390 N to lift a load of 130 kg.




3.1 Calculate the mechanical advantage of the lever that he is using.

3.2 What type of lever is he using? Give a reason for your answer.

3.3 If the force is applied 1 m from the pivot, calculate the distance between the pivot and the load.

4. A crowbar is used to lift a box of weight 400 N. The box is placed 75 cm from the pivot. A crow bar is a class 1 lever.

   4.1 Why is a crowbar a class 1 lever. Draw a diagram to explain your answer.

   4.2 What force F needs to be applied at a distance of 1,25 m from the pivot to balance the crowbar?

   4.3 If force F was applied at a distance of 2 m, what would the magnitude of F be?

5. A wheelbarrow is used to carry a load of 200 N. The load is 40 cm from the pivot and the force F is applied at a distance of 1,2 m from the pivot.

   5.1 What type of lever is a wheelbarrow?

   5.2 Calculate the force F that needs to be applied to lift the load.

6. The bolts holding a car wheel in place is tightened to a torque of 90 N · m. The mechanic has two spanners to undo the bolts, one with a length of 20 cm and one with a length of 30 cm. Which spanner should he use? Give a reason for your answer by showing calculations and explaining them.

## 12.7 Summary

**Newton's First Law** Every object will remain at rest or in uniform motion in a straight line unless it is made to change its state by the action of an *unbalanced force*.

**Newton's Second Law** The resultant force acting on a body will cause the body to accelerate in the direction of the resultant force The acceleration of the body is directly proportional to the magnitude of the resultant force and inversely proportional to the mass of the object.

**Newton's Third Law** If body A exerts a force on body B then body B will exert an equal but opposite force on body A.

**Newton's Law of Universal Gravitation** Every body in the universe exerts a force on every other body. The force is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.

**Equilibrium** Objects at rest or moving with constant velocity are in *equilibrium* and have a *zero resultant force*.

**Equilibrant** The *equilibrant* of any number of forces is the single force required to produce equilibrium.

**Triangle Law for Forces in Equilibrium** Three forces in equilibrium can be represented in magnitude and direction by the three sides of a triangle taken in order.

**Momentum** The *momentum* of an object is defined as its mass multiplied by its velocity.

**Momentum of a System** The *total momentum of a system* is the sum of the momenta of each of the objects in the system.

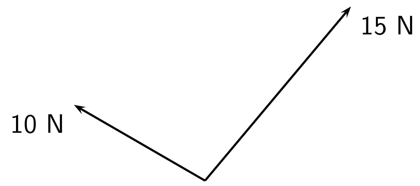
**Principle of Conservation of Linear Momentum:** 'The total linear momentum of an isolated system is constant' or 'In an isolated system the total momentum before a collision (or explosion) is equal to the total momentum after the collision (or explosion)'.

**Law of Momentum:** The applied resultant force acting on an object is equal to the rate of change of the object's momentum and this force is in the direction of the change in momentum.

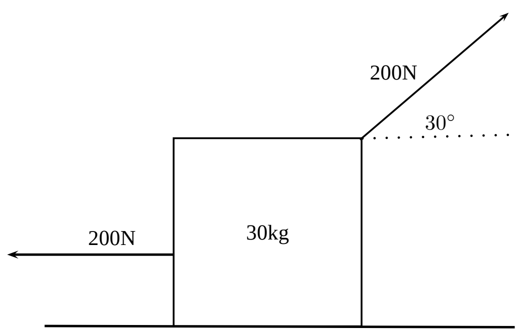
## 12.8 End of Chapter exercises

### Forces and Newton's Laws

1. [SC 2003/11] A constant, resultant force acts on a body which can move freely in a straight line. Which physical quantity will remain constant?
   1.1 acceleration
   1.2 velocity
   1.3 momentum
   1.4 kinetic energy
2. [SC 2005/11 SG1] Two forces, 10 N and 15 N, act at an angle at the same point.




   Which of the following **cannot** be the resultant of these two forces?
   A 2 N
   B 5 N
   C 8 N
   D 20 N
3. A concrete block weighing 250 N is at rest on an inclined surface at an angle of 20°. The magnitude of the normal force, in newtons, is
   A 250
   B 250 cos 20°
   C 250 sin 20°
   D 2500 cos 20°
4. A 30 kg box sits on a flat frictionless surface. Two forces of 200 N each are applied to the box as shown in the diagram. Which statement best describes the motion of the box?
   A The box is lifted off the surface.
   B The box moves to the right.
   C The box does not move.
   D The box moves to the left.



5. A concrete block weighing 200 N is at rest on an inclined surface at an angle of 20°. The normal reaction, in newtons, is

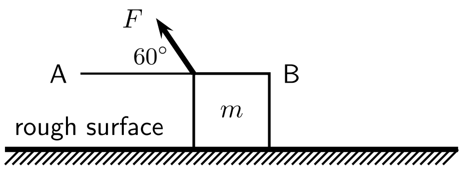
   A 200

   B 200 cos 20°

   C 200 sin 20°

   D 2000 cos 20°

6. [SC 2003/11]A box, mass $m$, is at rest on a rough horizontal surface. A force of constant magnitude $F$ is then applied on the box at an angle of 60°to the horizontal, as shown.




   If the box has a uniform horizontal acceleration of magnitude, $a$, the frictional force acting on the box is ...

   A $F\cos 60° - ma$ in the direction of A

   B $F\cos 60° - ma$ in the direction of B

   C $F\sin 60° - ma$ in the direction of A

   D $F\sin 60° - ma$ in the direction of B

7. [SC 2002/11 SG] Thabo stands in a train carriage which is moving eastwards. The train suddenly brakes. Thabo continues to move eastwards due to the effect of

   A his inertia.

   B the inertia of the train.

   C the braking force on him.

   D a resultant force acting on him.

8. [SC 2002/11 HG1] A body slides down a frictionless inclined plane. Which one of the following physical quantities will remain constant throughout the motion?

   A velocity

   B momentum

   C acceleration

   D kinetic energy

9. [SC 2002/11 HG1] A body moving at a *CONSTANT VELOCITY* on a horizontal plane, has a number of unequal forces acting on it. Which one of the following statements is TRUE?

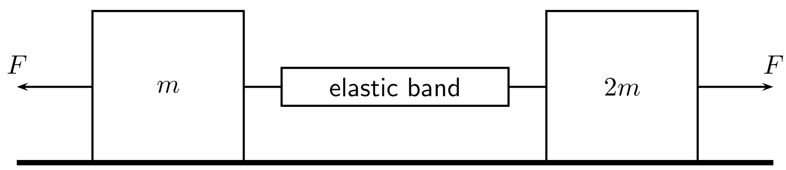
   A At least two of the forces must be acting in the same direction.

   B The resultant of the forces is zero.

   C Friction between the body and the plane causes a resultant force.

   D The vector sum of the forces causes a resultant force which acts in the direction of motion.

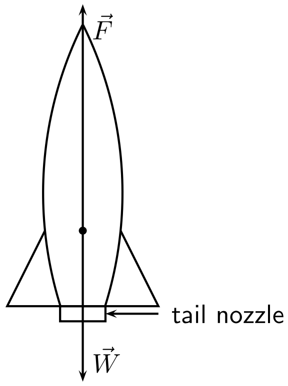
10. [IEB 2005/11 HG] Two masses of $m$ and $2m$ respectively are connected by an elastic band on a frictionless surface. The masses are pulled in opposite directions by two forces each of magnitude $F$, stretching the elastic band and holding the masses stationary.



Which of the following gives the magnitude of the tension in the elastic band?

A zero
B $\frac{1}{2}F$
C $F$
D $2F$

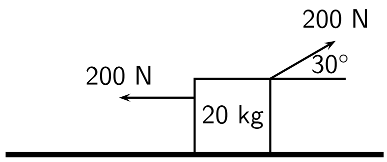
11. [IEB 2005/11 HG] A rocket takes off from its launching pad, accelerating up into the air.




The rocket accelerates because the magnitude of the upward force, $F$ is greater than the magnitude of the rocket's weight, $W$. Which of the following statements **best** describes how force $F$ arises?

A $F$ is the force of the air acting on the base of the rocket.
B $F$ is the force of the rocket's gas jet *pushing down* on the air.
C $F$ is the force of the rocket's gas jet *pushing down* on the ground.
D $F$ is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.

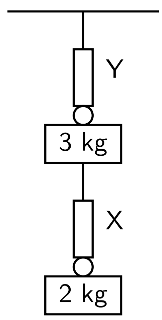
12. [SC 2001/11 HG1] A box of mass 20 kg rests on a smooth horizontal surface. What will happen to the box if two forces each of magnitude 200 N are applied simultaneously to the box as shown in the diagram.




The box will ...

A be lifted off the surface.
B move to the left.
C move to the right.
D remain at rest.

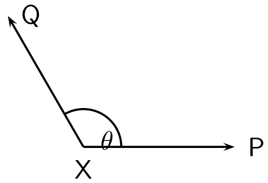
13. [SC 2001/11 HG1] A 2 kg mass is suspended from spring balance X, while a 3 kg mass is suspended from spring balance Y. Balance X is in turn suspended from the 3 kg mass. Ignore the weights of the two spring balances.



The readings (in N) on balances X and Y are as follows:

| | X | Y |
|---|---|---|
| (A) | 20 | 30 |
| (B) | 20 | 50 |
| (C) | 25 | 25 |
| (D) | 50 | 50 |

14. [SC 2002/03 HG1] $P$ and $Q$ are two forces of equal magnitude applied simultaneously to a body at X.




As the angle $\theta$ between the forces is **decreased** from 180° to 0°, the magnitude of the resultant of the two forces will

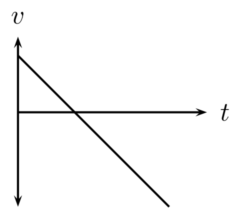
A initially increase and then decrease.

B initially decrease and then increase.

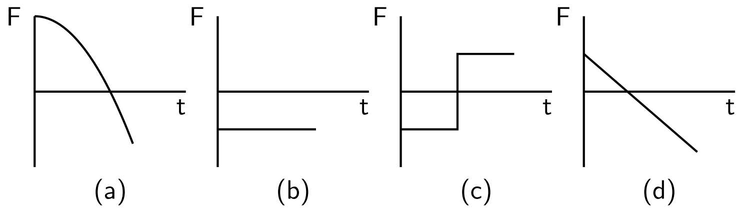
C increase only.

D decrease only.

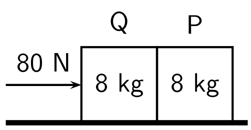
15. [SC 2002/03 HG1] The graph below shows the velocity-time graph for a moving object:




Which of the following graphs could best represent the relationship between the resultant force applied to the object and time?




16. [SC 2002/03 HG1] Two blocks each of mass 8 kg are in contact with each other and are accelerated along a frictionless surface by a force of 80 N as shown in the diagram. The force which block Q will exert on block P is equal to ...



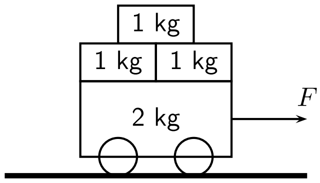

A 0 N

B 40 N

C 60 N

D 80 N

17. [SC 2002/03 HG1] Three 1 kg mass pieces are placed on top of a 2 kg trolley. When a force of magnitude $F$ is applied to the trolley, it experiences an acceleration $a$.




If one of the 1 kg mass pieces falls off while $F$ is still being applied, the trolley will accelerate at ...

A $\frac{1}{5}a$

B $\frac{4}{5}a$

C $\frac{5}{4}a$

D $5a$

18. [IEB 2004/11 HG1] A car moves along a horizontal road at constant velocity. Which of the following statements is true?

A The car is not in equilibrium.

B There are no forces acting on the car.

C There is zero resultant force.

D There is no frictional force.

19. [IEB 2004/11 HG1] A crane lifts a load vertically upwards at constant speed. The upward force exerted on the load is $F$. Which of the following statements is correct?

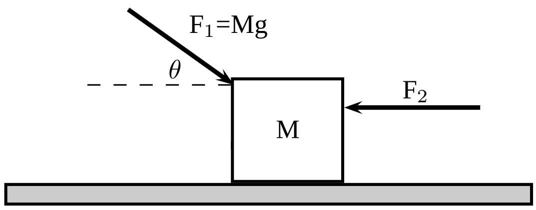
A The acceleration of the load is 9,8 m.s$^{-2}$ downwards.

B The resultant force on the load is F.

C The load has a weight equal in magnitude to F.

D The forces of the crane on the load, and the weight of the load, are an example of a Newton's third law 'action-reaction' pair.

20. [IEB 2004/11 HG1] A body of mass $M$ is at rest on a smooth horizontal surface with two forces applied to it as in the diagram below. Force $F_1$ is equal to $Mg$. The force $F_1$ is applied to the right at an angle $\theta$ to the horizontal, and a force of $F_2$ is applied horizontally to the left.




How is the body affected when the angle $\theta$ is increased?

A It remains at rest.

B It lifts up off the surface, and accelerates towards the right.

C It lifts up off the surface, and accelerates towards the left.

D It accelerates to the left, moving along the smooth horizontal surface.

21. [IEB 2003/11 HG1] Which of the following statements correctly explains why a passenger in a car, who is not restrained by the seat belt, continues to move forward when the brakes are applied suddenly?

A The braking force applied to the car exerts an equal and opposite force on the passenger.

B A forward force (called inertia) acts on the passenger.

C A resultant forward force acts on the passenger.

D A zero resultant force acts on the passenger.

22. [IEB 2004/11 HG1]

A rocket (mass 20 000 kg) accelerates from rest to 40 m·s$^{-1}$in the first 1,6 seconds of its journey upwards into space.
The rocket's propulsion system consists of exhaust gases, which are pushed out of an outlet at its base.

22.1 Explain, with reference to the appropriate law of Newton, how the escaping exhaust gases exert an upwards force (thrust) on the rocket.

22.2 What is the magnitude of the total thrust exerted on the rocket during the first 1,6 s?

22.3 An astronaut of mass 80 kg is carried in the space capsule. Determine the resultant force acting on him during the first 1,6 s.

22.4 Explain why the astronaut, seated in his chair, feels "heavier" while the rocket is launched.

23. [IEB 2003/11 HG1 - Sports Car]

23.1 State Newton's Second Law of Motion.

23.2 A sports car (mass 1 000 kg) is able to accelerate uniformly from rest to 30 m·s$^{-1}$in a minimum time of 6 s.

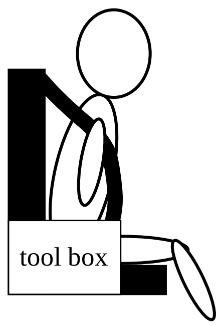
i. Calculate the magnitude of the acceleration of the car.

ii. What is the magnitude of the resultant force acting on the car during these 6 s?

23.3 The magnitude of the force that the wheels of the vehicle exert on the road surface as it accelerates is 7500 N. What is the magnitude of the retarding forces acting on this car?

23.4 By reference to a suitable Law of Motion, explain why a headrest is important in a car with such a rapid acceleration.

24. [IEB 2005/11 HG1] A child (mass 18 kg) is strapped in his car seat as the car moves to the right at constant velocity along a straight level road. A tool box rests on the seat beside him.




The driver brakes suddenly, bringing the car rapidly to a halt. There is negligible friction between the car seat and the box.

24.1 Draw a labelled free-body diagram of **the forces acting on the child** during the time that the car is being braked.

24.2 Draw a labelled free-body diagram of **the forces acting on the box** during the time that the car is being braked.

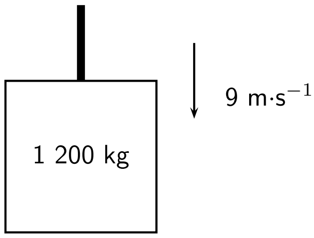
24.3 What is the rate of change of the child's momentum as the car is braked to a standstill from a speed of 72 $km.h^{-1}$ in 4 s.

*Modern cars are designed with safety features (besides seat belts) to protect drivers and passengers during collisions e.g. the crumple zones on the car's body. Rather than remaining rigid during a collision, the crumple zones allow the car's body to collapse steadily.*

24.4 State Newton's second law of motion.

24.5 Explain how the crumple zone on a car reduces the force of impact on it during a collision.

25. [SC 2003/11 HG1]The total mass of a lift together with its load is 1 200 kg. It is moving downwards at a constant velocity of 9 $m{\cdot}s^{-1}$.




25.1 What will be the magnitude of the force exerted by the cable on the lift while it is moving downwards at constant velocity? Give an explanation for your answer.
*The lift is now uniformly brought to rest over a distance of 18 m.*

25.2 Calculate the magnitude of the acceleration of the lift.

25.3 Calculate the magnitude of the force exerted by the cable while the lift is being brought to rest.

26. A driving force of 800 N acts on a car of mass 600 kg.

26.1 Calculate the car's acceleration.

26.2 Calculate the car's speed after 20 s.

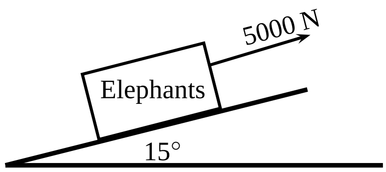
26.3 Calculate the new acceleration if a frictional force of 50 N starts to act on the car after 20 s.

26.4 Calculate the speed of the car after another 20 s (i.e. a total of 40 s after the start).

27. [IEB 2002/11 HG1 - A Crate on an Inclined Plane]

Elephants are being moved from the Kruger National Park to the Eastern Cape. They are loaded into crates that are pulled up a ramp (an inclined plane) on frictionless rollers.

The diagram shows a crate being held stationary on the ramp by means of a rope parallel to the ramp. The tension in the rope is 5 000 N.




27.1 Explain how one can deduce the following: "The forces acting on the crate are in equilibrium".

27.2 Draw a labelled free-body diagram of the forces acting on the crane and elephant. (Regard the crate and elephant as one object, and represent them as a dot. Also show the relevant angles between the forces.)

27.3 The crate has a mass of 800 kg. Determine the mass of the elephant.

27.4 The crate is now pulled up the ramp at a constant speed. How does the crate being pulled up the ramp at a constant speed affect the forces acting on the crate and elephant? Justify your answer, mentioning any law or principle that applies to this situation.

28. [IEB 2002/11 HG1 - Car in Tow]

Car A is towing Car B with a light tow rope. The cars move along a straight, horizontal road.

28.1 Write down a statement of Newton's Second Law of Motion (in words).

28.2 As they start off, Car A exerts a forwards force of 600 N at its end of the tow rope. The force of friction on Car B when it starts to move is 200 N. The mass of Car B is 1 200 kg. Calculate the acceleration of Car B.

28.3 After a while, the cars travel at constant velocity. The force exerted on the tow rope is now 300 N while the force of friction on Car B increases. What is the magnitude and direction of the force of friction on Car B now?

28.4 Towing with a rope is very dangerous. A solid bar should be used in preference to a tow rope. This is especially true should Car A suddenly apply brakes. What would be the advantage of the solid bar over the tow rope in such a situation?

28.5 The mass of Car A is also 1 200 kg. Car A and Car B are now joined by a solid tow bar and the total braking force is 9 600 N. Over what distance could the cars stop from a velocity of 20 m·s$^{-1}$?

29. [IEB 2001/11 HG1] - **Testing the Brakes of a Car**

A braking test is carried out on a car travelling at 20 m·s$^{-1}$. A braking distance of 30 m is measured when a braking force of 6 000 N is applied to stop the car.

29.1 Calculate the acceleration of the car when a braking force of 6 000 N is applied.

29.2 **Show** that the mass of this car is 900 kg.

29.3 How long (in s) does it take for this car to stop from 20 m·s$^{-1}$under the braking action described above?

29.4 A trailer of mass 600 kg is attached to the car and the braking test is repeated from 20 m·s$^{-1}$using the same braking force of 6 000 N. How much longer will it take to stop the car with the trailer in tow?

30. [IEB 2001/11 HG1] A rocket takes off from its launching pad, accelerating up into the air. Which of the following statements best describes the reason for the upward acceleration of the rocket?

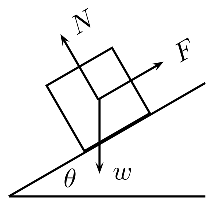
A The force that the atmosphere (air) exerts underneath the rocket is greater than the weight of the rocket.

B The force that the ground exerts on the rocket is greater than the weight of the rocket.

C The force that the rocket exerts on the escaping gases is less than the weight of the rocket.

D The force that the escaping gases exerts on the rocket is greater than the weight of the rocket.

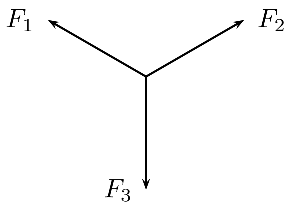
31. [IEB 2005/11 HG] A box is held stationary on a smooth plane that is inclined at angle $\theta$ to the horizontal.



$F$ is the force exerted by a rope on the box. $w$ is the weight of the box and $N$ is the normal force of the plane on the box. Which of the following statements is correct?

A $\tan\theta = \frac{F}{w}$
B $\tan\theta = \frac{F}{N}$
C $\cos\theta = \frac{F}{w}$
D $\sin\theta = \frac{N}{w}$

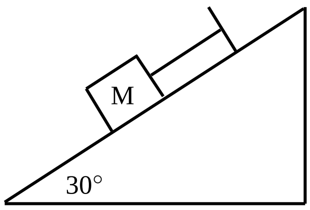
32. [SC 2001/11 HG1] As a result of three forces $F_1$, $F_2$ and $F_3$ acting on it, an object at point P is in equilibrium.




Which of the following statements is **not true** with reference to the three forces?

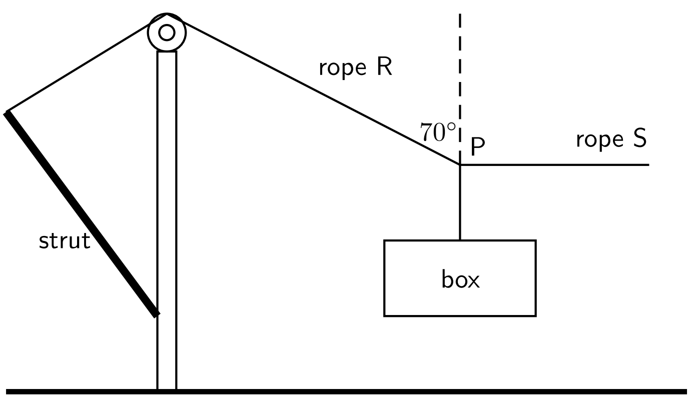
32.1 The resultant of forces $F_1$, $F_2$ and $F_3$ is zero.
32.2 Forces $F_1$, $F_2$ and $F_3$ lie in the same plane.
32.3 Forces $F_3$ is the resultant of forces $F_1$ and $F_2$.
32.4 The sum of the components of all the forces in any chosen direction is zero.

33. A block of mass M is held stationary by a rope of negligible mass. The block rests on a frictionless plane which is inclined at $30°$ to the horizontal.




33.1 Draw a labelled force diagram which shows all the forces acting on the block.
33.2 Resolve the force due to gravity into components that are parallel and perpendicular to the plane.
33.3 Calculate the weight of the block when the force in the rope is 8N.

34. [SC 2003/11] A heavy box, mass $m$, is lifted by means of a rope R which passes over a pulley fixed to a pole. A second rope S, tied to rope R at point P, exerts a horizontal force and pulls the box to the right. After lifting the box to a certain height, the box is held stationary as shown in the sketch below. Ignore the masses of the ropes. The tension in rope R is 5 850 N.



34.1 Draw a diagram (with labels) of all the forces acting at the point P, when P is in equilibrium.

34.2 By resolving the force exerted by rope R into components, calculate the ...

i. magnitude of the force exerted by rope S.

ii. mass, m, of the box.

34.3 Will the tension in rope R, increase, decrease or remain the same if rope S is pulled further to the right (the length of rope R remains the same)? Give a reason for your choice.

35. A tow truck attempts to tow a broken down car of mass 400 kg. The coefficient of static friction is 0,60 and the coefficient of kinetic (dynamic) friction is 0,4. A rope connects the tow truck to the car. Calculate the force required:

35.1 to just move the car if the rope is parallel to the road.

35.2 to keep the car moving at constant speed if the rope is parallel to the road.

35.3 to just move the car if the rope makes an angle of 30° to the road.

35.4 to keep the car moving at constant speed if the rope makes an angle of 30° to the road.

36. A crate weighing 2000 N is to be lowered at constant speed down skids 4 m long, from a truck 2 m high.

36.1 If the coefficient of sliding friction between the crate and the skids is 0,30, will the crate need to be pulled down or held back?

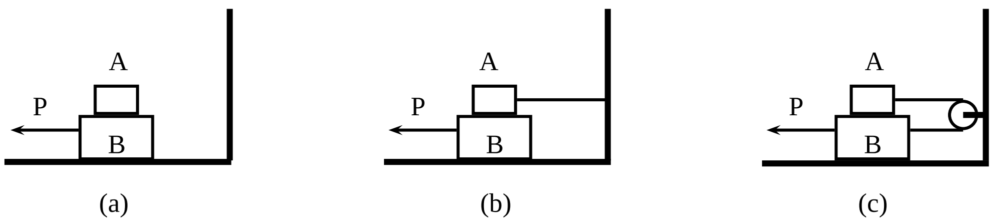
36.2 How great is the force needed parallel to the skids?

37. Block A in the figures below weighs 4 N and block B weighs 8 N. The coefficient of kinetic friction between all surfaces is 0,25. Find the force P necessary to drag block B to the left at constant speed if

37.1 A rests on B and moves with it

37.2 A is held at rest

37.3 A and B are connected by a light flexible cord passing around a fixed frictionless pulley




## Gravitation

1. [SC 2003/11]An object attracts another with a gravitational force $F$. If the distance between the centres of the two objects is now decreased to a third ($\frac{1}{3}$) of the original distance, the force of attraction that the one object would exert on the other would become...

A $\frac{1}{9}F$

B $\frac{1}{3}F$

C $3F$

D $9F$

2. [SC 2003/11] An object is dropped from a height of 1 km above the Earth. If air resistance is ignored, the acceleration of the object is dependent on the ...

A mass of the object

B radius of the earth

C mass of the earth

D weight of the object

3. A man has a mass of 70 kg on Earth. He is walking on a new planet that has a mass four times that of the Earth and the radius is the same as that of the Earth ( $M_E = 6 \times 10^{24}$ kg, $r_E = 6 \times 10^6$ m )

   3.1 Calculate the force between the man and the Earth.

   3.2 What is the man's mass on the new planet?

   3.3 Would his weight be bigger or smaller on the new planet? Explain how you arrived at your answer.

4. Calculate the distance between two objects, 5000 kg and $6 \times 10^{12}$ kg respectively, if the magnitude of the force between them is $3 \times 10^{?8}$ N.

5. Calculate the mass of the Moon given that an object weighing 80 N on the Moon has a weight of 480 N on Earth and the radius of the Moon is $1{,}6 \times 10^{16}$ m.

6. The following information was obtained from a free-fall experiment to determine the value of $g$ with a pendulum.
   Average falling distance between marks = 920 mm
   Time taken for 40 swings = 70 s
   Use the data to calculate the value of $g$.

7. An astronaut in a satellite 1600 km above the Earth experiences gravitational force of the magnitude of 700 N on Earth. The Earth's radius is 6400 km. Calculate

   7.1 The magnitude of the gravitational force which the astronaut experiences in the satellite.

   7.2 The magnitude of the gravitational force on an object in the satellite which weighs 300 N on Earth.

8. An astronaut of mass 70 kg on Earth lands on a planet which has half the Earth's radius and twice its mass. Calculate the magnitude of the force of gravity which is exerted on him on the planet.

9. Calculate the magnitude of the gravitational force of attraction between two spheres of lead with a mass of 10 kg and 6 kg respectively if they are placed 50 mm apart.

10. The gravitational force between two objects is 1200 N. What is the gravitational force between the objects if the mass of each is doubled and the distance between them halved?

11. Calculate the gravitational force between the Sun with a mass of $2 \times 10^{30}$ kg and the Earth with a mass of $6 \times 10^{24}$ kg if the distance between them is $1{,}4 \times 10^8$ km.

12. How does the gravitational force of attraction between two objects change when

    12.1 the mass of each object is doubled.

    12.2 the distance between the centres of the objects is doubled.

    12.3 the mass of one object is halved, and the distance between the centres of the objects is halved.

13. Read each of the following statements and say whether you agree or not. Give reasons for your answer and rewrite the statement if necessary:

    13.1 The gravitational acceleration g is constant.

    13.2 The weight of an object is independent of its mass.

    13.3 G is dependent on the mass of the object that is being accelerated.

14. An astronaut weighs 750 N on the surface of the Earth.

14.1 What will his weight be on the surface of Saturn, which has a mass 100 times greater than the Earth, and a radius 5 times greater than the Earth?

14.2 What is his mass on Saturn?

15. A piece of space garbage is at rest at a height 3 times the Earth's radius above the Earth's surface. Determine its acceleration due to gravity. Assume the Earth's mass is 6,0 x $10^{24}$ kg and the Earth's radius is 6400 km.

16. Your mass is 60 kg in Paris at ground level. How much less would you weigh after taking a lift to the top of the Eiffel Tower, which is 405 m high? Assume the Earth's mass is 6,0 x $10^{24}$ kg and the Earth's radius is 6400 km.

17. 17.1 State Newton's Law of Universal Gravitation.

    17.2 Use Newton's Law of Universal Gravitation to determine the magnitude of the acceleration due to gravity on the Moon.
    The mass of the Moon is 7,40 $\times$ $10^{22}$ kg.
    The radius of the Moon is 1,74 $\times$ $10^{6}$ m.

    17.3 Will an astronaut, kitted out in his space suit, jump higher on the Moon or on the Earth? Give a reason for your answer.

**Momentum**

1. [SC 2003/11]A projectile is fired vertically upwards from the ground. At the highest point of its motion, the projectile explodes and separates into two pieces of equal mass. If one of the pieces is projected vertically upwards after the explosion, the second piece will ...

   A drop to the ground at zero initial speed.

   B be projected downwards at the same initial speed at the first piece.

   C be projected upwards at the same initial speed as the first piece.

   D be projected downwards at twice the initial speed as the first piece.

2. [IEB 2004/11 HG1] A ball hits a wall horizontally with a speed of 15 m·s$^{-1}$. It rebounds horizontally with a speed of 8 m·s$^{-1}$. Which of the following statements about the system of the ball and the wall is **true**?

   A The total linear momentum of the system is not conserved during this collision.

   B The law of conservation of energy does not apply to this system.

   C The change in momentum of the wall is equal to the change in momentum of the ball.

   D Energy is transferred from the ball to the wall.

3. [IEB 2001/11 HG1] A block of mass M collides with a stationary block of mass 2M. The two blocks move off together with a velocity of v. What is the velocity of the block of mass M immediately **before** it collides with the block of mass 2M?

   A v

   B 2v

   C 3v

   D 4v

4. [IEB 2003/11 HG1] A cricket ball and a tennis ball move horizontally towards you with the <u>same momentum</u>. A cricket ball has greater mass than a tennis ball. You apply the same force in stopping each ball.
   How does the time taken to stop each ball compare?

   A It will take longer to stop the cricket ball.

   B It will take longer to stop the tennis ball.

   C It will take the same time to stop each of the balls.

D One cannot say how long without knowing the kind of collision the ball has when stopping.

5. [IEB 2004/11 HG1] Two identical billiard balls collide head-on with each other. The first ball hits the second ball with a speed of V, and the second ball hits the first ball with a speed of 2V. After the collision, the first ball moves off in the opposite direction with a speed of 2V. Which expression correctly gives the speed of the second ball after the collision?

   A V

   B 2V

   C 3V

   D 4V

6. [SC 2002/11 HG1] Which one of the following physical quantities is the same as the rate of change of momentum?

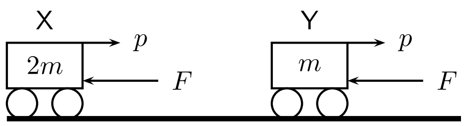
   A resultant force

   B work

   C power

   D impulse

7. [IEB 2005/11 HG] Cart X moves along a smooth track with momentum $p$. A resultant force $F$ applied to the cart stops it in time $t$. Another cart Y has only half the mass of X, but it has the same momentum $p$.




   In what time will cart Y be brought to rest when the same resultant force $F$ acts on it?

   A $\frac{1}{2}t$

   B $t$

   C $2t$

   D $4t$

8. [SC 2002/03 HG1] A ball with mass $m$ strikes a wall perpendicularly with a speed, $v$. If it rebounds in the opposite direction with the same speed, $v$, the magnitude of the change in momentum will be ...

   A $2mv$

   B $mv$

   C $\frac{1}{2}mv$

   D $0\ mv$

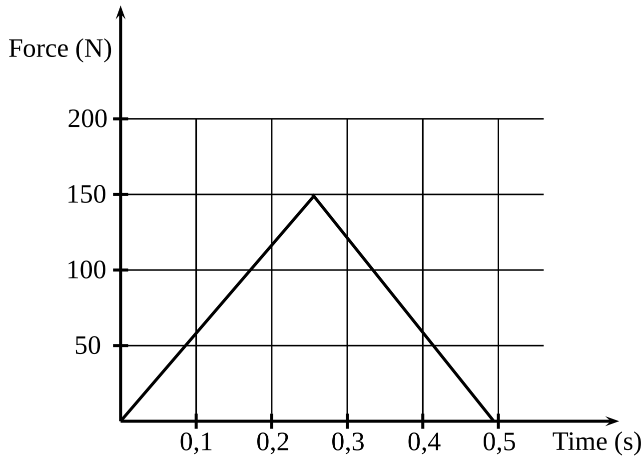
9. Show that impulse and momentum have the same units.

10. A golf club exerts an average force of 3 kN on a ball of mass 0,06 kg. If the golf club is in contact with the golf ball for 5 x $10^{-4}$ seconds, calculate

    10.1 the change in the momentum of the golf ball.

    10.2 the velocity of the golf ball as it leaves the club.

11. During a game of hockey, a player strikes a stationary ball of mass 150 g. The graph below shows how the force of the ball varies with the time.




11.1 What does the area under this graph represent?

11.2 Calculate the speed at which the ball leaves the hockey stick.

11.3 The same player hits a practice ball of the same mass, but which is made from a softer material. The hit is such that the ball moves off with the same speed as before. How will the **area**, the **height** and the **base** of the triangle that forms the graph, compare with that of the original ball?

12. The fronts of modern cars are deliberately designed in such a way that in case of a head-on collision, the front would crumple. Why is it desirable that the front of the car should crumple?

13. A ball of mass 100 g strikes a wall horizontally at 10 m·s$^{-1}$and rebounds at 8 m·s$^{-1}$. It is in contact with the wall for 0,01 s.

13.1 Calculate the average force exerted by the wall on the ball.

13.2 Consider a lump of putty also of mass 100 g which strikes the wall at 10 m·s$^{-1}$and comes to rest in 0,01 s against the surface. Explain qualitatively (no numbers) whether the force exerted on the putty will be less than, greater than of the same as the force exerted on the ball by the wall. Do not do any calculations.

14. Shaun swings his cricket bat and hits a stationary cricket ball vertically upwards so that it rises to a height of 11,25 m above the ground. The ball has a mass of 125 g. Determine

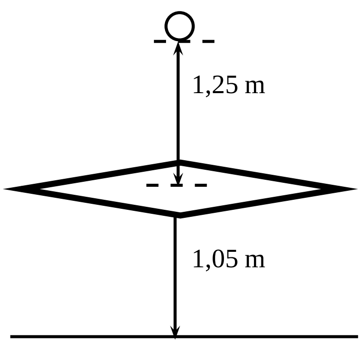
14.1 the speed with which the ball left the bat.

14.2 the impulse exerted by the bat on the ball.

14.3 the impulse exerted by the ball on the bat.

14.4 for how long the ball is in the air.

15. A glass plate is mounted horizontally 1,05 m above the ground. An iron ball of mass 0,4 kg is released from rest and falls a distance of 1,25 m before striking the glass plate and breaking it. The total time taken from release to hitting the ground is recorded as 0,80 s. Assume that the time taken to break the plate is negligible.



15.1 Calculate the speed at which the ball strikes the glass plate.

15.2 Show that the speed of the ball immediately after breaking the plate is 2,0 m·s$^{-1}$.

15.3 Calculate the magnitude and give the direction of the change of momentum which the ball experiences during its contact with the glass plate.

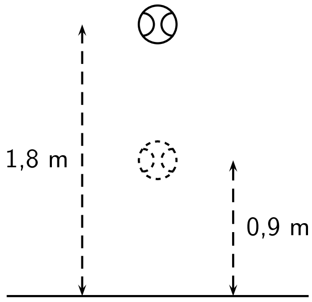
15.4 Give the magnitude and direction of the impulse which the glass plate experiences when the ball hits it.

16. [SC 2004/11 HG1]A cricket ball, mass 175 g is thrown directly towards a player at a velocity of 12 m·s$^{-1}$. It is hit back in the opposite direction with a velocity of 30 m·s$^{-1}$. The ball is in contact with the bat for a period of 0,05 s.

16.1 Calculate the impulse of the ball.

16.2 Calculate the magnitude of the force exerted by the bat on the ball.

17. [IEB 2005/11 HG1] A ball bounces to a vertical height of 0,9 m when it is dropped from a height of 1,8 m. It rebounds immediately after it strikes the ground, and the effects of air resistance are negligible.




17.1 How long (in s) does it take for the ball to hit the ground after it has been dropped?

17.2 At what speed does the ball strike the ground?

17.3 At what speed does the ball rebound from the ground?

17.4 How long (in s) does the ball take to reach its maximum height after the bounce?

17.5 Draw a velocity-time graph for the motion of the ball from the time it is dropped to the time when it rebounds to 0,9 m. Clearly, show the following on the graph:

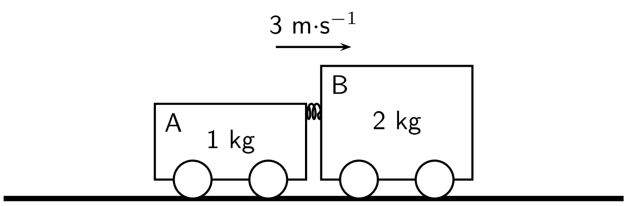
i. the time when the ball hits the ground
ii. the time when it reaches 0,9 m
iii. the velocity of the ball when it hits the ground, and
iv. the velocity of the ball when it rebounds from the ground.

18. [SC 2002/11 HG1] In a railway shunting yard, a locomotive of mass 4 000 kg, travelling due east at a velocity of 1,5 m·s$^{-1}$, collides with a stationary goods wagon of mass 3 000 kg in an attempt to couple with it. The coupling fails and instead the goods wagon moves due east with a velocity of 2,8 m·s$^{-1}$.

18.1 Calculate the magnitude and direction of the velocity of the locomotive immediately after collision.

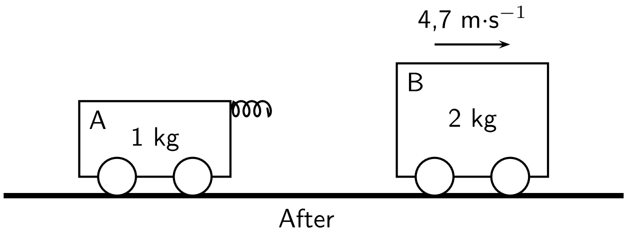
18.2 Name and state in words the law you used to answer question (18a)

19. [SC 2005/11 SG1] A combination of trolley A (fitted with a spring) of mass 1 kg, and trolley B of mass 2 kg, moves to the right at 3 m·s$^{-1}$ along a frictionless, horizontal surface. The spring is kept compressed between the two trolleys.




Before

While the combination of the two trolleys is moving at 3 m·s$^{-1}$ , the spring is released and when it has expanded completely, the 2 kg trolley is then moving to the right at 4,7 m·s$^{-1}$ as shown below.




19.1 State, in words, the principle of *conservation of linear momentum*.

19.2 Calculate the magnitude and direction of the velocity of the 1 kg trolley immediately after the spring has expanded completely.

20. [IEB 2002/11 HG1] A ball bounces back from the ground. Which of the following statements is true of this event?

20.1 The magnitude of the change in momentum of the ball is equal to the magnitude of the change in momentum of the Earth.

20.2 The magnitude of the impulse experienced by the ball is greater than the magnitude of the impulse experienced by the Earth.

20.3 The speed of the ball before the collision will always be equal to the speed of the ball after the collision.

20.4 Only the ball experiences a change in momentum during this event.

21. [SC 2002/11 SG] A boy is standing in a small stationary boat. He throws his schoolbag, mass 2 kg, horizontally towards the jetty with a velocity of 5 m·s$^{-1}$. The *combined mass* of the boy and the boat is 50 kg.

21.1 Calculate the magnitude of the horizontal momentum of the bag immediately after the boy has thrown it.

21.2 Calculate the velocity (magnitude and direction) of the *boat-and-boy* immediately after the bag is thrown.

### Torque and levers

1. State whether each of the following statements are true or false. If the statement is false, rewrite the statement correcting it.

1.1 The torque tells us what the turning effect of a force is.

1.2 To increase the mechanical advantage of a spanner you need to move the effort closer to the load.

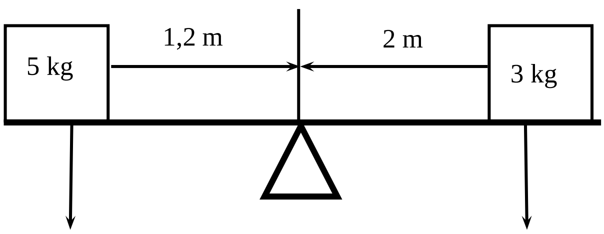
1.3 A class 2 lever has the effort between the fulcrum and the load.

1.4 An object will be in equilibrium if the clockwise moment and the anticlockwise moments are equal.

1.5 Mechanical advantage is a measure of the difference between the load and the effort.

1.6 The force times the perpendicular distance is called the mechanical advantage.

2. Study the diagram below and determine whether the seesaw is balanced. Show all your calculations.



3. Two children are playing on a seesaw. Tumi has a weight of 200 N and Thandi weighs 240 N. Tumi is sitting at a distance of 1,2 m from the pivot.

   3.1 What type of lever is a seesaw?

   3.2 Calculate the moment of the force that Tumi exerts on the seesaw.

   3.3 At what distance from the pivot should Thandi sit to balance the seesaw?

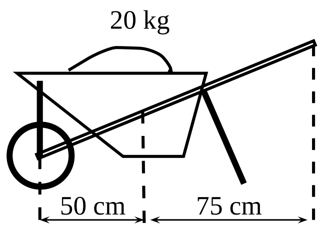
4. An applied force of 25 N is needed to open the cap of a glass bottle using a bottle opener. The distance between the applied force and the fulcrum is 10 cm and the distance between the load and the fulcrum is 1 cm.

   4.1 What type of lever is a bottle opener? Give a reason for your answer.

   4.2 Calculate the mechanical advantage of the bottle opener.

   4.3 Calculate the force that the bottle cap is exerting.

5. Determine the force needed to lift the 20 kg load in the wheelbarrow in the diagram below.




6. A body builder picks up a weight of 50 N using his right hand. The distance between the body builder's hand and his elbow is 45 cm. The distance between his elbow and where his muscles are attached to his forearm is 5 cm.

   6.1 What type of lever is the human arm? Explain your answer using a diagram.

   6.2 Determine the force his muscles need to apply to hold the weight steady.

# Chapter 13

# Geometrical Optics - Grade 11

## 13.1 Introduction

In Grade 10, we studied how light is reflected and refracted. This chapter builds on what you have learnt in Grade 10. You will learn about lenses, how the human eye works as well as how telescopes and microscopes work.

## 13.2 Lenses

In this section we will discuss properties of **thin lenses**. In Grade 10, you learnt about two kinds of mirrors: concave mirrors which were also known as converging mirrors and convex mirrors which were also known as diverging mirrors. Similarly, there are two types of lenses: converging and diverging lenses.

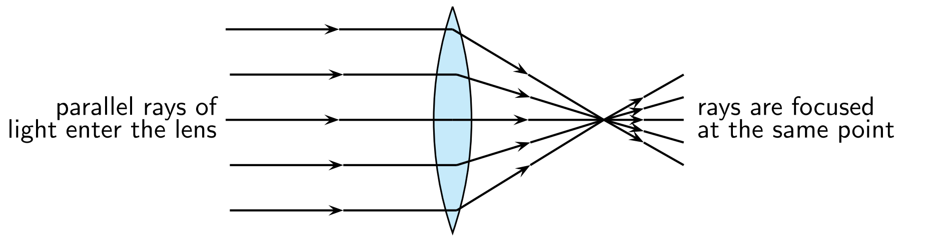
We have learnt how light travels in different materials, and we are now ready to learn how we can control the direction of light rays. We use *lenses* to control the direction of light. When light enters a lens, the light rays bend or change direction as shown in Figure 13.1.

**Definition: Lens**
A lens is any transparent material (e.g. glass) of an appropriate shape that can take parallel rays of incident light and either converge the rays to a point or diverge the rays from a point.

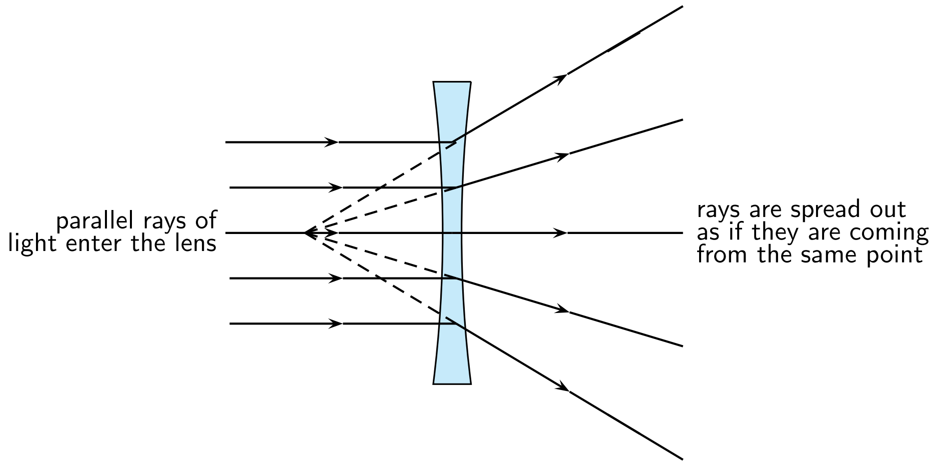
Some lenses will focus light rays to a single point. These lenses are called converging or concave lenses. Other lenses spread out the light rays so that it looks like they all come from the same point. These lenses are called diverging or convex lenses. Lenses change the direction of light rays by *refraction*. They are designed so that the image appears in a certain place or as a certain size. Lenses are used in eyeglasses, cameras, microscopes, and telescopes. You also have lenses in your eyes!

**Definition: Converging Lenses**
Converging lenses converge parallel rays of light and are thicker in the middle than at the edges.

**Definition: Diverging Lenses**
Diverging lenses diverge parallel rays of light and are thicker at the edges than in the middle.




(a) A converging lens will focus the rays that enter the lens




(b) A diverging lens will spread out the rays that enter the lens

Figure 13.1: The behaviour of parallel light rays entering either a converging or diverging lens.

Examples of converging and diverging lenses are shown in Figure 13.2.

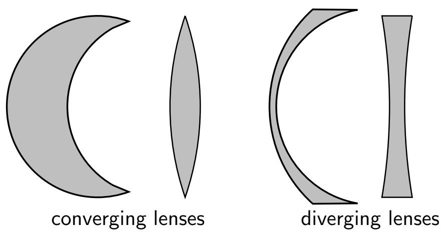
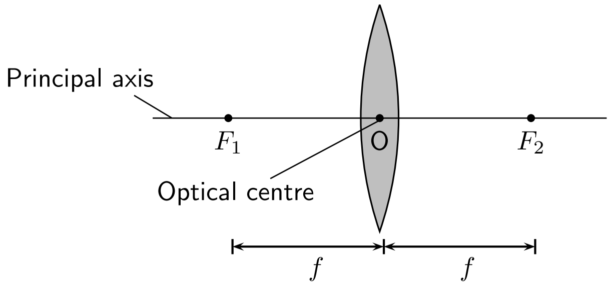


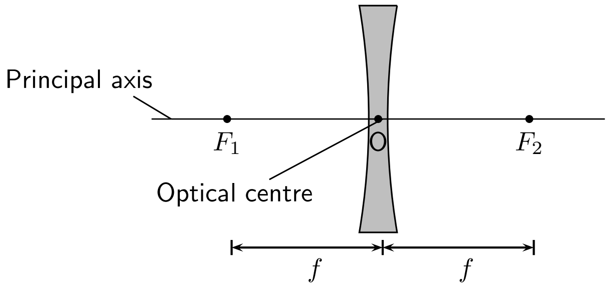

Figure 13.2: Types of lenses

Before we study lenses in detail, there are a few important terms that must be defined. Figure 13.3 shows important lens properties:

- The **principal axis** is the line which runs horizontally straight through the *optical centre* of the lens. It is also sometimes called the *optic axis*.
- The **optical centre** (O) of a convex lens is usually the centre point of the lens. The direction of all light rays which pass through the optical centre, remains unchanged.
- The **focus** or **focal point** of the lens is the position on the *principal axis* where all light rays which run *parallel* to the principal axis through the lens converge (come together) at a point. Since light can pass through the lens either from right to left or left to right, there is a focal point on each side of the lens ($F_1$ and $F_2$), at the same distance from the optical centre in each direction. (**Note:** the plural form of the word focus is *foci*.)
- The **focal length** ($f$) is the distance between the *optical centre* and the *focal point*.




(a) converging lens




(b) diverging lens

Figure 13.3: Properties of lenses.

### 13.2.1 Converging Lenses

We will only discuss double convex converging lenses as shown in Figure 13.4. Converging lenses are thinner on the outside and thicker on the inside.

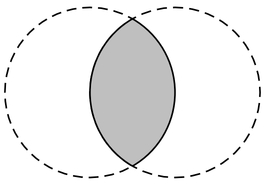


Figure 13.4: A double convex lens is a converging lens.

Figure 13.5 shows a convex lens. Light rays traveling through a **convex** lens are bent **towards** the principal axis. For this reason, convex lenses are called **converging** lenses.

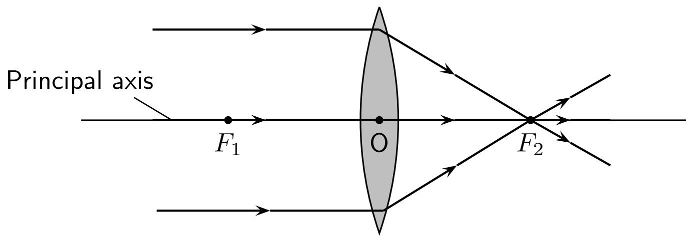



Figure 13.5: Light rays bend towards each other or *converge* when they travel through a convex lens. $F_1$ and $F_2$ are the foci of the lens.

The type of images created by a convex lens is dependent on the position of the object. We will examine the following cases:

1. the object is placed at a distance greater than $2f$ from the lens
2. the object is placed at a distance equal to $2f$ from the lens
3. the object is placed at a distance between $2f$ and $f$ from the lens
4. the object is placed at a distance less than $f$ from the lens

We examine the properties of the image in each of these cases by drawing ray diagrams. We can find the image by tracing the path of three light rays through the lens. Any two of these rays will show us the location of the image. You can use the third ray to check the location.

---

**Activity :: Experiment : Lenses A**

**Aim:**
To determine the focal length of a convex lens.

**Method:**

1. Using a distant object from outside, adjust the position of the convex lens so that it gives the smallest possible focus on a sheet of paper that is held parallel to the lens.
2. Measure the distance between the lens and the sheet of paper as accurately as possible.

**Results:**
The focal length of the lens is ________________ cm

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**Activity :: Experiment : Lenses B**

**Aim:**
To investigate the position, size and nature of the image formed by a convex lens.

**Method:**

1. Set up the candle, the lens from Experiment Lenses A in its holder and the screen in a straight line on the metre rule. Make sure the lens holder is on the 50 cm mark.

   From your knowledge of the focal length of your lens, note where $f$ and $2f$ are on both sides of the lens.
2. Using the position indicated on the table below, start with the candle at a position that is greater than $2f$ and adjust the position of the screen until a sharp focused image is obtained. Note that there are two positions for which a sharp focused image will not be obtained on the screen. When this is so, remove the screen and look at the candle through the lens.
3. Fill in the relevant information on the table below

**Results:**

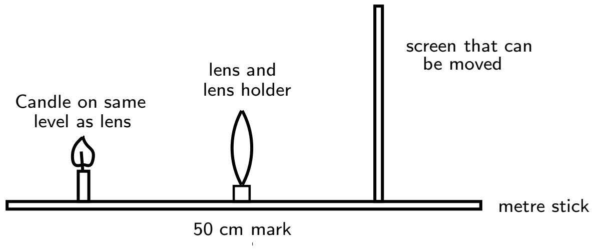



Figure 13.6: Experimental setup for investigation.

| Relative position of object | Relative position of image | Image upright or inverted | Relative size of image | Nature of image |
|---|---|---|---|---|
| Beyond $2f$ _______ cm | | | | |
| At $2f$ _______ cm | | | | |
| Between $2f$ and $f$ _______ cm | | | | |
| At $f$ _______ cm | | | | |
| Between $f$ and the lens _______ cm | | | | |

**QUESTIONS:**

1. When a convex lens is being used:
   1.1 A real inverted image is formed when an object is placed ________________
   1.2 No image is formed when an object is placed ________________
   1.3 An upright, enlarged, virtual image is formed when an object is placed ________________
2. Write a conclusion for this investigation.

________________________________________________

________________________________________________

**Activity :: Experiment : Lenses C**

**Aim:**

To determine the mathematical relationship between $d_0, d_i$ and $f$ for a lens.

**Method:**

1. Using the same arrangement as in Experiment Lenses B, place the object (candle) at the distance indicated from the lens.
2. Move the screen until a clear sharp image is obtained. Record the results on the table below.

**Results:**

$f$ = focal length of lens

$d_0$ = object distance

$d_i$ = image distance

| Object distance $d_0$ (cm) | Image distance $d_i$ (cm) | $\frac{1}{d_0}$ (cm$^{-1}$) | $\frac{1}{d_i}$ (cm$^{-1}$) | $\frac{1}{d_0}+\frac{1}{d_i}$ (cm$^{-1}$) |
|---|---|---|---|---|
| 25,0 | | | | |
| 20,0 | | | | |
| 18,0 | | | | |
| 15,0 | | | | |
| | | | | Average = ______ |

$$\text{Reciprocal of average} = \left(\frac{1}{\frac{1}{d_0}+\frac{1}{d_i}}\right) = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \text{ (a)}$$

$$\text{Focal length of lens} = \text{\_\_\_\_\_\_\_\_\_\_\_\_} \text{ (b)}$$

**QUESTIONS:**

1. Compare the values for (a) and (b) above and explain any similarities or differences
2. What is the name of the mathematical relationship between $d_0$, $d_i$ and $f$?
3. Write a conclusion for this part of the investigation.

________________________________________________

**Drawing Ray Diagrams for Converging Lenses**

The three rays are labelled $R_1$, $R_2$ and $R_3$. The ray diagrams that follow will use this naming convention.

1. The first ray ($R_1$) travels from the object to the lens *parallel* to the principal axis. This ray is bent by the lens and travels through the **focal point**.
2. *Any* ray travelling parallel to the principal axis is bent through the focal point.

3. If a light ray passes through a focal point *before* it enters the lens, then it will leave the lens *parallel* to the principal axis. The second ray ($R_2$) is therefore drawn to pass through the focal point before it enters the lens.

4. A ray that travels through the centre of the lens does not change direction. The third ray ($R_3$) is drawn through the centre of the lens.

5. The point where all three of the rays ($R_1$, $R_2$ and $R_3$) intersect is the **image** of the point where they all started. The image will form at this point.

**Important:** In ray diagrams, lenses are drawn like this:

Convex lens: 

Concave lens: 

**CASE 1:**
**Object placed at a distance greater than $2f$ from the lens**

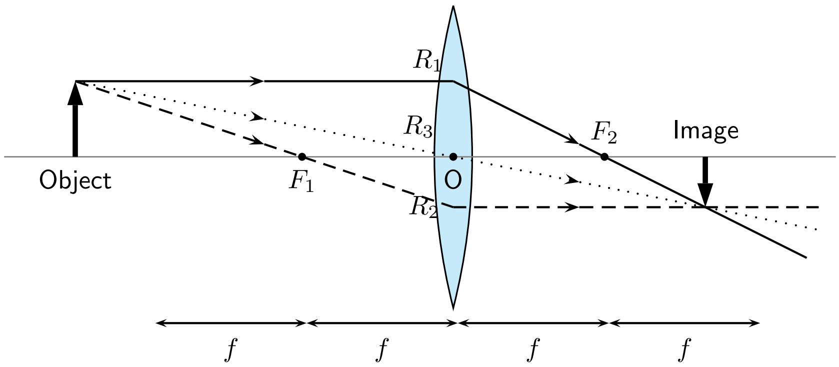



Figure 13.7: An object is placed at a distance greater than $2f$ away from the converging lens. Three rays are drawn to locate the image, which is real, smaller than the object and inverted.

We can locate the position of the image by drawing our three rays. $R_1$ travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. $R_2$ passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. $R_3$ travels through the center of the lens and does not change direction. The point where $R_1$, $R_2$ and $R_3$ intersect is the image of the point where they all started.

The image of an object placed at a distance greater than $2f$ from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a **real image** because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is also smaller than the object and is located closer to the lens than the object.

**Important:** In reality, light rays come from *all* points along the length of the object. In ray diagrams we only draw three rays (all starting at the top of the object) to keep the diagram clear and simple.

**CASE 2:**
**Object placed at a distance equal to $2f$ from the lens**

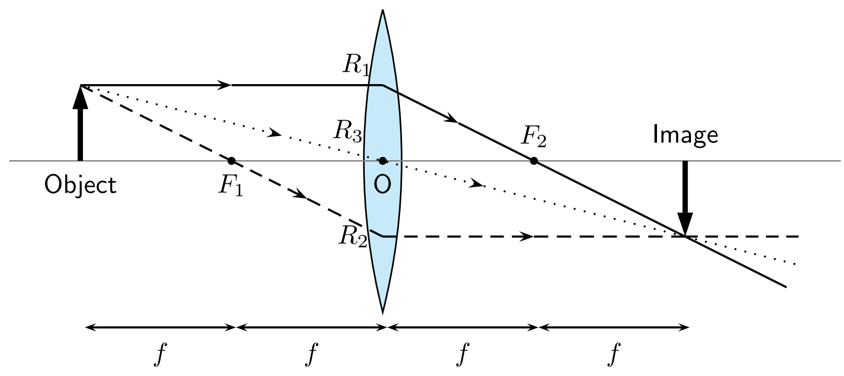



Figure 13.8: An object is placed at a distance equal to $2f$ away from the converging lens. Three rays are drawn to locate the image, which is real, the same size as the object and inverted.

We can locate the position of the image by drawing our three rays. $R_1$ travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. $R_2$ passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. $R_3$ travels through the center of the lens and does not change direction. The point where $R_1$, $R_2$ and $R_3$ intersect is the image of the point where they all started.

The image of an object placed at a distance equal to $2f$ from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a **real image** because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is the same size as the object and is located at a distance $2f$ away from the lens.

**CASE 3:**
**Object placed at a distance between $2f$ and $f$ from the lens**

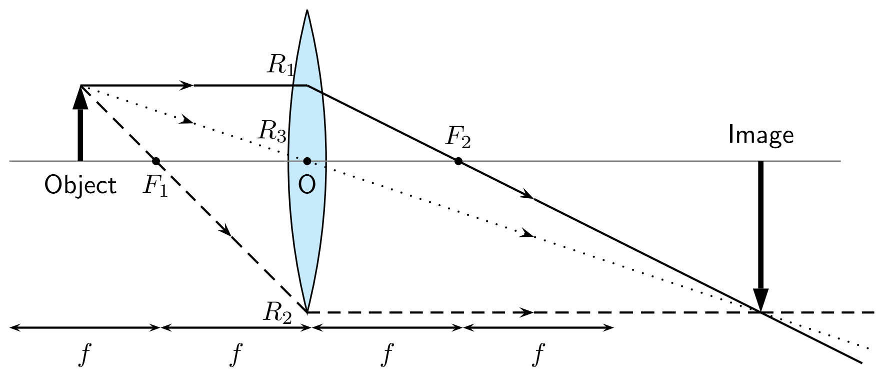



Figure 13.9: An object is placed at a distance between $2f$ and $f$ away from the converging lens. Three rays are drawn to locate the image, which is real, larger than the object and inverted.

We can locate the position of the image by drawing our three rays. $R_1$ travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. $R_2$ passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. $R_3$ travels through the center of the lens and does not change direction. The point where $R_1$, $R_2$ and $R_3$ intersect is the image of the point where they all started.

The image of an object placed at a distance between $2f$ and $f$ from the lens is upside down or *inverted*. This is because the rays which began at the top of the object, *above* the principal axis, after passing through the lens end up *below* the principal axis. The image is called a real image because it is on the opposite side of the lens to the object and you can trace all the light rays directly from the image back to the object.

The image is larger than the object and is located at a distance greater than $2f$ away from the lens.

**CASE 4:**
**Object placed at a distance less than $f$ from the lens**

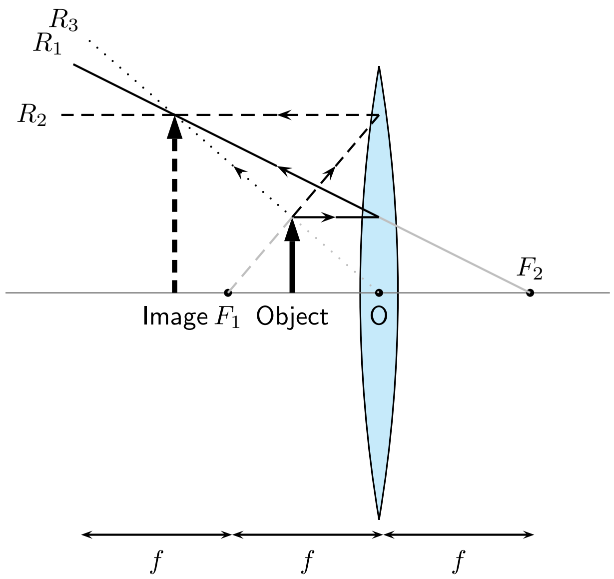



Figure 13.10: An object is placed at a distance less than $f$ away from the converging lens. Three rays are drawn to locate the image, which is virtual, larger than the object and upright.

We can locate the position of the image by drawing our three rays. $R_1$ travels from the object to the lens parallel to the principal axis and is bent by the lens and then travels through the focal point. $R_2$ passes through the focal point before it enters the lens and therefore must leave the lens parallel to the principal axis. $R_3$ travels through the center of the lens and does not change direction. The point where $R_1$, $R_2$ and $R_3$ intersect is the image of the point where they all started.

The image of an object placed at a distance less than $f$ from the lens is upright. The image is called a **virtual image** because it is on the same side of the lens as the object and you *cannot* trace all the light rays directly from the image back to the object.

The image is larger than the object and is located further away from the lens than the object.

*Extension: The thin lens equation and magnification*

**The Thin Lens Equation**

We can find the position of the image of a lens mathematically as there is mathematical relation between the object distance, image distance, and focal length. The equation is:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where $f$ is the focal length, $d_o$ is the object distance and $d_i$ is the image distance.

The object distance $d_o$ is the distance from the object to the lens. $d_o$ is positive if the object is on the same side of the lens as the light rays enter the lens. This should make sense: we expect the light rays to travel from the object to the lens. The image distance $d_i$ is the distance from the lens to the image. Unlike mirrors, which reflect light back, lenses refract light through them. We expect to find the image on the same side of the lens as the light leaves the lens. If this is the case, then $d_i$ is *positive* and the image is **real** (see Figure 13.9). Sometimes the image will be on the same side of the lens as the light rays enter the lens. Then $d_i$ is *negative* and the image is **virtual** (Figure 13.10). If we know any two of the three quantities above, then we can use the Thin Lens Equation to solve for the third quantity.

**Magnification**

It is possible to calculate the magnification of an image. The magnification is how much *bigger* or *smaller* the image is than the object.

$$m = -\frac{d_i}{d_o}$$

where $m$ is the magnification, $d_o$ is the object distance and $d_i$ is the image distance.

If $d_i$ and $d_o$ are both positive, the magnification is negative. This means the image is inverted, or upside down. If $d_i$ is negative and $d_o$ is positive, then the image is not inverted, or right side up. If the absolute value of the magnification is *greater than one*, the image is *larger* than the object. For example, a magnification of -2 means the image is *inverted* and *twice as big* as the object.

**Worked Example 97: Using the lens equation**

**Question:** An object is placed 6 cm from a converging lens with a focal point of 4 cm.

1. Calculate the position of the image
2. Calculate the magnification of the lens
3. Identify three properties of the image

**Answer**

**Step 1 : Identify what is given and what is being asked**

$$\begin{aligned} f &= 4 \text{ cm} \\ d_o &= 6 \text{ cm} \\ d_i &= ? \\ m &= ? \end{aligned}$$

Properties of the image are required.

**Step 2 : Calculate the image distance ($d_i$)**

$$\begin{aligned} \frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\ \frac{1}{4} &= \frac{1}{6} + \frac{1}{d_i} \\ \frac{1}{4} - \frac{1}{6} &= \frac{1}{d_i} \\ \frac{3-2}{12} &= \frac{1}{d_i} \\ d_i &= 12 \text{ cm} \end{aligned}$$

**Step 3 : Calculate the magnification**

$$\begin{aligned} m &= -\frac{d_i}{d_o} \\ &= -\frac{12}{6} \\ &= -2 \end{aligned}$$

**Step 4 : Write down the properties of the image**

The image is real, $d_i$ is positive, inverted (because the magnification is negative) and enlarged (magnification is $> 1$)

**Worked Example 98: Locating the image position of a convex lens: I**

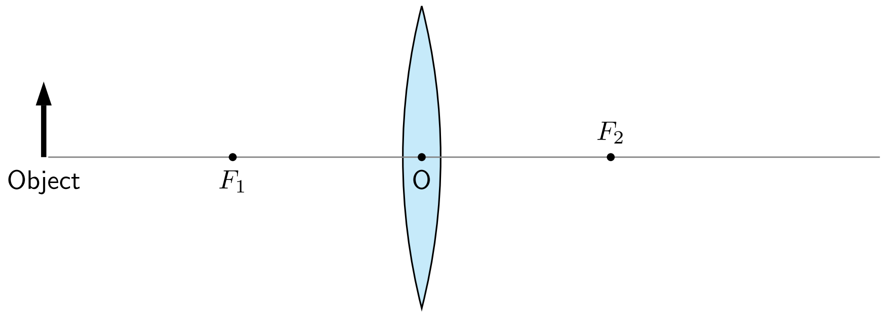
**Question:** An object is placed 5 cm to the left of a converging lens which has a focal length of 2,5 cm.

1. What is the position of the image?
2. Is the image real or virtual?

**Answer**

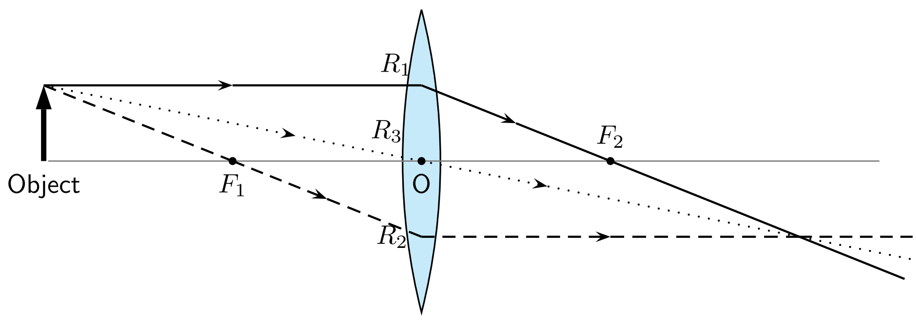
**Step 1 : Set up the ray diagram**

Draw the lens, the object and mark the focal points.



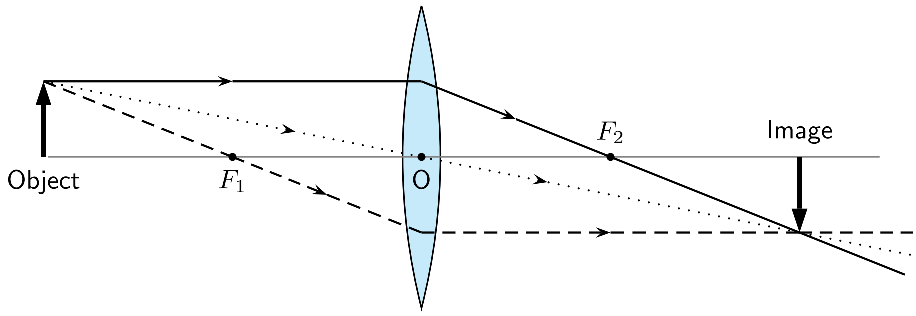

**Step 2 : Draw the three rays**

- $R_1$ goes from the top of the object parallel to the principal axis, through the lens and through the focal point $F_2$ on the other side of the lens.
- $R_2$ goes from the top of the object through the focal point $F_1$, through the lens and out parallel to the principal axis.
- $R_3$ goes from the top of the object through the optical centre with its direction unchanged.




**Step 3 : Find the image**

The image is at the place where all the rays intersect. Draw the image.



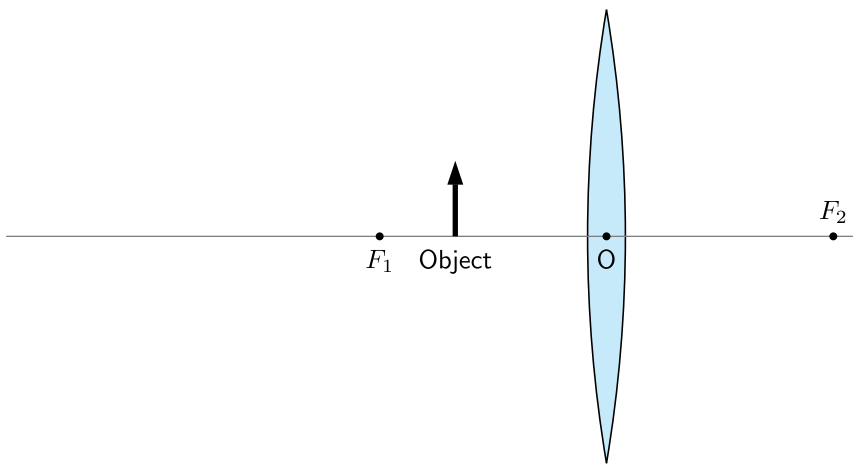

**Step 4 : Measure the distance between the lens and the image**
The image is 5 cm away from the lens, on the opposite side of the lens to the object.
**Step 5 : Is the image virtual or real?**
Since the image is on the opposite side of the lens to the object, the image is real.

**Worked Example 99: Locating the image position of a convex lens: II**

**Question:** An object, 1 cm high, is placed 2 cm to the left of a converging lens which has a focal length of 3,0 cm. The image is found also on the left side of the lens.

1. Is the image real or virtual?
2. What is the position and height of the image?

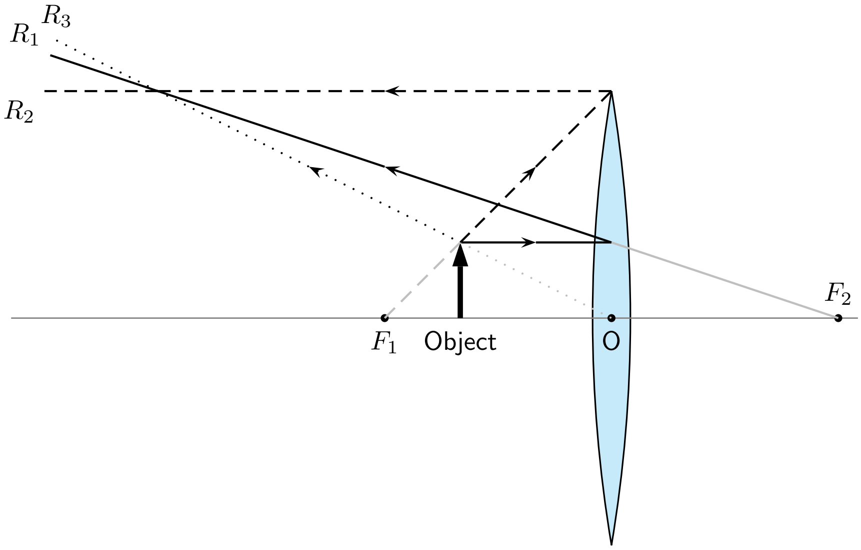
**Answer**
**Step 1 : Draw the picture to set up the problem**
Draw the lens, principal axis, focal points and the object.



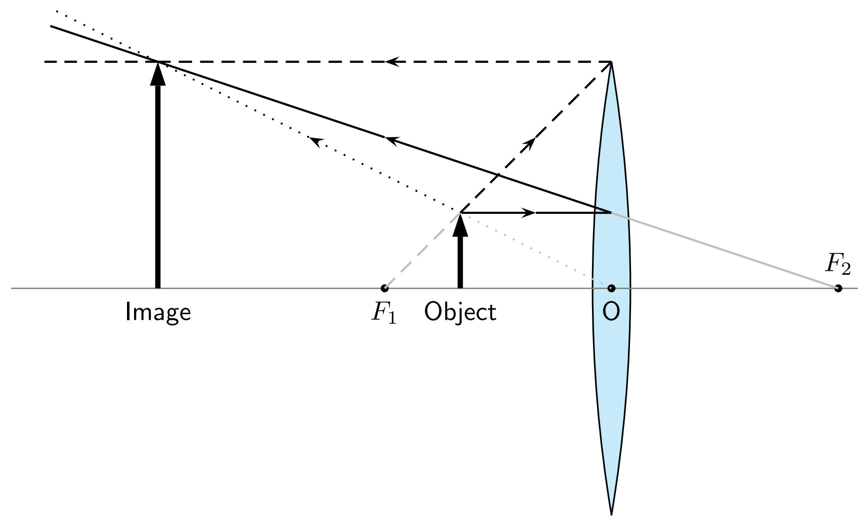

**Step 2 : Draw the three rays to locate image**

- $R_1$ goes from the top of the object parallel to the principal axis, through the lens and through the focal point $F_2$ on the other side of the lens.
- $R_2$ is the light ray which should go through the focal point $F_1$ but the object is placed *after* the focal point! This is not a problem, just trace the line from the focal point $F_1$, through the top of the object, to the lens. This ray then leaves the lens parallel to the principal axis.
- $R_3$ goes from the top of the object through the optical centre with its direction unchanged.

- Do not write $R_1$, $R_2$ and $R_3$ on your diagram, otherwise it becomes too cluttered.
- Since the rays do not intersect on the right side of the lens, we need to trace them backwards to find the place where they do come together (these are the light gray lines). Again, this is the position of the image.




**Step 3 : Draw the image**




**Step 4 : Measure distance to image**
The image is 6 cm away from the lens, on the same side as the object.
**Step 5 : Measure the height of the image**
The image is 3 cm high.
**Step 6 : Is image real or virtual?**
Since the image is on the same side of the lens as the object, the image is virtual.

?

**Exercise: Converging Lenses**

1. Which type of lens can be used as a magnifying glass? Draw a diagram to show how it works. An image of the sun is formed at the principal focus of a magnifying glass.

2. In each case state whether a real or virtual image is formed:
   2.1 Much further than $2f$
   2.2 Just further than $2f$
   2.3 At $2f$
   2.4 Between $2f$ and $f$
   2.5 At $f$
   2.6 Between $f$ and 0

   Is a virtual image always inverted?
3. An object stands 50 mm from a lens (focal length 40 mm). Draw an accurate sketch to determine the position of the image. Is it enlarged or shrunk; upright or inverted?
4. Draw a scale diagram (scale: 1 cm = 50 mm) to find the position of the image formed by a convex lens with a focal length of 200 mm. The distance of the object is 100 mm and the size of the object is 50 mm. Determine whether the image is enlarged or shrunk. What is the height of the image? What is the magnification?
5. An object, 20 mm high, is 80 mm from a convex lens with focal length 50 mm. Draw an accurate scale diagram and find the position and size of the image, and hence the ratio between the image size and object size.
6. An object, 50 mm high, is placed 100 mm from a convex lens with a focal length of 150 mm. Construct an accurate ray diagram to determine the nature of the image, the size of the image and the magnification. Check your answer for the magnification by using a calculation.
7. What would happen if you placed the object right at the focus of a converging lens? Hint: Draw the picture.

### 13.2.2 Diverging Lenses

We will only discuss double concave diverging lenses as shown in Figure 13.11. Concave lenses are thicker on the outside and thinner on the inside.

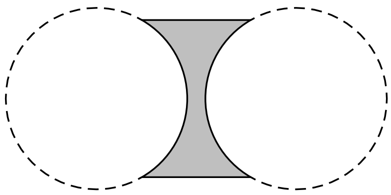


Figure 13.11: A double concave lens is a diverging lens.

Figure 13.12 shows a concave lens with light rays travelling through it. You can see that concave lenses have the opposite curvature to convex lenses. This causes light rays passing through a concave lens to **diverge** or be spread out *away* from the principal axis. For this reason, concave lenses are called **diverging lenses.** Images formed by concave lenses are *always* virtual.

Unlike converging lenses, the type of images created by a concave lens is not dependent on the position of the object. The image is *always* upright, smaller than the object, and located closer to the lens than the object.

We examine the properties of the image by drawing ray diagrams. We can find the image by tracing the path of three light rays through the lens. Any two of these rays will show us the

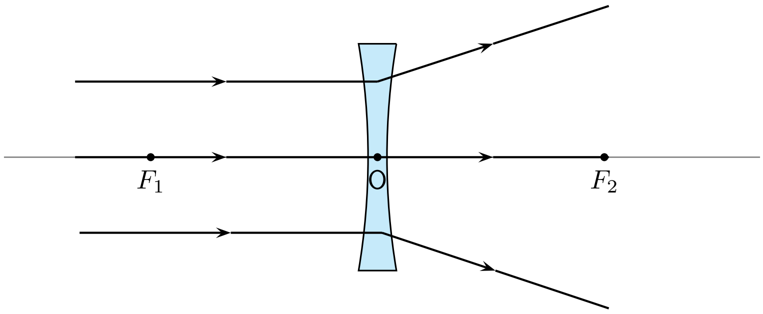



Figure 13.12: Light rays bend *away from* each other or *diverge* when they travel through a concave lens. $F_1$ and $F_2$ are the foci of the lens.

location of the image. You can use the third ray to check the location, but it is not necessary to show it on your diagram.

**Drawing Ray Diagrams for Diverging Lenses**
Draw the three rays starting at the top of the object.

1. Ray $R_1$ travels parallel to the principal axis. The ray bends and lines up with a focal point. However, the concave lens is a *diverging* lens, so the ray must line up with the focal point on the same side of the lens where light rays enter it. This means that we must project an imaginary line backwards through that focal point ($F_1$) (shown by the dashed line extending from $R_1$).

2. Ray $R_2$ points towards the focal point $F_2$ on the opposite side of the lens. When it hits the lens, it is bent parallel to the principal axis.

3. Ray $R_3$ passes through the optical center of the lens. Like for the convex lens, this ray passes through with its direction unchanged.

4. We find the image by locating the point where the rays meet. Since the rays diverge, they will only meet if projected backward to a point on the same side of the lens as the object. This is why concave lenses *always* have virtual images. (Since the light rays do not actually meet at the image, the image cannot be real.)

Figure 13.13 shows an object placed at an arbitrary distance from the diverging lens.

We can locate the position of the image by drawing our three rays for a diverging lens.

Figure 13.13 shows that the image of an object is upright. The image is called a **virtual image** because it is on the same side of the lens as the object.

The image is smaller than the object and is closer to the lens than the object.

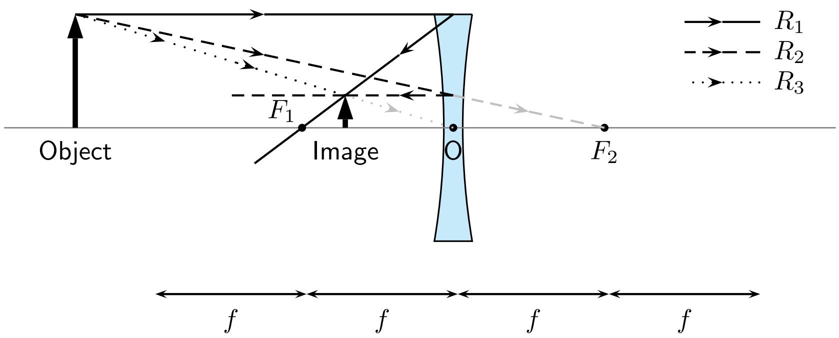


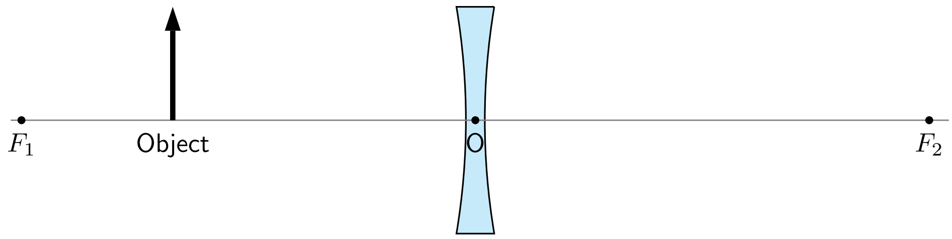

Figure 13.13: Three rays are drawn to locate the image, which is virtual, smaller than the object and upright.

**Worked Example 100: Locating the image position for a diverging lens: I**

**Question:** An object is placed 4 cm to the left of a diverging lens which has a focal length of 6 cm.

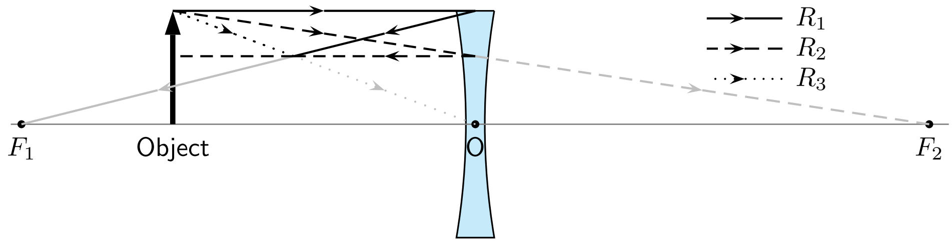
1. What is the position of the image?
2. Is the image real or virtual?

**Answer**
**Step 1 : Set up the problem**
Draw the lens, object, principal axis and focal points.

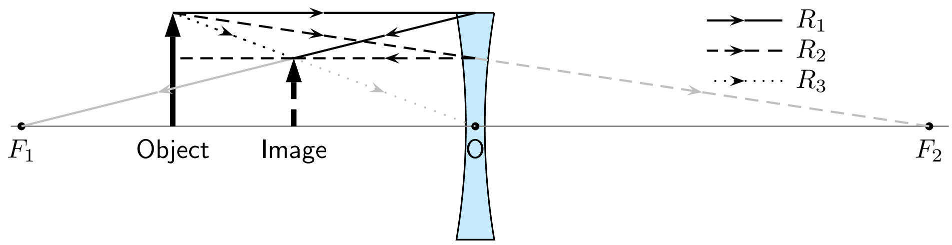



**Step 2 : Draw the three light rays to locate the image**

- $R_1$ goes from the top of the object parallel to the principal axis. To determine the angle it has when it leaves the lens on the other side, we draw the dashed line from the focus $F_1$ through the point where $R_1$ hits the lens. (Remember: for a diverging lens, the light ray on the opposite side of the lens to the object has to bend *away* from the principal axis.)
- $R_2$ goes from the top of the object in the direction of the other focal point $F_2$. After it passes through the lens, it travels parallel to the principal axis.
- $R_3$ goes from the top of the lens, straight through the optical centre with its direction unchanged.
- Just like for converging lenses, the image is found at the position where all the light rays intersect.




**Step 3 : Draw the image**
Draw the image at the point where all three rays intersect.




**Step 4 : Measure the distance to the object**
The distance to the object is 2,4 cm.
**Step 5 : Determine type of object**
The image is on the same side of the lens as the object, and is upright. Therefore it is virtual. (*Remember:* The image from a diverging lens is *always* virtual.)

### 13.2.3 Summary of Image Properties

The properties of the images formed by converging and diverging lenses depend on the position of the object. The properties are summarised in the Table 13.1.

Table 13.1: Summary of image properties for converging and diverging lenses

| | | Image Properties | | | |
|---|---|---|---|---|---|
| **Lens type** | **Object Position** | **Position** | **Orientation** | **Size** | **Type** |
| Converging | $> 2f$ | $< 2f$ | inverted | smaller | real |
| Converging | $2f$ | $2f$ | inverted | same size | real |
| Converging | $> f, < 2f$ | $> 2f$ | inverted | larger | real |
| Converging | $f$ | no image formed | | | |
| Converging | $< f$ | $> f$ | upright | larger | virtual |
| Diverging | any position | $< f$ | upright | smaller | virtual |

?

**Exercise: Diverging Lenses**

1. An object 3 cm high is at right angles to the principal axis of a concave lens of focal length 15 cm. If the distance from the object to the lens is 30 cm, find the distance of the image from the lens, and its height. Is it real or virtual?
2. The image formed by a concave lens of focal length 10 cm is 7,5 cm from the lens and is 1,5 cm high. Find the distance of the object from the lens, and its height.
3. An object 6 cm high is 10 cm from a concave lens. The image formed is 3 cm high. Find the focal length of the lens and the distance of the image from the lens.

## 13.3 The Human Eye

**Activity :: Investigation : Model of the Human Eye**

This demonstration shows that:

1. The eyeball has a spherical shape.
2. The pupil is a small hole in the front and middle of the eye that lets light into the eye.
3. The retina is at the back of the eyeball.
4. The images that we see are formed on the retina.
5. The images on the retina are upside down. The brain inverts the images so that what we see is the right way up.

You will need:

1. a round, clear glass bowl
2. water
3. a sheet of cardboard covered with black paper
4. a sheet of cardboard covered with white paper
5. a small desk lamp with an incandescent light-bulb or a candle and a match

You will have to:

1. Fill the glass bowl with water.
2. Make a small hole in the middle of the black cardboard.
3. Place the black cardboard against one side of the bowl and the white cardboard on the other side of the bowl so that it is opposite the black cardboard.
4. Turn on the lamp (or light the candle).

5. Place the lamp so it is shining through the hole in the black cardboard.
6. Make the room as dark as possible.
7. Move the white cardboard until an image of the light bulb or candle appears on it.

You now have a working model of the human eye.

1. The hole in the black cardboard represents the pupil. The pupil is a small hole in the front of the eyeball that lets light into the eye.
2. The round bowl of water represents the eyeball.
3. The white cardboard represents the retina. Images are projected onto the retina and are then sent to the brain via the optic nerve.

**Tasks**

1. Is the image on the retina right-side up or upside down? Explain why.
2. Draw a simple labelled diagram of the model of the eye showing which part of the eye each part of the model represents.

### 13.3.1 Structure of the Eye

Eyesight begins with lenses. As light rays enter your eye, they pass first through the **cornea** and then through the **crystalline lens**. These form a double lens system and focus light rays onto the back wall of the eye, called the **retina**. **Rods** and **cones** are nerve cells on the retina that transform light into electrical signals. These signals are sent to the brain via the **optic nerve**. A cross-section of the eye is shown in Figure 13.14.

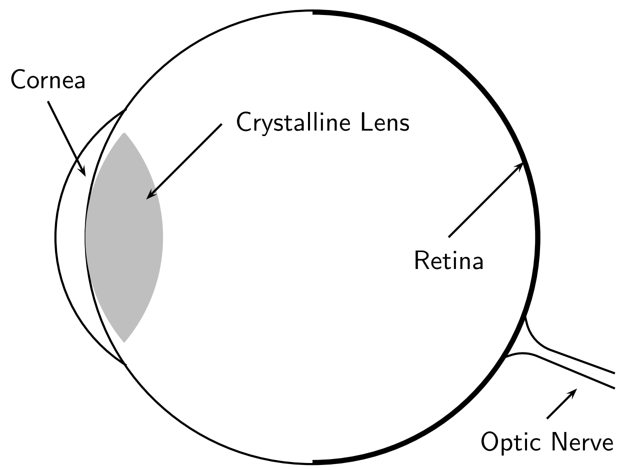



Figure 13.14: A cross-section of the human eye.

For clear vision, the image must be formed right on the retina, not in front of or behind it. To accomplish this, you may need a long or short focal length, depending on the object distance. How do we get the exact right focal length we need? Remember that the lens system has two parts. The cornea is fixed in place but the crystalline lens is flexible – it can change shape. When the shape of the lens changes, its focal length also changes. You have muscles in your eye called **ciliary muscles** that control the shape of the crystalline lens. When you focus your gaze on something, you are squeezing (or relaxing) these muscles. This process of **accommodation** changes the focal length of the lens and allows you to see an image clearly.

The lens in the eye creates a real image that is smaller than the object and is inverted (Figure 13.15).

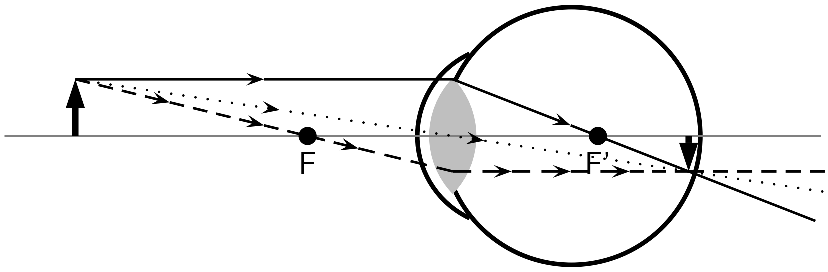



Figure 13.15: Normal eye

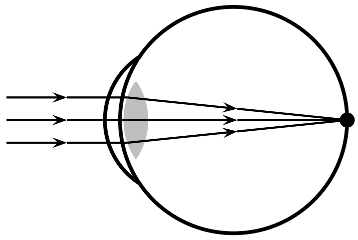


Figure 13.16: Normal eye

### 13.3.2 Defects of Vision

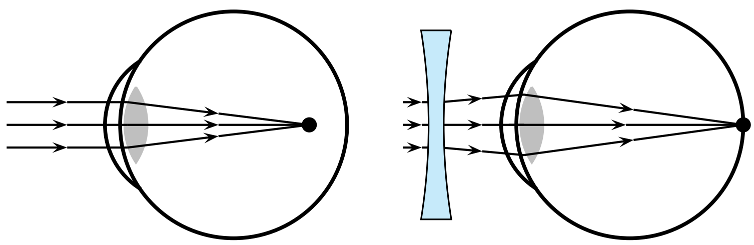
In a normal eye the image is focused on the retina.

If the muscles in the eye are unable to accommodate adequately, the image will not be in focus. This leads to problems with vision. There are three basic conditions that arise:

1. short-sightedness
2. long-sightedness
3. astigmatism

#### Short-sightedness

Short-sightedness or **myopia** is a defect of vision which means that the image is focused in front of the retina. Close objects are seen clearly but distant objects appear blurry. This condition can be corrected by placing a diverging lens in front of the eye. The diverging lens spreads out light rays before they enter the eye. The situation for short-sightedness and how to correct it is shown in Figure 13.17.



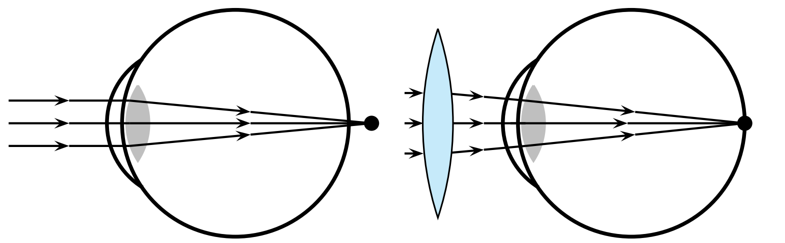
(a) Short-sightedness : Light rays are focused in front of the retina.

(b) Short-sightedness corrected by a diverging lens.

Figure 13.17: Short-sightedness

#### Long-sightedness

Long-sightedness or **hyperopia** is a defect of vision which means that the image is focused in behind the retina. People with this condition can see distant objects clearly, but not close ones. A converging lens in front of the eye corrects long-sightedness by converging the light rays slightly before they enter the eye. Reading glasses are an example of a converging lens used to correct long-sightedness.



(a) Long-sightedness : Light rays are focused in behind the retina. (b) Long-sightedness corrected by a converging lens.

Figure 13.18: Long-sightedness

#### Astigmatism

Astigmatism is characterised by a cornea or lens that is not spherical, but is more curved in one plane compared to another. This means that horizontal lines may be focused at a different point to vertical lines. Astigmatism is corrected by a special lens, which has different focal lengths in the vertical and horizontal planes.

## 13.4 Gravitational Lenses

Einstein's Theory of General Relativity predicts that light that passes close to very heavy objects like galaxies, black holes and massive stars will be bent. These massive objects therefore act as a kind of lens that is known as a *gravitational lens*. Gravitational lenses distort and change the apparent position of the image of stars.

If a heavy object is acting as a gravitational lens, then an observer from Earth will see many images of a distant star (Figure 13.19).

## 13.5 Telescopes

We have seen how a simple lens can be used to correct eyesight. Lenses and mirrors are also combined to magnify (or make bigger) objects that are far away.

Telescopes use combinations of lenses to gather and focus light. However, telescopes collect light from objects that are large but far away, like planets and galaxies. For this reason, telescopes are the tools of astronomers. **Astronomy** is the study of objects outside the Earth, like stars, planets, galaxies, comets, and asteroids.

Usually the object viewed with a telescope is very far away. There are two types of objects: those with a detectable diameter, such as the moon, and objects that appear as points of light, like stars.

There are many kinds of telescopes, but we will look at two basic types: reflecting and refracting.

### 13.5.1 Refracting Telescopes

A **refracting telescope** like the one pictured in Figure 13.20 uses two convex lenses to enlarge an image. The refracting telescope has a large primary lens with a long focal length to gather a

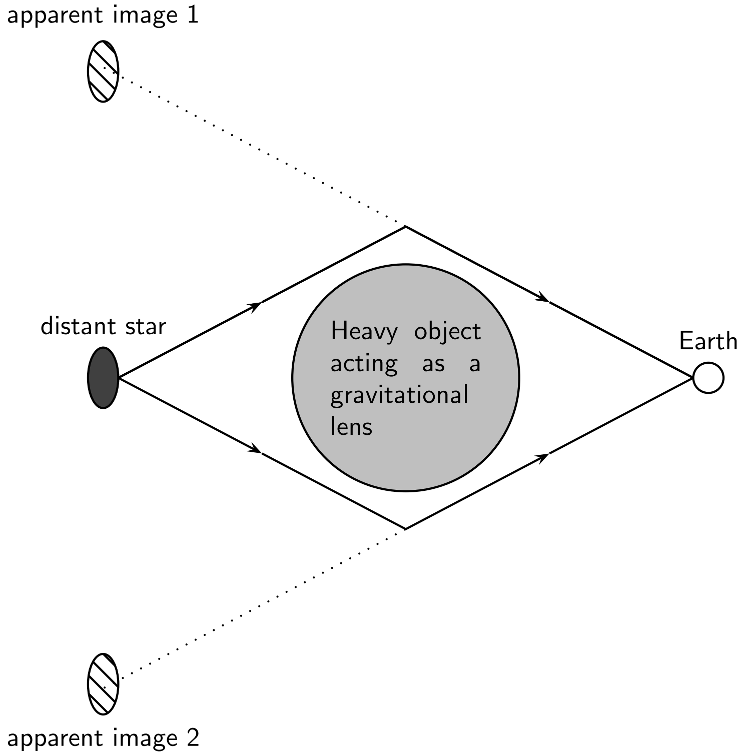



Figure 13.19: Effect of a gravitational lens.

lot of light. The lenses of a refracting telescope share a focal point. This ensures that parallel rays entering the telescope are again parallel when they reach your eye.

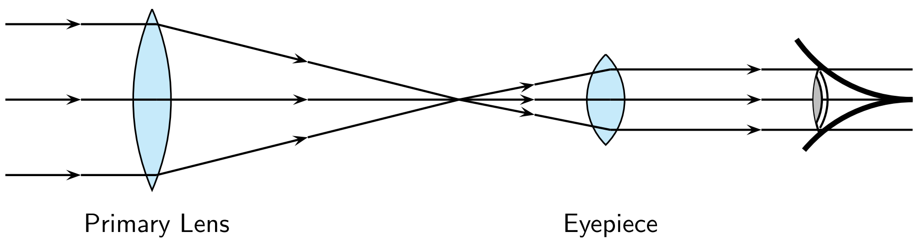



Figure 13.20: Layout of lenses in a refracting telescope

### 13.5.2 Reflecting Telescopes

Some telescopes use mirrors as well as lenses and are called reflecting telescopes. Specifically, a **reflecting telescope** uses a convex lens and two mirrors to make an object appear larger. (Figure 13.21.)

Light is collected by the primary mirror, which is large and concave. Parallel rays traveling toward this mirror are reflected and focused to a point. The secondary plane mirror is placed within the focal length of the primary mirror. This changes the direction of the light. A final eyepiece lens diverges the rays so that they are parallel when they reach your eye.

### 13.5.3 Southern African Large Telescope

The Southern African Large Telescope (SALT) is the largest single optical telescope in the southern hemisphere, with a hexagonal mirror array 11 metres across. SALT is located in Sutherland in the Northern Cape. SALT is able to record distant stars, galaxies and quasars a billion times too faint to be seen with the unaided eye. This is equivalent to a person being able to see a candle flame at on the moon.

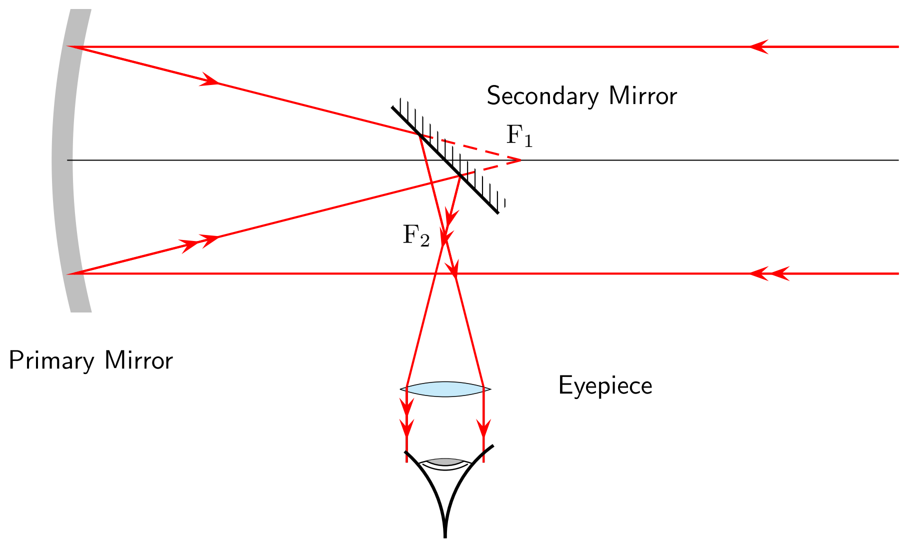



Figure 13.21: Lenses and mirrors in a reflecting telescope.

SALT was completed in 2005 and is a truly international initiative, because the money to build it came from South Africa, the United States, Germany, Poland, the United Kingdom and New Zealand.

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**Activity :: SALT : Investigate what the South African Astronomical Observatory (SAAO) does. SALT is part of SAAO. Write your investigation as a short 5-page report.**

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## 13.6 Microscopes

We have seen how lenses and mirrors are combined to magnify objects that are far away in a telescope. Lenses can also be used to make very small objects bigger.

Figure 13.10 shows that when an object is placed at a distance less than $f$ from the lens, the image formed is virtual, upright and is larger than the object. This set-up is a simple magnifier.

If you want to look at something very small, two lenses may work better than one. Microscopes and telescopes often use two lenses to make an image large enough to see.

A **compound microscope** uses two lenses to achieve high magnification (Figure 13.22). Both lenses are convex, or converging. Light from the object first passes through the **objective lens**. The lens that you look through is called the **eyepiece**. The focus of the system can be adjusted by changing the length of the tube between the lenses.

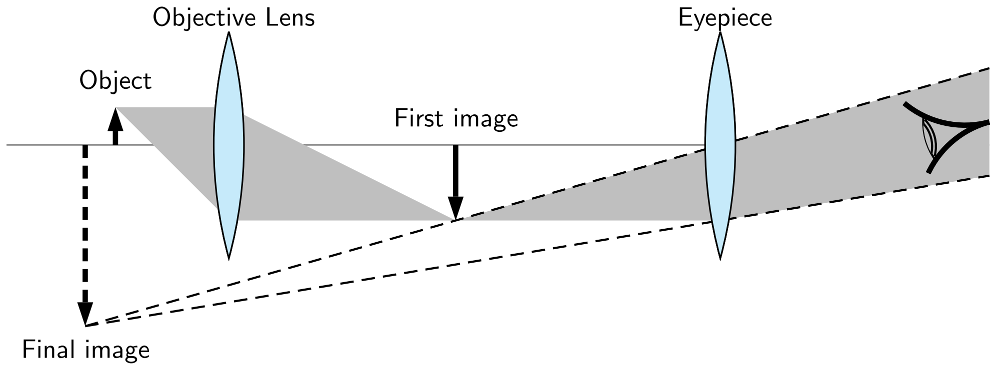



Figure 13.22: Compound microscope

**Drawing a Ray Diagram for a Two-Lens System**
You already have all the tools to analyze a two-lens system. Just consider one lens at a time.

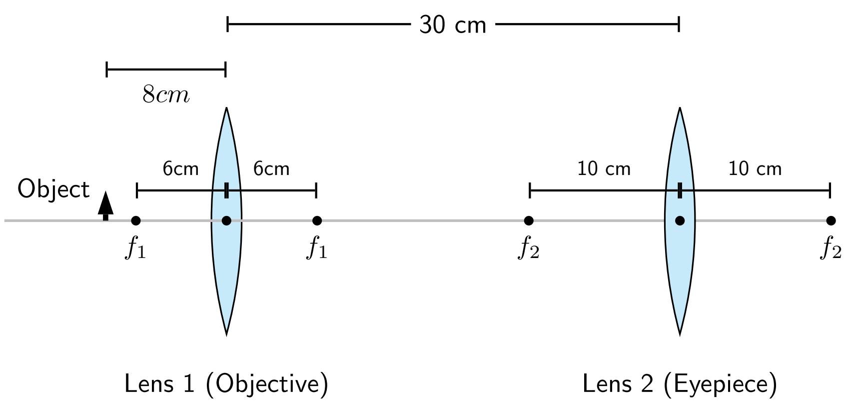
1. Use ray tracing or the lens equation to find the image for the first lens.
2. Use the image of the first lens as the object of the second lens.
3. To find the magnification, multiply: $m_{total} = m_1 \times m_2 \times m_3 \times ...$

**Worked Example 101: The Compound Microscope**

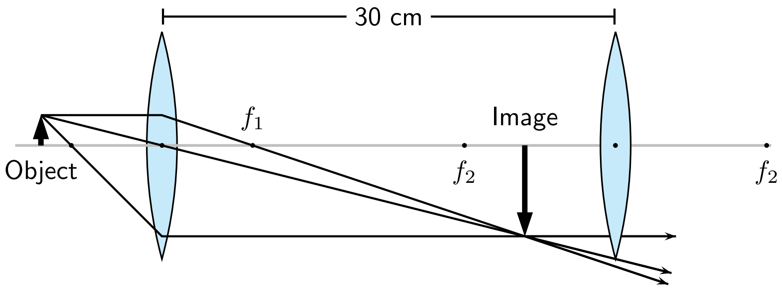
**Question:** A compound microscope consists of two convex lenses. The eyepiece has a focal length of 10 cm. The objective lens has a focal length of 6 cm. The two lenses are 30 cm apart. A 2 cm-tall object is placed 8 cm from the objective lens.

1. Where is the final image?
2. Is the final image real or virtual?

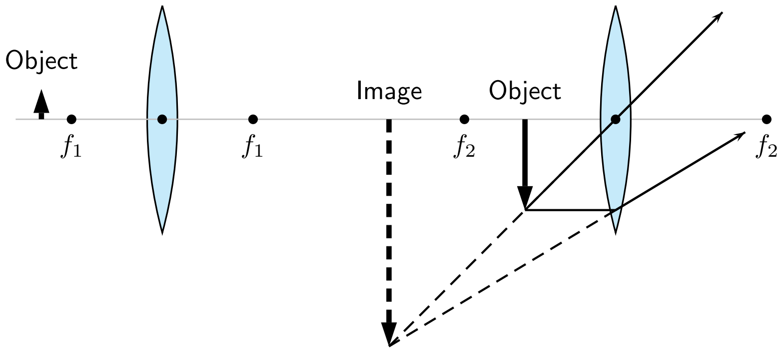
**Answer**
We can use ray tracing to follow light rays through the microscope, one lens at a time.
**Step 1 : Set up the system**
To prepare to trace the light rays, make a diagram. In the diagram here, we place the image on the left side of the microscope. Since the light will pass through the objective lens first, we'll call this Lens 1. The eyepiece will be called Lens 2. Be sure to include the focal points of both lenses in your diagram.




**Step 2 : Find the image for the objective lens.**




**Step 3 : Find the image for the eyepiece.**
The image we just found becomes the object for the second lens.




## 13.7 Summary

1. A lens is any transparent material that is shaped in such a way that it will converge parallel incident rays to a point or diverge incident rays from a point.
2. Converging lenses are thicker in the middle than on the edge and will bend incoming light rays towards the principal axis.
3. Diverging lenses are thinner in the middle than on the edge and will bend incoming light rays away from the principal axis.
4. The principal axis of a lens is the horizontal line through the centre of the lens.
5. The centre of the lens is called the optical centre.
6. The focus or focal point is a point on the principal axis where parallel rays converge through or diverge from.
7. The focal length is the distance between the focus and the optical centre.
8. Ray diagrams are used to determine the position and height of an image formed by a lens. The properties of images formed by converging and diverging lenses are summarised in Table 13.1.
9. The human eye consists of a lens system that focuses images on the retina where the optic nerve transfers the messages to the brain.
10. Defects of vision are short-sightedness, long-sightedness and astigmatism.
11. Massive bodies act as gravitational lenses that change the apparent positions of the images of stars.
12. Microscopes and telescopes use systems of lenses to create visible images of different objects.

## 13.8 Exercises

1. Select the correct answer from the options given:

   1.1 A ............ (*convex/concave*) lens is thicker in the center than on the edges.

   1.2 When used individually, a (*diverging/converging*) lens usually forms real images.

   1.3 When formed by a single lens, a ............ (*real/virtual*) image is always inverted.

   1.4 When formed by a single lens, a ............ (*real/virtual*) image is always upright.

   1.5 Virtual images formed by converging lenses are ............ (*bigger/the same size/smaller*) compared to the object.

   1.6 A ............ (*real/virtual*) image can be projected onto a screen.

   1.7 A ............ (*real/virtual*) image is said to be "trapped" in the lens.

   1.8 When light passes through a lens, its frequency ............ (*decreases/remains the same/increases*).

   1.9 A ray that starts from the top of an object and runs parallel to the axis of the lens, would then pass through the ............ (*principal focus of the lens/center of the lens/secondary focus of the lens*).

   1.10 A ray that starts from the top of an object and passes through the ............ (*principal focus of the lens/center of the lens/secondary focus of the lens*) would leave the lens running parallel to its axis.

   1.11 For a converging lens, its ............ (*principal focus/center/secondary focus*) is located on the same side of the lens as the object.

   1.12 After passing through a lens, rays of light traveling parallel to a lens' axis are refracted to the lens' ............ (*principal focus/center/secondary focus*).

   1.13 Real images are formed by ............ (*converging/parallel/diverging*) rays of light that have passed through a lens.

   1.14 Virtual images are formed by ............ (*converging/parallel/diverging*) rays of light that have passed through a lens.

   1.15 Images which are closer to the lens than the object are ............ (*bigger/the same size/smaller*) than the object.

   1.16 ............ (*Real/Virtual*) images are located on the same side of the lens as the object - that is, by looking in one direction, the observer can see both the image and the object.

   1.17 ............ (*Real/Virtual*) images are located on the opposite side of the lens as the object.

   1.18 When an object is located greater than two focal lengths in front of a converging lens, the image it produces will be ............ (*real and enlarged/virtual and enlarged/real and reduced/virtual and reduced*).

2. An object 1 cm high is placed 1,8 cm in front of a converging lens with a focal length of 0,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

3. An object 1 cm high is placed 2,10 cm in front of a diverging lens with a focal length of 1,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

4. An object 1 cm high is placed 0,5 cm in front of a converging lens with a focal length of 0,5 cm. Draw a ray diagram to show where the image is formed. Is the final image real or virtual?

5. An object is at right angles to the principal axis of a convex lens. The object is 2 cm high and is 5 cm from the centre of the lens, which has a focal length of 10 cm. Find the distance of the image from the centre of the lens, and its height. Is it real or virtual?

6. A convex lens of focal length 15 cm produces a real image of height 4 cm at 45 cm from the centre of the lens. Find the distance of the object from the lens and its height.

7. An object is 20 cm from a concave lens. The virtual image formed is three times smaller than the object. Find the focal length of the lens.

8. A convex lens produces a virtual image which is four times larger than the object. The image is 15 cm from the lens. What is the focal length of the lens?

9. A convex lens is used to project an image of a light source onto a screen. The screen is 30 cm from the light source, and the image is twice the size of the object. What focal length is required, and how far from the source must it be placed?

10. An object 6 cm high is place 20 cm from a converging lens of focal length 8 cm. Find by scale drawing the position, size and nature of the image produced. (Advanced: check your answer by calculation).

11. An object is placed in front of a converging lens of focal length 12 cm. By scale diagram, find the nature, position and magnification of the image when the object distance is

    11.1 16 cm

    11.2 8 cm

12. A concave lens produces an image three times smaller than the object. If the object is 18 cm away from the lens, determine the focal length of the lens by means of a scale diagram. (Advanced: check your answer by calculation).

13. You have seen how the human eye works, how telescopes work and how microscopes work. Using what you have learnt, describe how you think a camera works.

14. Describe 3 common defects of vision and discuss the various methods that are used to correct them.

# Chapter 14

# Longitudinal Waves - Grade 11

## 14.1 Introduction

In Grade 10 we studied pulses and waves. We looked at transverse waves more closely. In this chapter we look at another type of wave called *longitudinal* waves. In transverse waves, the motion of the particles in the medium were perpendicular to the direction of the wave. In longitudinal waves, the particles in the medium move *parallel* (in the *same* direction as) to the motion of the wave. Examples of transverse waves are water waves or light waves. An example of a longitudinal wave is a sound wave.

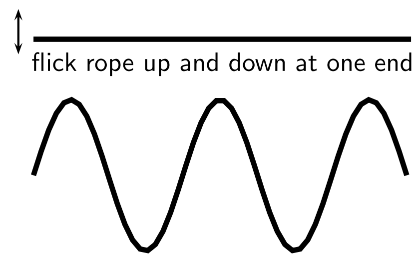
## 14.2 What is a longitudinal wave?

**Definition: Longitudinal waves**
A longitudinal wave is a wave where the particles in the medium move parallel to the direction of propagation of the wave.

When we studied transverse waves we looked at two different motions: the motion of the particles of the medium and the motion of the save itself. We will do the same for longitudinal waves.

The question is how do we construct such a wave?

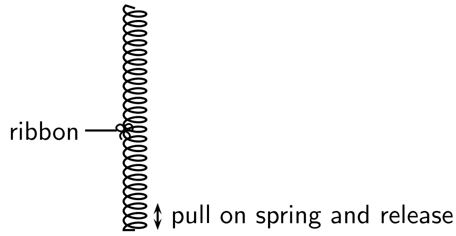
To create a transverse wave, we flick the end of for example a rope up and down. The particles move up and down and return to their equilibrium position. The wave moves from left to right and will be displaced.




A longitudinal wave is seen best in a spring that is hung from a ceiling. Do the following investigation to find out more about longitudinal waves.

**Activity :: Investigation : Investigating longitudinal waves**

1. Take a spring and hang it from the ceiling. Pull the free end of the spring and release it. Observe what happens.




2. In which direction does the disturbance move?
3. What happens when the disturbance reaches the ceiling?
4. Tie a ribbon to the middle of the spring. Watch carefully what happens to the ribbon when the free end of the spring is pulled and released. Describe the motion of the ribbon.

---

From the investigation you will have noticed that the disturbance moves in the same direction as the direction in which the spring was pulled. The spring was pulled up and down and the wave also moved up and down. The ribbon in the investigation represents one particle in the medium. The particles in the medium move in the same direction as the wave. The ribbon moves from rest upwards, then back to its original position, then down and then back to its original position.

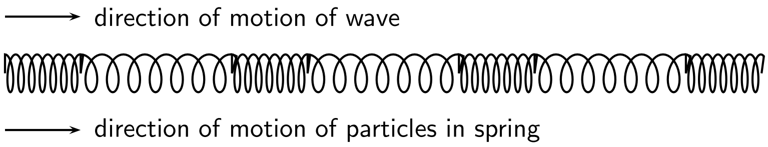



Figure 14.1: Longitudinal wave through a spring

## 14.3 Characteristics of Longitudinal Waves

As for transverse waves the following can be defined for longitudinal waves: wavelength, amplitude, period, frequency and wave speed. However instead of peaks and troughs, longitudinal waves have *compressions* and *rarefactions*.

**Definition: Compression**
A **compression** is a region in a longitudinal wave where the particles are closer together.

**Definition: Rarefaction**
A **rarefaction** is a region in a longitudinal wave where the particles are further apart.

### 14.3.1 Compression and Rarefaction

As seen in Figure 14.2, there are regions where the medium is compressed and other regions where the medium is spread out in a longitudinal wave.

The region where the medium is compressed is known as a **compression** and the region where the medium is spread out is known as a **rarefaction**.

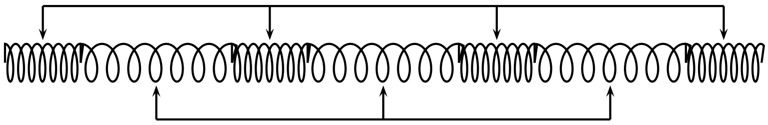



Figure 14.2: Compressions and rarefactions on a longitudinal wave

### 14.3.2 Wavelength and Amplitude

**Definition: Wavelength**
The **wavelength** in a longitudinal wave is the distance between two consecutive points that are in phase.

The wavelength in a longitudinal wave refers to the distance between two consecutive compressions or between two consecutive rarefactions.

**Definition: Amplitude**
The **amplitude** is the maximum displacement from a position of rest.

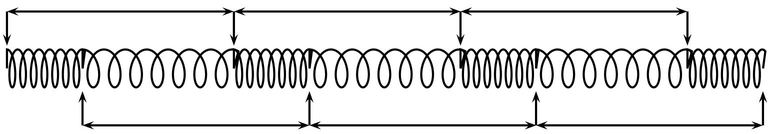



Figure 14.3: Wavelength on a longitudinal wave

The amplitude is the distance from the equilibrium position of the medium to a compression or a rarefaction.

### 14.3.3 Period and Frequency

**Definition: Period**
The **period** of a wave is the time taken by the wave to move one wavelength.

**Definition: Frequency**
The **frequency** of a wave is the number of wavelengths per second.

The *period* of a longitudinal wave is the time taken by the wave to move one wavelength. As for transverse waves, the symbol $T$ is used to represent period and period is measured in seconds (s).

The *frequency* $f$ of a wave is the number of wavelengths per second. Using this definition and the fact that the period is the time taken for 1 wavelength, we can define:

$$f = \frac{1}{T}$$

or alternately,

$$T = \frac{1}{f}.$$

### 14.3.4 Speed of a Longitudinal Wave

The speed of a longitudinal wave is defined as:

$$v = f \cdot \lambda$$

where
$v$ = speed in m.s$^{-1}$
$f$ = frequency in Hz
$\lambda$ = wavelength in m

**Worked Example 102: Speed of longitudinal waves**

**Question:** The musical note A is a sound wave. The note has a frequency of 440 Hz and a wavelength of 0,784 m. Calculate the speed of the musical note.
**Answer**
**Step 1 : Determine what is given and what is required**

$$\begin{aligned} f &= 440 \text{ Hz} \\ \lambda &= 0{,}784 \text{ m} \end{aligned}$$

We need to calculate the speed of the musical note "A".
**Step 2 : Determine how to approach based on what is given**
We are given the frequency and wavelength of the note. We can therefore use:

$$v = f \cdot \lambda$$

**Step 3 : Calculate the wave speed**

$$\begin{aligned} v &= f \cdot \lambda \\ &= (440\,\text{Hz})(0{,}784\,\text{m}) \\ &= 345\,\text{m} \cdot \text{s}^{-1} \end{aligned}$$

**Step 4 : Write the final answer**
The musical note "A" travels at 345 m·s$^{-1}$.

**Worked Example 103: Speed of longitudinal waves**

**Question:** A longitudinal wave travels into a medium in which its speed increases. How does this affect its... (write only *increases, decreases, stays the same*).

1. period?
2. wavelength?

**Answer**
**Step 1 : Determine what is required**
We need to determine how the period and wavelength of a longitudinal wave change when its speed increases.

**Step 2 : Determine how to approach based on what is given**
We need to find the link between period, wavelength and wave speed.
**Step 3 : Discuss how the period changes**
We know that the frequency of a longitudinal wave is dependent on the frequency of the vibrations that lead to the creation of the longitudinal wave. Therefore, the frequency is always unchanged, irrespective of any changes in speed. Since the period is the inverse of the frequency, the period remains the same.
**Step 4 : Discuss how the wavelength changes**
The frequency remains unchanged. According to the wave equation

$$v = f\lambda$$

if $f$ remains the same and $v$ increases, then $\lambda$, the wavelength, must also increase.

## 14.4 Graphs of Particle Position, Displacement, Velocity and Acceleration

When a longitudinal wave moves through the medium, the particles in the medium **only** move back and forth relative to the direction of motion of the wave. We can see this in Figure 14.4 which shows the motion of the particles in a medium as a longitudinal wave moves through the medium.

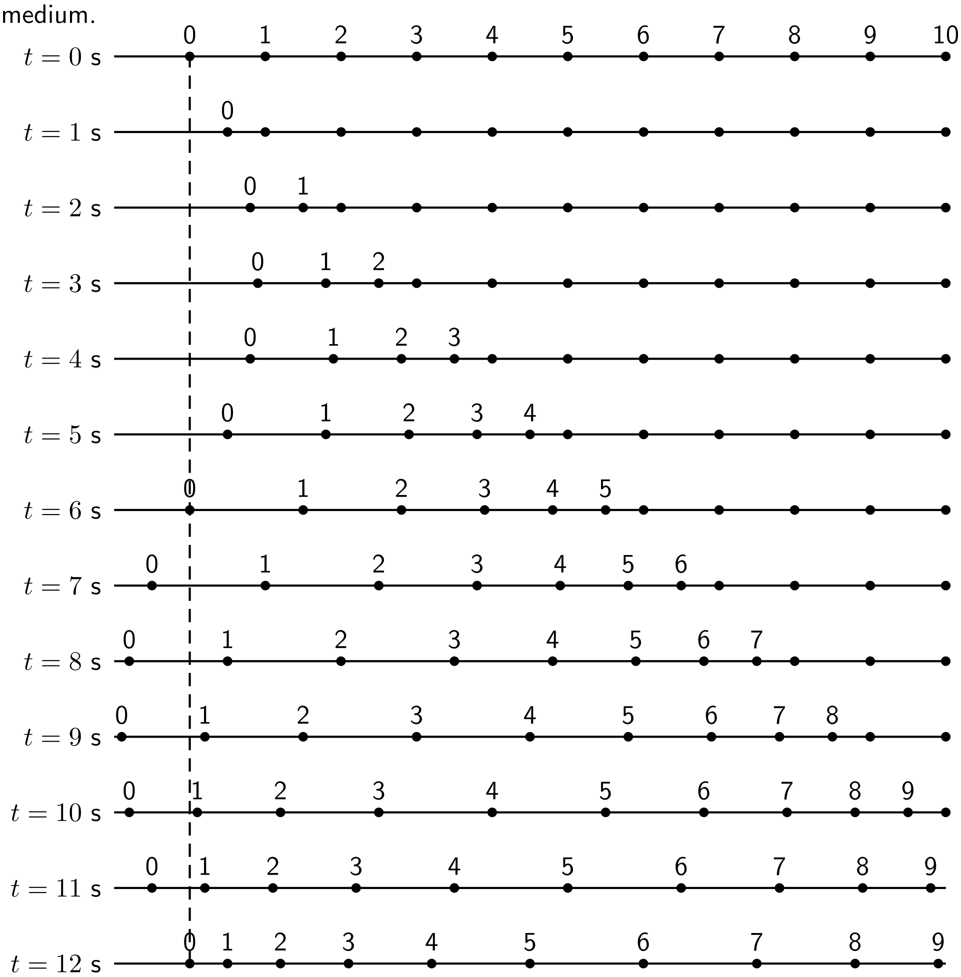



Figure 14.4: Positions of particles in a medium at different times as a longitudinal wave moves through it. The wave moves to the right. The dashed line shows the equilibrium position of particle 0.

**Important:** A particle in the medium **only** moves back and forth when a longitudinal wave moves through the medium.

As in Chapter 6, we can draw a graph of the particle's position as a function of time. For the wave shown in Figure 14.4, we can draw the graph shown in Figure 14.5 for particle 0. The graph for each of the other particles will be identical.

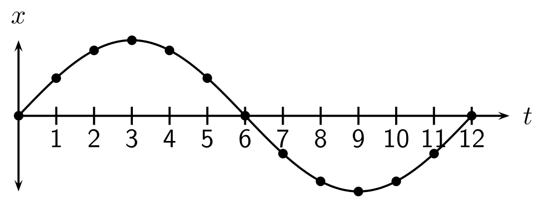



Figure 14.5: Graph of particle displacement as a function of time for the longitudinal wave shown in Figure 14.4.

The graph of the particle's velocity as a function of time is obtained by taking the gradient of the position vs. time graph. The graph of velocity vs. time for the position vs. time graph shown in Figure 14.5 is shown is Figure 14.6.

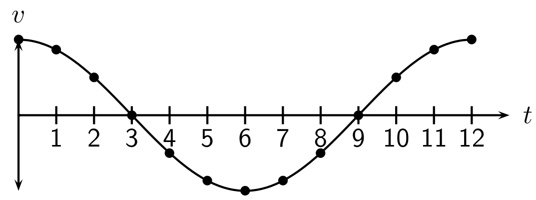



Figure 14.6: Graph of velocity as a function of time.

The graph of the particle's acceleration as a function of time is obtained by taking the gradient of the velocity vs. time graph. The graph of acceleration vs. time for the position vs. time graph shown in Figure 14.5 is shown is Figure 14.7.

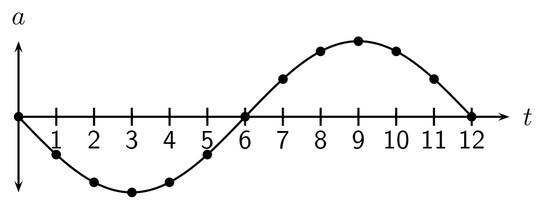



Figure 14.7: Graph of acceleration as a function of time.

## 14.5 Sound Waves

Sound waves coming from a tuning fork cause the tuning fork to vibrate and push against the air particles in front of it. As the air particles are pushed together a compression is formed. The particles behind the compression move further apart causing a rarefaction. As the particles continue to push against each other, the sound wave travels through the air. Due to this motion of the particles, there is a constant variation in the pressure in the air. Sound waves are therefore pressure waves. This means that in media where the particles are closer together, sound waves will travel quicker.

Sound waves travel faster through liquids, like water, than through the air because water is denser than air (the particles are closer together). Sound waves travel faster in solids than in liquids.

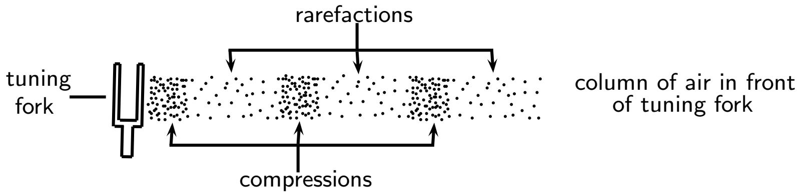



Figure 14.8: Sound waves are pressure waves and need a medium through which to travel.

**Important:** A sound wave is different from a light wave.

- A sound wave is produced by an oscillating object while a light wave is not.
- A sound wave cannot be diffracted while a light wave can be diffracted.

Also, because a sound wave is a mechanical wave (i.e. that it needs a medium) it is not capable of traveling through a vacuum, whereas a light wave can travel through a vacuum.

**Important:** A sound wave is a pressure wave. This means that regions of high pressure (compressions) and low pressure (rarefactions) are created as the sound source vibrates. These compressions and rarefactions arise because sound vibrates longitudinally and the longitudinal motion of air produces pressure fluctuations.

Sound will be studied in more detail in Chapter 15.

## 14.6 Seismic Waves

Seismic waves are waves from vibrations in the Earth (core, mantle, oceans). Seismic waves also occur on other planets, for example the moon and can be natural (due to earthquakes, volcanic eruptions or meteor strikes) or man-made (due to explosions or anything that hits the earth hard). Seismic P-waves (P for pressure) are longitudinal waves which can travel through solid and liquid.

## 14.7 Summary - Longitudinal Waves

1. A longitudinal wave is a wave where the particles in the medium move parallel to the direction in which the wave is travelling.
2. Longitudinal waves consist of areas of higher pressure, where the particles in the medium are closer together (compressions) and areas of lower pressure, where the particles in the medium are further apart (rarefactions).
3. The wavelength of a longitudinal wave is the distance between two consecutive compressions, or two consecutive rarefactions.
4. The relationship between the period ($T$) and frequency ($f$) is given by

$$T = \frac{1}{f} \text{ or } f = \frac{1}{T}.$$

5. The relationship between wave speed ($v$), frequency ($f$) and wavelength ($\lambda$) is given by

$$v = f\lambda.$$

6. Graphs of position vs time, velocity vs time and acceleration vs time can be drawn and are summarised in figures

7. Sound waves are examples of longitudinal waves. The speed of sound depends on the medium, temperature and pressure. Sound waves travel faster in solids than in liquids, and faster in liquids than in gases. Sound waves also travel faster at higher temperatures and higher pressures.

## 14.8 Exercises - Longitudinal Waves

1. Which of the following is not a longitudinal wave?

    1.1 seismic P-wave
    1.2 light
    1.3 sound
    1.4 ultrasound

2. Which of the following media can sound not travel through?

    2.1 solid
    2.2 liquid
    2.3 gas
    2.4 vacuum

3. Select a word from Column B that best fits the description in Column A:

| Column A | Column B |
|---|---|
| waves in the air caused by vibrations | longitudinal waves |
| waves that move in one direction, but medium moves in another | frequency |
| waves and medium that move in the same direction | white noise |
| the distance between one wave and the next wave | amplitude |
| how often a single wave goes by | sound waves |
| difference between high points and low points of waves | standing waves |
| the distance a wave covers per time interval | transverse waves |
| the time taken for one wavelength to pass a point | wavelength |
| | music |
| | sounds |
| | wave speed |

4. A longitudinal wave has a crest to crest distance of 10 m. It takes the wave 5 s to pass a point.

    4.1 What is the wavelength of the longitudinal wave?
    4.2 What is the speed of the wave?

5. A flute produces a musical sound travelling at a speed of 320 m.s$^{-1}$. The frequency of the note is 256 Hz. Calculate:

    5.1 the period of the note
    5.2 the wavelength of the note

6. A person shouts at a cliff and hears an echo from the cliff 1 s later. If the speed of sound is 344 m·s$^{-1}$, how far away is the cliff?

7. A wave travels from one medium to another and the speed of the wave decreases. What will the effect be on the ... (write only *increases, decreases* or *remains the same*)

    7.1 wavelength?
    7.2 period?

# Chapter 15

# Sound - Grade 11

## 15.1 Introduction

Now that we have studied the basics of longitudinal waves, we are ready to study sound waves in detail.

Have you ever thought about how amazing your sense of hearing is? It is actually pretty remarkable. There are many types of sounds: a car horn, a laughing baby, a barking dog, and somehow your brain can sort it all out. Though it seems complicated, it is rather simple to understand once you learn a very simple fact. Sound is a wave. So you can use everything you know about waves to explain sound.

## 15.2 Characteristics of a Sound Wave

Since sound is a wave, we can relate the properties of sound to the properties of a wave. The basic properties of sound are: pitch, loudness and tone.

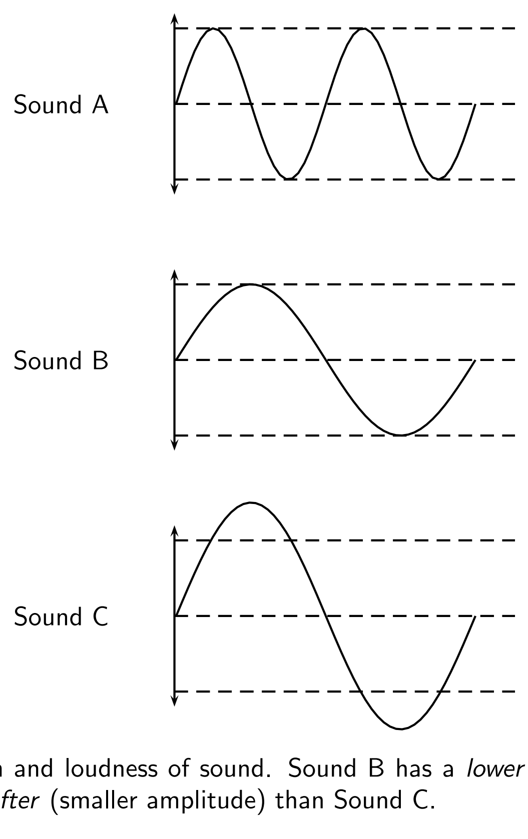



Figure 15.1: Pitch and loudness of sound. Sound B has a *lower* pitch (lower frequency) than Sound A and is *softer* (smaller amplitude) than Sound C.

### 15.2.1 Pitch

The frequency of a sound wave is what your ear understands as pitch. A higher frequency sound has a higher pitch, and a lower frequency sound has a lower pitch. In Figure 15.1 sound A has a higher pitch than sound B. For instance, the chirp of a bird would have a high pitch, but the roar of a lion would have a low pitch.

The human ear can detect a wide range of frequencies. Frequencies from 20 to 20 000 Hz are audible to the human ear. Any sound with a frequency below 20 Hz is known as an **infrasound** and any sound with a frequency above 20 000 Hz is known as an **ultrasound**.

Table 15.1 lists the ranges of some common animals compared to humans.

Table 15.1: Range of frequencies

| | lower frequency (Hz) | upper frequency (Hz) |
|---|---|---|
| Humans | 20 | 20 000 |
| Dogs | 50 | 45 000 |
| Cats | 45 | 85 000 |
| Bats | | 120 000 |
| Dolphins | | 200 000 |
| Elephants | 5 | 10 000 |

**Activity :: Investigation : Range of Wavelengths**
Using the information given in Table 15.1, calculate the lower and upper wavelengths that each species can hear. Assume the speed of sound in air is 344 $m \cdot s^{-1}$.

### 15.2.2 Loudness

The amplitude of a sound wave determines its loudness or volume. A larger amplitude means a louder sound, and a smaller amplitude means a softer sound. In Figure 15.1 sound C is louder than sound B. The vibration of a source sets the amplitude of a wave. It transmits energy into the medium through its vibration. More energetic vibration corresponds to larger amplitude. The molecules move back and forth more vigorously.

The loudness of a sound is also determined by the sensitivity of the ear. The human ear is more sensitive to some frequencies than to others. Loudness thus depends on both the amplitude of a sound wave and its frequency whether it lies in a region where the ear is more or less sensitive.

### 15.2.3 Tone

Tone is a measure of the quality of the sound wave. For example, the quality of the sound produced in a particular musical instruments depends on which harmonics are superposed and in which proportions. The harmonics are determined by the standing waves that are produced in the instrument. Chapter 16 will explain the physics of music in greater detail.

The quality (timbre) of the sound heard depends on the pattern of the incoming vibrations, i.e. the *shape* of the sound wave. The more irregular the vibrations, the more jagged is the shape of the sound wave and the harsher is the sound heard.

## 15.3 Speed of Sound

The speed of sound depends on the medium the sound is travelling in. Sound travels faster in solids than in liquids, and faster in liquids than in gases. This is because the density of solids is higher than that of liquids which means that the particles are closer together. Sound can be transmitted more easily.

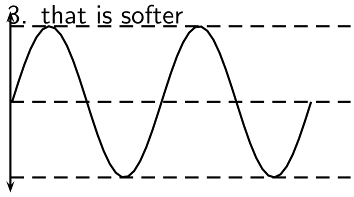
The speed of sound also depends on the temperature of the medium. The hotter the medium is, the faster its particles move and therefore the quicker the sound will travel through the medium. When we heat a substance, the particles in that substance have more kinetic energy and vibrate or move faster. Sound can therefore be transmitted more easily and quickly in hotter substances.

Sound waves are pressure waves. The speed of sound will therefore be influenced by the pressure of the medium through which it is travelling. At sea level the air pressure is higher than high up on a mountain. Sound will travel faster at sea level where the air pressure is higher than it would at places high above sea level.

**Definition: Speed of sound**
The speed of sound in air, at sea level, at a temperature of 21°C and under normal atmospheric conditions, is 344 m·s$^{-1}$.

**Exercise: Sound frequency and amplitude**
Study the following diagram representing a musical note. Redraw the diagram for a note

1. with a higher pitch
2. that is louder
3. that is softer



## 15.4 Physics of the Ear and Hearing

Figure 15.2: Diagram of the human ear.

The human ear is divided into three main sections: the outer, middle, and inner ear. Let's follow the journey of a sound wave from the pinna to the auditory nerve which transmits a signal to the brain. The pinna is the part of the ear we typically think of when we refer to the ear. Its main function is to collect and focus an incident sound wave. The wave then travels through the ear canal until it meets the eardrum. The pressure fluctuations of the sound wave make the eardrum vibrate. The three very small bones of the middle ear, the malleus (hammer), the incus (anvil), and the stapes (stirrup), transmit the signal through to the elliptical window. The elliptical window is the beginning of the inner ear. From the elliptical window the sound waves are transmitted through the liquid in the inner ear and interpreted as sounds by the brain. The inner ear, made of the semicircular canals, the cochlea, and the auditory nerve, is filled with fluid. The fluid allows the body to detect quick movements and maintain balance. The snail-shaped cochlea is covered in nerve cells. There are more than 25 000 hairlike nerve cells. Different nerve cells vibrate with different frequencies. When a nerve cell vibrates, it releases electrical impulses to the auditory nerve. The impulses are sent to the brain through the auditory nerve and understood as sound.

### 15.4.1 Intensity of Sound

Intensity is one indicator of amplitude. Intensity is the energy transmitted over a unit of area each second.

*Extension: Intensity*
Intensity is defined as:

$$\text{Intensity} = \frac{\text{energy}}{\text{time} \times \text{area}} = \frac{\text{power}}{\text{area}}$$

By the definition of intensity, we can see that the units of intensity are

$$\frac{\text{Joules}}{\text{s} \cdot \text{m}^2} = \frac{\text{Watts}}{\text{m}^2}$$

The unit of intensity is the **decibel** (symbol: dB). This reduces to an SI equivalent of $\text{W} \cdot \text{m}^{-2}$.

The threshold of hearing is $10^{-12}\ \text{W} \cdot \text{m}^{-2}$. Below this intensity, the sound is too soft for the ear to hear. The threshold of pain is $1.0\ \text{W} \cdot \text{m}^{-2}$. Above this intensity a sound is so loud it becomes uncomfortable for the ear.

Notice that there is a factor of $10^{12}$ between the thresholds of hearing and pain. This is one reason we define the decibel (dB) scale.

*Extension: dB Scale*
The intensity in dB of a sound of intensity $I$, is given by:

$$\beta = 10\ \log \frac{I}{I_o} \qquad I_o = 10^{-12}\ \text{W} \cdot \text{m}^{-2} \tag{15.1}$$

In this way we can compress the whole hearing intensity scale into a range from 0 dB to 120 dB.

Table 15.2: Examples of sound intensities.

| **Source** | **Intensity** (dB) | **Times greater than hearing threshold** |
|---|---|---|
| Rocket Launch | 180 | $10^{18}$ |
| Jet Plane | 140 | $10^{14}$ |
| Threshold of Pain | 120 | $10^{12}$ |
| Rock Band | 110 | $10^{11}$ |
| Subway Train | 90 | $10^{9}$ |
| Factory | 80 | $10^{8}$ |
| City Traffic | 70 | $10^{7}$ |
| Normal Conversation | 60 | $10^{6}$ |
| Library | 40 | $10^{4}$ |
| Whisper | 20 | $10^{2}$ |
| Threshold of hearing | 0 | 0 |

Notice that there are sounds which exceed the threshold of pain. Exposure to these sounds can cause immediate damage to hearing. In fact, exposure to sounds from 80 dB and above can damage hearing over time. Measures can be taken to avoid damage, such as wearing earplugs or ear muffs. Limiting exposure time and increasing distance between you and the source are also important steps to protecting your hearing.

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**Activity :: Discussion : Importance of Safety Equipment**
Working in groups of 5, discuss the importance of safety equipment such as ear protectors for workers in loud environments, e.g. those who use jack hammers or direct aeroplanes to their parking bays. Write up your conclusions in a one page report. Some prior research into the importance of safety equipment might be necessary to complete this group discussion.

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## 15.5 Ultrasound

Ultrasound is sound with a frequency that is higher than 20 kHz. Some animals, such as dogs, dolphins, and bats, have an upper limit that is greater than that of the human ear and can hear ultrasound.

The most common use of ultrasound is to create images, and has industrial and medical applications. The use of ultrasound to create images is based on the reflection and transmission of a wave at a boundary. When an ultrasound wave travels inside an object that is made up of different materials such as the human body, each time it encounters a boundary, e.g. between bone and muscle, or muscle and fat, part of the wave is reflected and part of it is transmitted. The reflected rays are detected and used to construct an image of the object.

Ultrasound in medicine can visualise muscle and soft tissue, making them useful for scanning the organs, and is commonly used during pregnancy. Ultrasound is a safe, non-invasive method of looking inside the human body.

Ultrasound sources may be used to generate local heating in biological tissue, with applications in physical therapy and cancer treatment. Focussed ultrasound sources may be used to break up kidney stones.

Ultrasonic cleaners, sometimes called supersonic cleaners, are used at frequencies from 20-40 kHz for jewellery, lenses and other optical parts, watches, dental instruments, surgical instruments and industrial parts. These cleaners consist of containers with a fluid in which the object to be cleaned is placed. Ultrasonic waves are then sent into the fluid. The main mechanism for cleaning action in an ultrasonic cleaner is actually the energy released from the collapse of millions of microscopic bubbles occurring in the liquid of the cleaner.

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**Interesting Fact**

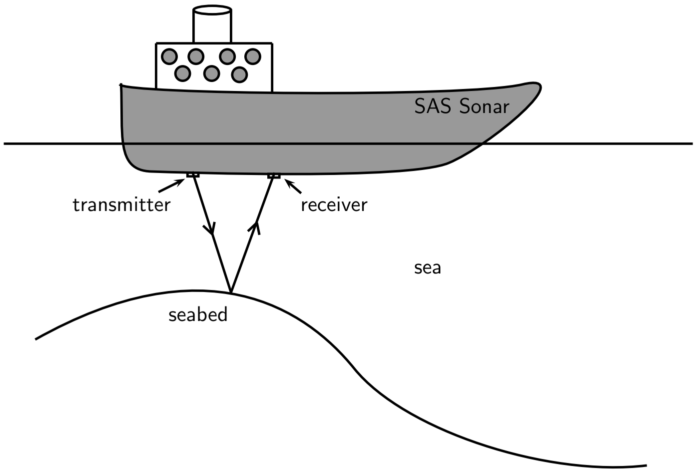
Ultrasound generator/speaker systems are sold with claims that they frighten away rodents and insects, but there is no scientific evidence that the devices work; controlled tests have shown that rodents quickly learn that the speakers are harmless.

---

Interesting Fact

In echo-sounding the reflections from ultrasound pulses that are bounced off objects (for example the bottom of the sea, fish etc.) are picked up. The reflections are timed and since their speed is known, the distance to the object can be found. This information can be built into a picture of the object that reflects the ultrasound pulses.

## 15.6 SONAR




Ships on the ocean make use of the reflecting properties of sound waves to determine the depth of the ocean. A sound wave is transmitted and bounces off the seabed. Because the speed of sound is known and the time lapse between sending and receiving the sound can be measured, the distance from the ship to the bottom of the ocean can be determined, This is called sonar, which stands from **So**und **N**avigation **A**nd **R**anging.

### 15.6.1 Echolocation

Animals like dolphins and bats make use of sounds waves to find their way. Just like ships on the ocean, bats use sonar to navigate. Ultrasound waves that are sent out are reflected off the objects around the animal. Bats, or dolphins, then use the reflected sounds to form a "picture" of their surroundings. This is called echolocation.

**Worked Example 104: SONAR**

**Question:** A ship sends a signal to the bottom of the ocean to determine the depth of the ocean. The speed of sound in sea water is 1450 $\text{m.s}^{-1}$ If the signal is received 1,5 seconds later, how deep is the ocean at that point?

**Answer**

**Step 1 : Identify what is given and what is being asked:**

$$\begin{aligned} s &= 1450\ \text{m.s}^{-1} \\ t &= 1{,}5\ \text{s there and back} \\ \therefore t &= 0{,}75\ \text{s one way} \\ d &= ? \end{aligned}$$

**Step 2 : Calculate the distance:**

$$\begin{aligned} \text{Distance} &= \text{speed} \times \text{time} \\ d &= s \times t \\ &= 1450 \times 0{,}75 \\ &= 1087{,}5 \text{ m} \end{aligned}$$

## 15.7 Summary

1. Sound waves are longitudinal waves
2. The **frequency** of a sound is an indication of how high or low the *pitch* of the sound is.
3. The human ear can hear frequencies from 20 to 20 000 Hz.
   **Infrasound** waves have frequencies lower than 20 Hz.
   **Ultrasound** waves have frequencies higher than 20 000 Hz.
4. The **amplitude** of a sound determines its *loudness* or volume.
5. The **tone** is a measure of the *quality* of a sound wave.
6. The speed of sound in air is around 340 $m.s^{-1}$. It is dependent on the temperature, height above sea level and the phase of the medium through which it is travelling.
7. Sound travels faster when the medium is hot.
8. Sound travels faster in a solid than a liquid and faster in a liquid than in a gas.
9. Sound travels faster at sea level where the air pressure is higher.
10. The intensity of a sound is the energy transmitted over a certain area. Intensity is a measure of frequency.
11. Ultrasound can be used to form pictures of things we cannot see, like unborn babies or tumors.
12. Echolocation is used by animals such as dolphins and bats to "see" their surroundings by using ultrasound.
13. Ships use sonar to determine how deep the ocean is or to locate shoals of fish.

## 15.8 Exercises

1. Choose a word from column B that best describes the concept in column A.

| **Column A** | **Column B** |
|---|---|
| pitch of sound | amplitude |
| loudness of sound | frequncy |
| quality of sound | speed |
| | waveform |

2. A tuning fork, a violin string and a loudspeaker are producing sounds. This is because they are all in a state of:

   A compression

   B rarefaction

   C rotation

   D tension

E vibration

3. What would a drummer do to make the sound of a drum give a note of lower pitch?

    A hit the drum harder

    B hit the drum less hard

    C hit the drum near the edge

    D loosen the drum skin

    E tighten the drum skin

4. What is the approximate range of audible frequencies for a healthy human?

    A 0.2 Hz → 200 Hz

    B 2 Hz → 2 000 Hz

    C 20 Hz → 20 000 Hz

    D 200 Hz → 200 000 Hz

    E 2 000 Hz → 2 000 000 Hz

5. X and Y are different wave motions. In air, X travels much faster than Y but has a much shorter wavelength. Which types of wave motion could X and Y be?

| | X | Y |
|---|---|---|
| A | microwaves | red light |
| B | radio | infra red |
| C | red light | sound |
| D | sound | ultraviolet |
| E | ultraviolet | radio |

6. Astronauts are in a spaceship orbiting the moon. They see an explosion on the surface of the moon. Why can they not hear the explosion?

    A explosions do not occur in space

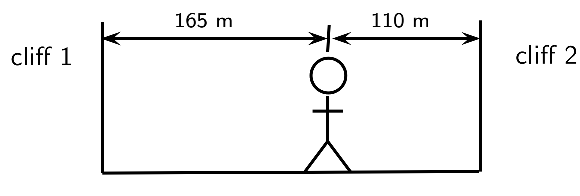
    B sound cannot travel through a vacuum

    C sound is reflected away from the spaceship

    D sound travels too quickly in space to affect the ear drum

    E the spaceship would be moving at a supersonic speed

7. A man stands between two cliffs as shown in the diagram and claps his hands once.




Assuming that the velocity of sound is 330 $m.s^{-1}$, what will be the time interval between the two loudest echoes?

    A $\frac{1}{6}$ s

    B $\frac{5}{6}$ s

    C $\frac{1}{3}$ s

    D 1 s

    E $\frac{2}{3}$ s

8. A dolphin emits an ultrasonic wave with frequency of 0,15 MHz. The speed of the ultrasonic wave in water is 1 500 $m.s^{-1}$. What is the wavelength of this wave in water?

    A 0.1 mm

    B 1 cm

C 10 cm

D 10 m

E 100 m

9. The amplitude and frequency of a sound wave are both increased. How are the loudness and pitch of the sound affected?

| | loudness | pitch |
|---|---|---|
| A | increased | raised |
| B | increased | unchanged |
| C | increased | lowered |
| D | decreased | raised |
| E | decreased | lowered |

10. A jet fighter travels slower than the speed of sound. Its speed is said to be:

A Mach 1

B supersonic

C isosonic

D hypersonic

E infrasonic

11. A sound wave is different from a light wave in that a sound wave is:

A produced by a vibrating object and a light wave is not.

B not capable of travelling through a vacuum.

C not capable of diffracting and a light wave is.

D capable of existing with a variety of frequencies and a light wave has a single frequency.

12. At the same temperature, sound waves have the fastest speed in:

A rock

B milk

C oxygen

D sand

13. Two sound waves are traveling through a container of nitrogen gas. The first wave has a wavelength of 1,5 m, while the second wave has a wavelength of 4,5 m. The velocity of the second wave must be:

A $\frac{1}{9}$ the velocity of the first wave.

B $\frac{1}{3}$ the velocity of the first wave.

C the same as the velocity of the first wave.

D three times larger than the velocity of the first wave.

E nine times larger than the velocity of the first wave.

14. Sound travels at a speed of 340 m·s$^{-1}$. A straw is 0,25 m long. The standing wave set up in such a straw with one end closed has a wavelength of 1,0 m. The standing wave set up in such a straw with both ends open has a wavelength of 0,50 m.

(a) calculate the frequency of the sound created when you blow across the straw with the bottom end closed.

(b) calculate the frequency of the sound created when you blow across the straw with the bottom end open.

15. A lightning storm creates both lightning and thunder. You see the lightning almost immediately since light travels at $3 \times 10^8\,\mathrm{m \cdot s^{-1}}$. After seeing the lightning, you count 5 s and then you hear the thunder. Calculate the distance to the location of the storm.

16. A person is yelling from a second story window to another person standing at the garden gate, 50 m away. If the speed of sound is 344 m·s$^{-1}$, how long does it take the sound to reach the person standing at the gate?

17. A piece of equipment has a warning label on it that says, "Caution! This instrument produces 140 decibels." What safety precaution should you take before you turn on the instrument?

18. What property of sound is a measure of the amount of energy carried by a sound wave?

19. How is intensity related to loudness?

20. Person 1 speaks to person 2. Explain how the sound is created by person 1 and how it is possible for person 2 to hear the conversation.

21. Sound cannot travel in space. Discuss what other modes of communication astronauts can use when they are outside the space shuttle?

22. An automatic focus camera uses an ultrasonic sound wave to focus on objects. The camera sends out sound waves which are reflected off distant objects and return to the camera. A sensor detects the time it takes for the waves to return and then determines the distance an object is from the camera. If a sound wave (speed = 344 m·s$^{-1}$) returns to the camera 0,150 s after leaving the camera, how far away is the object?

23. Calculate the frequency (in Hz) and wavelength of the annoying sound made by a mosquito when it beats its wings at the average rate of 600 wing beats per second. Assume the speed of the sound waves is 344 m·s$^{-1}$.

24. Does halving the frequency of a wave source halve or double the speed of the waves?

25. Humans can detect frequencies as high as 20 000 Hz. Assuming the speed of sound in air is 344 m·s$^{-1}$, calculate the wavelength of the sound corresponding to the upper range of audible hearing.

26. An elephant trumpets at 10 Hz. Assuming the speed of sound in air is 344 m·s$^{-1}$, calculate the wavelength of this infrasonic sound wave made by the elephant.

27. A ship sends a signal out to determine the depth of the ocean. The signal returns 2,5 seconds later. If sound travels at 1450 m.s$^{-1}$ in sea water, how deep is the ocean at that point?

# Chapter 16

# The Physics of Music - Grade 11

## 16.1 Introduction

What is your favorite musical instrument? How do you play it? Do you pluck a string, like a guitar? Do you blow through it, like a flute? Do you hit it, like a drum? All of these work by making standing waves. Each instrument has a unique sound because of the special waves made in it. These waves could be in the strings of a guitar or violin. They could also be in the skin of a drum or a tube of air in a trumpet. These waves are picked up by the air and later reach your ear as sound.

In Grade 10, you learned about standing waves and boundary conditions. We saw a rope that was:

- fixed at both ends
- fixed at one end and free at the other

We also saw a pipe:

- closed at both ends
- open at both ends
- open at one end, closed at the other

String and wind instruments are good examples of standing waves on strings and pipes.

One way to describe standing waves is to count nodes. Recall that a node is a point on a string that does not move as the wave changes. The anti-nodes are the highest and lowest points on the wave. There is a node at each end of a fixed string. There is also a node at the closed end of a pipe. But an open end of a pipe has an anti-node.

What causes a standing wave? There are incident and reflected waves traveling back and forth on our string or pipe. For some frequencies, these waves combine in just the right way so that the whole wave appears to be standing still. These special cases are called harmonic frequencies, or **harmonics**. They depend on the length and material of the medium.

**Definition: Harmonic**
A **harmonic** frequency is a frequency at which standing waves can be made.

## 16.2 Standing Waves in String Instruments

Let us look at a basic "instrument": a string pulled tight and fixed at both ends. When you pluck the string, you hear a certain pitch. This pitch is made by a certain frequency. What causes the string to emit sounds at this pitch?

You have learned that the frequency of a standing wave depends on the length of the wave. The wavelength depends on the nodes and anti-nodes. The longest wave that can "fit" on the string is shown in Figure 16.1. This is called the **fundamental** or **natural frequency** of the string. The string has nodes at both ends. The wavelength of the fundamental is twice the length of the string.

Now put your finger on the center of the string. Hold it down gently and pluck it. The standing wave now has a node in the middle of the string. There are three nodes. We can fit a whole wave between the ends of the string. This means the wavelength is equal to the length of the string. This wave is called the first harmonic. As we add more nodes, we find the second harmonic, third harmonic, and so on. We must keep the nodes equally spaced or we will lose our standing wave.

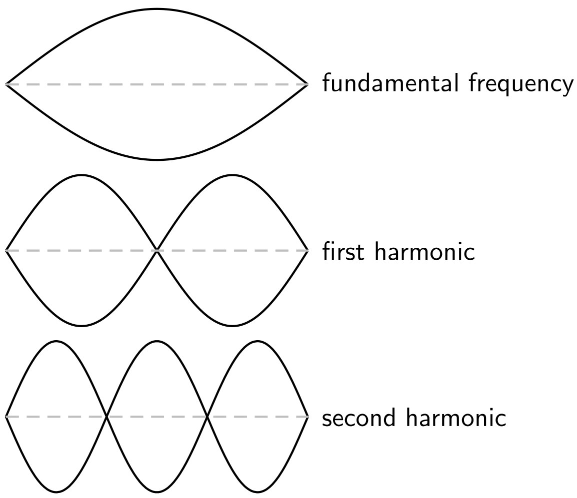



Figure 16.1: Harmonics on a string fixed at both ends.

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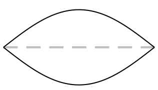
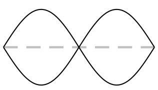
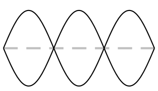
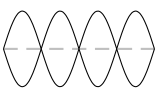
**Activity :: Investigation : Waves on a String Fixed at Both Ends**
This chart shows various waves on a string. The string length $L$ is the dashed line.

1. Fill in the:
   - number of nodes
   - number of anti-nodes
   - wavelength in terms of $L$

   The first and last waves are done for you.

| Wave | Nodes | Antinodes | Wavelength |
|---|---|---|---|
|  | 2 | 1 | $2L$ |
|  | | | |
|  | | | |
|  | 5 | 4 | $\frac{L}{2}$ |

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

You should have found this formula:

$$\lambda = \frac{2L}{n-1}$$

Here, $n$ is the number of nodes. $L$ is the length of the string. The frequency $f$ is:

$$f = \frac{v}{\lambda}$$

Here, $v$ is the velocity of the wave. This may seem confusing. The wave is a *standing* wave, so how can it have a velocity? But one standing wave is made up of many waves that travel back and forth on the string. Each of these waves has the same velocity. This speed depends on the mass and tension of the string.

**Worked Example 105: Harmonics on a String**

**Question:** We have a standing wave on a string that is 65 cm long. The wave has a velocity of 143 m.s$^{-1}$ Find the frequencies of the fundamental, first, second, and third harmonics.

**Answer**

**Step 1 : Identify what is given and what is asked:**

$$\begin{aligned} \text{L} &= 65\ \text{cm} = 0.65\ \text{m} \\ v &= 143\ \text{m.s}^{-1} \\ f &= ? \end{aligned}$$

To find the frequency we will use $f = \frac{v}{\lambda}$

**Step 2 : Find the wavelength for each harmonic:**

To find $f$ we need the wavelength of each harmonic ($\lambda = \frac{2L}{n-1}$). The wavelength is then substituted into $f = \frac{v}{\lambda}$ to find the harmonics. Table **??** below shows the calculations.

| | **Nodes** | **Wavelength** $\lambda = \frac{2L}{n-1}$ | **Frequency** $f = \frac{v}{\lambda}$ |
|---|---|---|---|
| Fundamental frequency $f_o$ | 2 | $\frac{2(0{,}65)}{2-1} = 1{,}3$ | $\frac{143}{1{,}3} = 110$ Hz |
| First harmonic $f_1$ | 3 | $\frac{2(0{,}65)}{3-1} =$ | $\frac{143}{} = 220$ Hz |
| Second harmonic $f_2$ | 4 | $\frac{2(0{,}65)}{4-1} =$ | $\frac{143}{} = 330$ Hz |
| Third harmonic $f_3$ | 5 | $\frac{2(0{,}65)}{5-1} =$ | $\frac{143}{} = 440$ Hz |

110 Hz is the natural frequency of the A string on a guitar. The third harmonic, at 440 Hz, is the note that orchestras use for tuning.

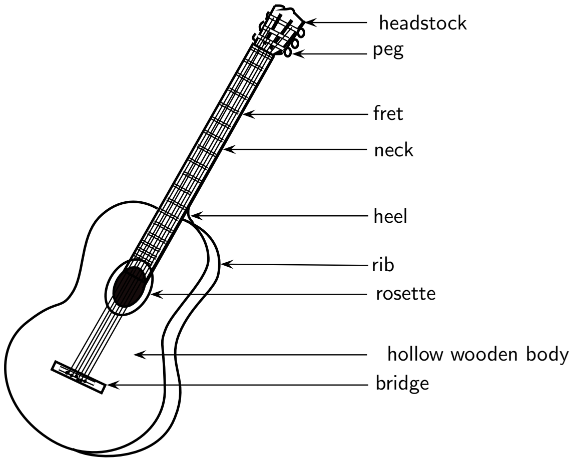
*Extension: Guitar*

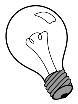
Guitars use strings with high tension. The length, tension and mass of the strings affect the pitches you hear. High tension and short strings make high frequencies; Low tension and long strings make low frequencies. When a string is first plucked, it vibrates at many frequencies. All of these except the harmonics are quickly filtered out. The harmonics make up the tone we hear.

The body of a guitar acts as a large wooden soundboard. Here is how a soundboard works: the body picks up the vibrations of the strings. It then passes these vibrations to the air. A sound hole allows the soundboard of the guitar to vibrate more freely. It also helps sound waves to get out of the body.

The neck of the guitar has thin metal bumps on it called frets. Pressing a string against a fret shortens the length of that string. This raises the natural frequency and the pitch of that string.

Most guitars use an "equal tempered" tuning of 12 notes per octave. A 6 string guitar has a range of 4 $\frac{1}{2}$ octaves with pitches from 82.407 Hz (low E) to 2093 kHz (high C). Harmonics may reach over 20 kHz, in the inaudible range.



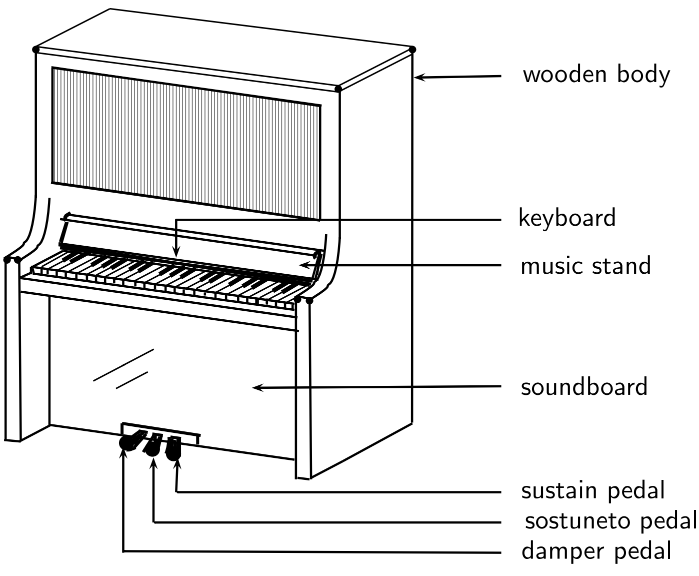



*Extension: Piano*

Let us look at another stringed instrument: the piano. The piano has strings that you can not see. When a key is pressed, a felt-tipped hammer hits a string inside the piano. The pitch depends on the length, tension and mass of the string. But there are many more strings than keys on a piano. This is because the short and thin strings are not as loud as the long and heavy strings. To make up for this, the higher keys have groups of two to four strings each.

The soundboard in a piano is a large cast iron plate. It picks up vibrations from the strings. This heavy plate can withstand over 200 tons of pressure from string tension! Its mass also allows the piano to sustain notes for long periods of time.

The piano has a wide frequency range, from 27,5 Hz (low A) to 4186,0 Hz (upper C). But these are just the fundamental frequencies. A piano plays complex, rich tones with over 20 harmonics per note. Some of these are out of the range of human hearing. Very low piano notes can be heard mostly because of their higher harmonics.



## 16.3 Standing Waves in Wind Instruments

A wind instrument is an instrument that is usually made with a a pipe or thin tube. Examples of wind instruments are recorders, clarinets, flutes, organs etc.

When one plays a wind instrument, the air that is pushed through the pipe vibrates and standing waves are formed. Just like with strings, the wavelengths of the standing waves will depend on the length of the pipe and whether it is open or closed at each end. Let's consider each of the following situations:

- A pipe with both ends open, like a flute or organ pipe.

- A pipe with one end open and one closed, like a clarinet.

If you blow across a small hole in a pipe or reed, it makes a sound. If both ends are open, standing waves will form according to figure 16.2. You will notice that there is an anti-node at each end. In the next activity you will find how this affects the wavelengths.

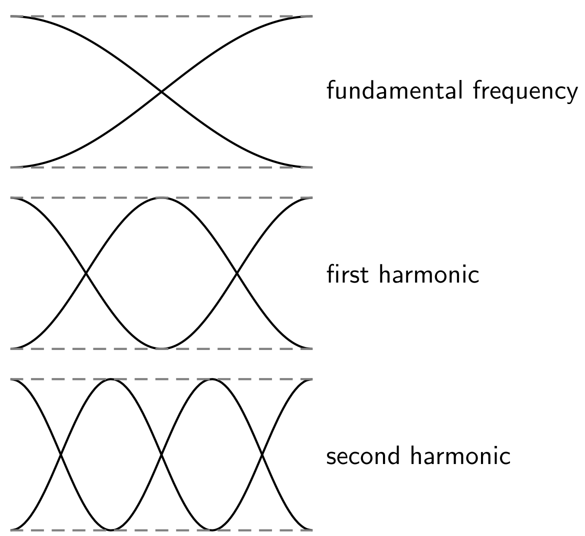



Figure 16.2: Harmonics in a pipe open at both ends.

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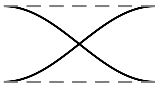
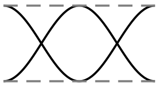
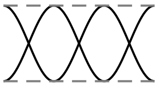
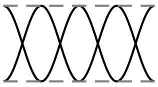
**Activity :: Investigation : Waves in a Pipe Open at Both Ends**

This chart shows some standing waves in a pipe open at both ends. The pipe (shown with dashed lines) has length L.

1. Fill in the:
   - number of nodes
   - number of anti-nodes
   - wavelength in terms of $L$

   The first and last waves are done for you.

| Wave | Nodes | Antinodes | Wavelength |
|---|---|---|---|
|  | 1 | 2 | $2L$ |
|  | | | |
|  | | | |
|  | 4 | 5 | $\frac{L}{2}$ |

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

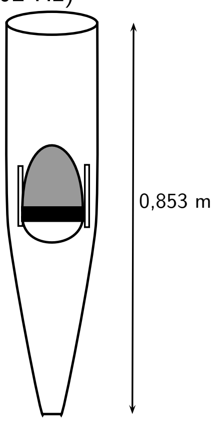
The formula is different because there are more anti-nodes than nodes. The right formula is:

$$\lambda_n = \frac{2L}{n}$$

Here, $n$ is still the number of nodes.

**Worked Example 106: The Organ Pipe**

**Question:** An open organ pipe is 0,853 m long. The speed of sound in air is 345 m.s$^{-1}$. Can this pipe play middle C? (Middle C has a frequency of about 262 Hz)




**Answer**

The main frequency of a note is the fundamental frequency. The fundamental frequency of the open pipe has one node.

**Step 1 : To find the frequency we will use the equation:**

$$f = \frac{v}{\lambda}$$

We need to find the wavelength first.

$$\begin{aligned} \lambda &= \frac{2L}{n} \\ &= \frac{2(0{,}853)}{1} \\ &= 1{,}706 \text{ m} \end{aligned}$$

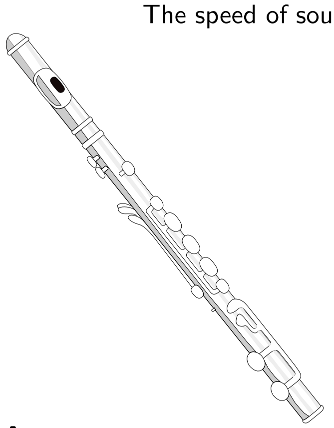
**Step 2 : Now we can calculate the frequency:**

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{345}{1{,}706} \\ &= 202 \text{ Hz} \end{aligned}$$

This is lower than 262 Hz, so this pipe will not play middle C. We will need a shorter pipe for a higher pitch.

**Worked Example 107: The Flute**

**Question:** A flute can be modeled as a metal pipe open at both ends. (One end looks closed but the flute has an *embouchure*, or hole for the player to blow across. This hole is large enough for air to escape on that side as well.) If the fundamental note of a flute is middle C, how long is the flute? The speed of sound in air is 345 m.s$^{-1}$.



**Answer**

We can calculate the length of the flute from $\lambda = \frac{2L}{n}$ but

**Step 1 : We need to calculate the wavelength first:**

$$\begin{aligned} f &= \frac{v}{\lambda} \\ 262 &= \frac{345}{\lambda} \\ \lambda &= \frac{345}{262} = 1{,}32 \text{ m} \end{aligned}$$

**Step 2 : Using the wavelength, we can now solve for $L$:**

$$\begin{aligned} \lambda &= \frac{2L}{n} \\ &= \frac{2L}{1} \\ L &= \frac{1{,}32}{2} = 0{,}66 \text{ m} \end{aligned}$$

Now let's look at a pipe that is open on one end and closed on the other. This pipe has a node at one end and an antinode at the other. An example of a musical instrument that has a node at one end and an antinode at the other is a clarinet. In the activity you will find out how the wavelengths are affected.

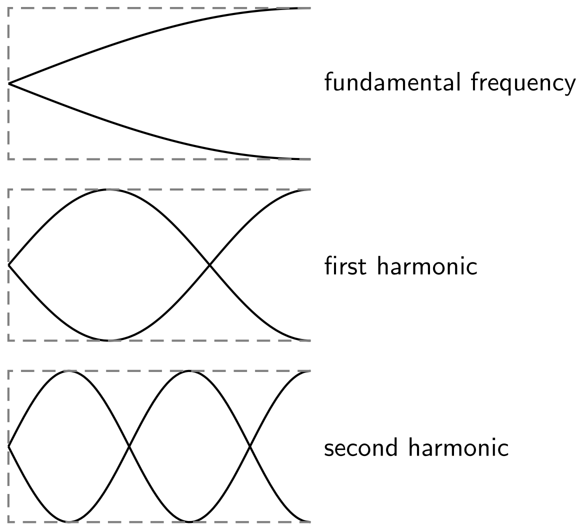


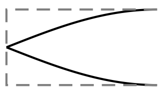
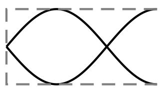
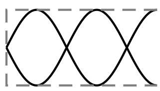
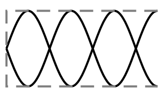

Figure 16.3: Harmonics in a pipe open at one end.

---

**Activity :: Investigation : Waves in a Pipe open at One End**
This chart shows some standing waves in a pipe open at *one* end. The pipe (shown as dashed lines) has length L.

1. Fill in the:
   - number of nodes
   - number of anti-nodes
   - wavelength in terms of $L$

   The first and last waves are done for you.

| Wave | Nodes | Antinodes | Wavelength |
|---|---|---|---|
|  | 1 | 1 | $4L$ |
|  | | | |
|  | | | |
|  | 4 | 4 | $\frac{4L}{7}$ |

2. Use the chart to find a formula for the wavelength in terms of the number of nodes.

---

The right formula for this pipe is:

$$\lambda_n = \frac{4L}{2n-1}$$

A long wavelength has a low frequency and low pitch. If you took your pipe from the last example and covered one end, you should hear a much lower note! Also, the wavelengths of the harmonics for this tube are *not* integer multiples of each other.

**Worked Example 108: The Clarinet**

**Question:** A clarinet can be modeled as a wooden pipe closed on one end and open on the other. The player blows into a small slit on one end. A reed then vibrates in the mouthpiece. This makes the standing wave in the air. What is the fundamental frequency of a clarinet 60 cm long? The speed of sound in air is 345 m.s$^{-1}$.

**Answer**

**Step 1 : Identify what is given and what is asked:**

We are given:

$$\begin{aligned} L &= 60 \text{ cm} \\ v &= 345 \text{ m.s}^{-1} \\ f &= ? \end{aligned}$$

**Step 2 : To find the frequency we will use the equation $f = \frac{v}{\lambda}$ but we need to find the wavelength first:**

$$\begin{aligned} \lambda &= \frac{4L}{2n-1} \\ &= \frac{4(0{,}60)}{2(1)-1} \\ &= 2{,}4 \text{ m} \end{aligned}$$

**Step 3 : Now, using the wavelength you have calculated, find the frequency:**

$$\begin{aligned} f &= \frac{v}{\lambda} \\ &= \frac{345}{2{,}4} \\ &= 144 \text{ Hz} \end{aligned}$$

This is closest to the D below middle C. This note is one of the lowest notes on a clarinet.

*Extension: Musical Scale*

The 12 tone scale popular in Western music took centuries to develop. This scale is also called the 12-note Equal Tempered scale. It has an octave divided into 12 steps. (An **octave** is the main interval of most scales. If you double a frequency, you have raised the note one octave.) All steps have equal ratios of frequencies. But this scale is not perfect. If the octaves are in tune, all the other intervals are slightly mistuned. No interval is badly out of tune. But none is perfect.

For example, suppose the base note of a scale is a frequency of 110 Hz ( a low A). The first harmonic is 220 Hz. This note is also an A, but is one octave higher. The second harmonic is at 330 Hz (close to an E). The third is 440 Hz (also an A). But not all the notes have such simple ratios. Middle C has a frequency of about 262 Hz. This is not a simple multiple of 110 Hz. So the interval between C and A is a little out of tune.

Many other types of tuning exist. Just Tempered scales are tuned so that all intervals are simple ratios of frequencies. There are also equal tempered scales with more or less notes per octave. Some scales use as many as 31 or 53 notes.

## 16.4 Resonance

Resonance is the tendency of a system to vibrate at a maximum amplitude at the natural frequency of the system.

Resonance takes place when a system is made to vibrate at its natural frequency as a result of vibrations that are received from another source of the same frequency. In the following investigation you will measure the speed of sound using resonance.

---

**Activity :: Experiment : Using resonance to measure the speed of sound**

**Aim:**

To measure the speed of sound using resonance

**Apparatus:**

- one measuring cylinder
- a high frequency (512 Hz) tuning fork
- some water
- a ruler or tape measure

**Method:**

1. Make the tuning fork vibrate by hitting it on the sole of your shoe or something else that has a rubbery texture. A hard surface is not ideal as you can more easily damage the tuning fork.
2. Hold the vibrating tuning fork about 1 cm above the cylinder mouth and start adding water to the cylinder at the same time. Keep doing this until the first resonance occurs. Pour out or add a little water until you find the level at which the loudest sound (i.e. the resonance) is made.
3. When the water is at the resonance level, use a ruler or tape measure to measure the distance ($L_A$) between the top of the cylinder and the water level.
4. Repeat the steps **??** above, this time adding more water until you find the next resonance. Remember to hold the tuning fork at the same height of about 1 cm above the cylinder mouth and adjust the water level to get the loudest sound.
5. Use a ruler or tape measure to find the new distance ($L_B$) from the top of the cylinder to the new water level.

**Conclusions:**

The difference between the two resonance water levels (i.e. $L = L_A - L_B$) is half a wavelength, or the same as the distance between a compression and rarefaction. Therefore, since you know the wavelength, and you know the frequency of the tuning fork, it is easy to calculate the speed of sound!

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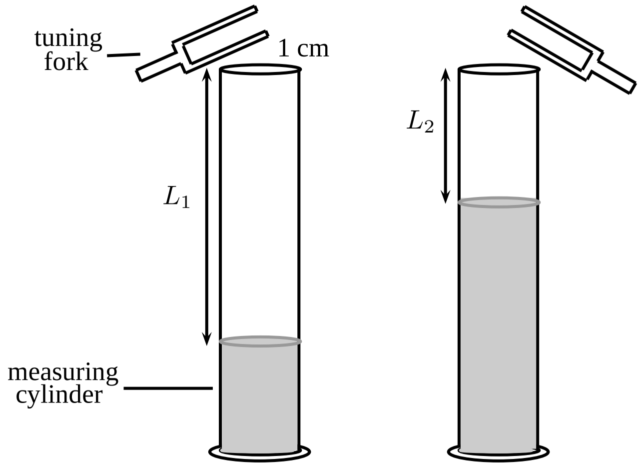



Interesting fact: Soldiers march out of time on bridges to avoid stimulating the bridge to vibrate at its natural frequency.

### Worked Example 109: Resonance

**Question:** A 512 Hz tuning fork can produce a resonance in a cavity where the air column is 18,2 cm long. It can also produce a second resonance when the length of the air column is 50,1 cm. What is the speed of sound in the cavity?

**Answer**

**Step 1 : Identify what is given and what is asked:**

$$\begin{aligned} L_1 &= 18{,}2 \text{ cm} \\ L_2 &= 50{,}3 \text{ cm} \\ f &= 512 \text{ Hz} \\ v &= ? \end{aligned}$$

Remember that:

$$v = f \times \lambda$$

We have values for $f$ and so to calculate $v$, we need to first find $\lambda$. You know that the difference in the length of the air column between two resonances is half a wavelength.

**Step 2 : Calculate the difference in the length of the air column between the two resonances:**

$$L_2 - L_1 = 32{,}1 \text{ cm}$$

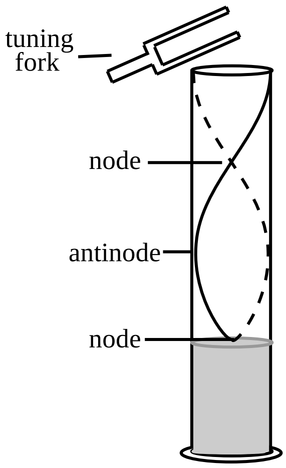
Therefore 32,1 cm $= \frac{1}{2} \times \lambda$
So,

$$\begin{aligned} \lambda &= 2 \times 32{,}1 \text{ cm} \\ &= 64{,}2 \text{ cm} \\ &= 0{,}642 \text{ m} \end{aligned}$$

**Step 3 : Now you can substitute into the equation for $v$ to find the speed of sound:**

$$
\begin{aligned}
v &= f \times \lambda \\
&= 512 \times 0{,}642 \\
&= 328{,}7\ \text{m.s}^{-1}
\end{aligned}
$$

From the investigation you will notice that the column of air will make a sound at a certain length. This is where resonance takes place.




## 16.5 Music and Sound Quality

In the sound chapter, we referred to the quality of sound as its tone. What makes the tone of a note played on an instrument? When you pluck a string or vibrate air in a tube, you hear mostly the fundamental frequency. Higher harmonics are present, but are fainter. These are called **overtones**. The tone of a note depends on its mixture of overtones. Different instruments have different mixtures of overtones. This is why the same note sounds different on a flute and a piano.

Let us see how overtones can change the shape of a wave:

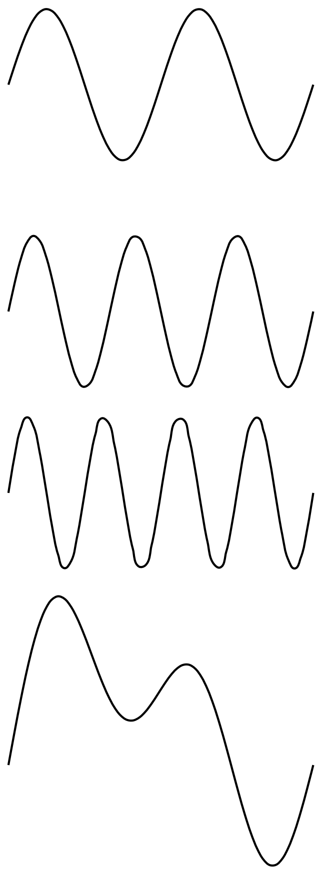



Figure 16.4: The quality of a tone depends on its mixture of harmonics.

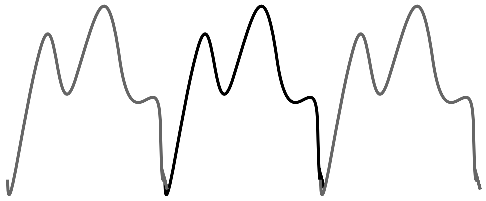
The resultant waveform is very different from the fundamental frequency. Even though the two waves have the same main frequency, they do not sound the same!

## 16.6 Summary - The Physics of Music

1. Instruments produce sounds because they form standing waves in strings or pipes.
2. The fundamental frequency of a string or a pipe is its natural frequency. The wavelength of the fundamental frequency is twice the length of the string or pipe.
3. The first harmonic is formed when the standing wave forms one whole wavelength in the string or pipe. The second harmonic is formed when the standing wave forms $1\frac{1}{2}$ wavelengths in the string or pipe.
4. The frequency of a standing wave can be calculated with the equation $f = \frac{v}{\lambda}$.
5. The wavelength of a standing wave in a string fixed at both ends can be calculated using $\lambda_n = \frac{2L}{n-1}$.
6. The wavelength of a standing wave in a pipe with both ends open can be calculated using $\lambda_n = \frac{2L}{n}$.
7. The wavelength of a standing wave in a pipe with one end open can be calculated using $\lambda_n = \frac{4L}{2n-1}$.
8. Resonance takes place when a system is made to vibrate at its own natural frequency as a result of vibrations received from another source of the same frequency.

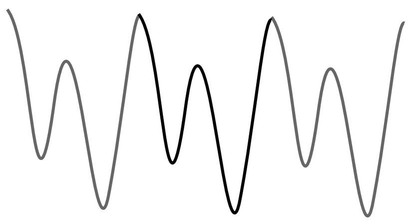
*Extension: Waveforms*

Below are some examples of the waveforms produced by a flute, clarinet and saxophone for different frequencies (i.e. notes):



Flute waveform
$B_4$, 247 Hz

Clarinet waveform
$E_\flat$, 156 Hz



Saxophone waveform
$C_4$, 256 Hz

## 16.7 End of Chapter Exercises

1. A guitar string with a length of 70 cm is plucked. The speed of a wave in the string is 400 m·s$^{-1}$. Calculate the frequency of the first, second, and third harmonics.
2. A pitch of Middle D (first harmonic = 294 Hz) is sounded out by a vibrating guitar string. The length of the string is 80 cm. Calculate the speed of the standing wave in the guitar string.
3. A frequency of the first harmonic is 587 Hz (pitch of D5) is sounded out by a vibrating guitar string. The speed of the wave is 600 m·s$^{-1}$. Find the length of the string.
4. Two notes which have a frequency ratio of 2:1 are said to be separated by an octave. A note which is separated by an octave from middle C (256 Hz) is

   A 254 Hz

   B 128 Hz

   C 258 Hz

   D 512 Hz
5. Playing a middle C on a piano keyboard generates a sound at a frequency of 256 Hz. If the speed of sound in air is 345 m·s$^{-1}$, calculate the wavelength of the sound corresponding to the note of middle C.
6. What is resonance? Explain how you would demonstrate what resonance is if you have a measuring cylinder, tuning fork and water available.
7. A tuning fork with a frequency of 256 Hz produced resonance with an air column of length 25,2 cm and at 89,5 cm. Calculate the speed of sound in the air column.

# Chapter 17

# Electrostatics - Grade 11

## 17.1 Introduction

In Grade 10, you learnt about the force between charges. In this chapter you will learn exactly how to determine this force and about a basic law of electrostatics.

## 17.2 Forces between charges - Coulomb's Law

Like charges repel each other while opposite charges attract each other. If the charges are at rest then the force between them is known as the **electrostatic force**. The electrostatic force between charges increases when the magnitude of the charges increases or the distance between the charges decreases.
The electrostatic force was first studied in detail by Charles Coulomb around 1784. Through his observations he was able to show that the electrostatic force between two point-like charges is inversely proportional to the square of the distance between the objects. He also discovered that the force is proportional to the product of the charges on the two objects.

$$F \propto \frac{Q_1 Q_2}{r^2},$$

where $Q_1$ is the charge on the one point-like object, $Q_2$ is the charge on the second, and $r$ is the distance between the two. The magnitude of the electrostatic force between two point-like charges is given by *Coulomb's Law*.

**Definition: Coulomb's Law**
Coulomb's Law states that the magnitude of the electrostatic force between two point charges is directly proportional to the magnitudes of each charge and inversely proportional to the square of the distance between the charges.

$$F = k\frac{Q_1 Q_2}{r^2}$$

and the proportionality constant $k$ is called the *electrostatic constant* and has the value:

$$k = 8{,}99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{C}^{-2}.$$

*Extension: Similarity of Coulomb's Law to the Newton's Universal Law of Gravitation.*

Notice how similar Coulomb's Law is to the form of Newton's Universal Law of Gravitation between two point-like particles:

$$F_G = G\frac{m_1 m_2}{r^2},$$

where $m_1$ and $m_2$ are the masses of the two particles, $r$ is the distance between them, and $G$ is the gravitational constant.

Both laws represent the force exerted by particles (masses or charges) on each other that interact by means of a field.

It is very interesting that Coulomb's Law has been shown to be correct no matter how small the distance, nor how large the charge. For example it still applies inside the atom (over distances smaller than $10^{-10}\mathrm{m}$).

**Worked Example 110: Coulomb's Law I**

**Question:** Two point-like charges carrying charges of $+3 \times 10^{-9}\mathrm{C}$ and $-5 \times 10^{-9}\mathrm{C}$ are $2\,\mathrm{m}$ apart. Determine the magnitude of the force between them and state whether it is attractive or repulsive.

**Answer**

**Step 1 : Determine what is required**

We are required to find the force between two point charges given the charges and the distance between them.

**Step 2 : Determine how to approach the problem**

We can use Coulomb's Law to find the force.

$$F = k\frac{Q_1 Q_2}{r^2}$$

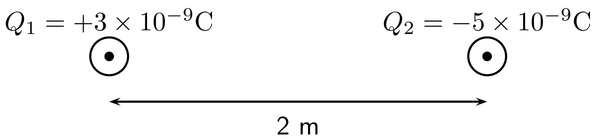
**Step 3 : Determine what is given**

We are given:

- $Q_1 = +3 \times 10^{-9}\mathrm{C}$
- $Q_2 = -5 \times 10^{-9}\mathrm{C}$
- $r = 2\,\mathrm{m}$

We know that $k = 8{,}99 \times 10^{9}\mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{C}^{-2}$.

We can draw a diagram of the situation.




**Step 4 : Check units**

All quantities are in SI units.

**Step 5 : Determine the magnitude of the force**

Using Coulomb's Law we have

$$\begin{aligned} F &= k\frac{Q_1 Q_2}{r^2} \\ &= (8{,}99 \times 10^{9}\mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2)\frac{(3 \times 10^{-9}\mathrm{C})(5 \times 10^{-9}\mathrm{C})}{(2\mathrm{m})^2} \\ &= 3{,}37 \times 10^{-8}\mathrm{N} \end{aligned}$$

Thus the *magnitude* of the force is $3{,}37 \times 10^{-8}\mathrm{N}$. However since both point charges have opposite signs, the force will be attractive.

Next is another example that demonstrates the difference in magnitude between the gravitational force and the electrostatic force.

**Worked Example 111: Coulomb's Law II**

**Question:** Determine the electrostatic force and gravitational force between two electrons $10^{-10}\text{m}$ apart (i.e. the forces felt inside an atom)

**Answer**

**Step 1 : Determine what is required**

We are required to calculate the electrostatic and gravitational forces between two electrons, a given distance apart.

**Step 2 : Determine how to approach the problem**

We can use:

$$F_e = k\frac{Q_1Q_2}{r^2}$$

to calculate the electrostatic force and

$$F_g = G\frac{m_1m_2}{r^2}$$

to calculate the gravitational force.

**Step 3 : Determine what is given**

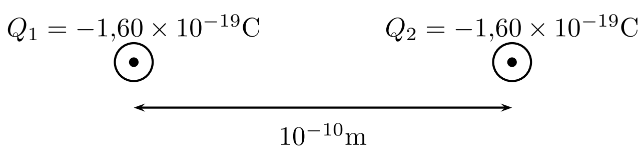
- $Q_1 = Q_2 = 1{,}6 \times 10^{-19}$ C(The charge on an electron)
- $m_1 = m_2 = 9{,}1 \times 10^{-31}$ kg(The mass of an electron)
- $r = 1 \times 10^{-10}\text{m}$

We know that:

- $k = 8{,}99 \times 10^9\ \text{N} \cdot \text{m}^2 \cdot \text{C}^{-2}$
- $G = 6{,}67 \times 10^{-11}\ \text{N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$

All quantities are in SI units.

We can draw a diagram of the situation.




**Step 4 : Calculate the electrostatic force**

$$\begin{aligned} F_e &= k\frac{Q_1Q_2}{r^2} \\ &= (8{,}99 \times 10^9)\frac{(-1{,}60 \times 10^{-19})(-1{,}60 \times 10^{-19})}{(10^{-10})^2} \\ &= 2{,}30 \times 10^{-8}\ \text{N} \end{aligned}$$

Hence the *magnitude* of the electrostatic force between the electrons is $2{,}30 \times 10^{-8}\text{N}$. Since electrons carry the same charge, the force is repulsive.

**Step 5 : Calculate the gravitational force**

$$\begin{aligned} F_g &= G\frac{m_1m_2}{r^2} \\ &= (6{,}67 \times 10^{-11}\text{N} \cdot \text{m}^2/\text{kg}^2)\frac{(9.11 \times 10^{-31}\text{C})(9.11 \times 10^{-31}\text{kg})}{(10^{-10}\text{m})^2} \\ &= 5{,}54 \times 10^{-51}\text{N} \end{aligned}$$

The magnitude of the gravitational force between the electrons is $5{,}54 \times 10^{-51}\text{N}$. This is an attractive force.

Notice that the gravitational force between the electrons is much smaller than the electrostatic force. For this reason, the gravitational force is usually neglected when determining the force between two charged objects.

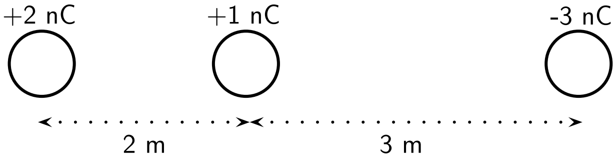
**Important:** We can apply Newton's Third Law to charges because, two charges exert forces of equal magnitude on one another in opposite directions.

**Important:** Coulomb's Law

When substituting into the Coulomb's Law equation, it is not necessary to include the signs of the charges. Instead, select a positive direction. Then forces that tend to move the charge in this direction are added, while forces that act in the opposite direction are subtracted.

**Worked Example 112: Coulomb's Law III**

**Question:** Three point charges are in a straight line. Their charges are $Q_1 = +2 \times 10^{-9}\text{C}$, $Q_2 = +1 \times 10^{-9}\text{C}$ and $Q_3 = -3 \times 10^{-9}\text{C}$. The distance between $Q_1$ and $Q_2$ is $2 \times 10^{-2}\text{m}$ and the distance between $Q_2$ and $Q_3$ is $4 \times 10^{-2}\text{m}$. What is the net electrostatic force on $Q_2$ from the other two charges?




**Answer**

**Step 1 : Determine what is required**

We are needed to calculate the net force on $Q_2$. This force is the sum of the two electrostatic forces - the forces between $Q_1$ on $Q_2$ and $Q_3$ on $Q_2$.

**Step 2 : Determine how to approach the problem**

- We need to calculate the two electrostatic forces on $Q_2$, using Coulomb's Law equation.
- We then need to add up the two forces using our rules for adding vector quantities, because force is a vector quantity.

**Step 3 : Determine what is given**

We are given all the charges and all the distances.

**Step 4 : Calculate the forces.**

Force of $Q_1$ on $Q_2$:

$$\begin{aligned} F &= k\frac{Q_1 Q_2}{r^2} \\ &= (8{,}99 \times 10^9)\frac{(2 \times 10^{-9})(1 \times 10^{-9})}{(2 \times 10^{-9})} \\ &= 4{,}5 \times 10^{-5}\text{N} \end{aligned}$$

Force of $Q_3$ on $Q_2$:

$$\begin{aligned} F &= k\frac{Q_2 Q_3}{r^2} \\ &= (8{,}99 \times 10^9)\frac{(1 \times 10^{-9})(3 \times 10^{-9})}{(4 \times 10^{-9}} \\ &= 1{,}69 \times 10^{-5}\text{N} \end{aligned}$$

Both forces act in the same direction because the force between $Q_1$ and $Q_2$ is repulsive (like charges) and the force between $Q_2$ on $Q_3$ is attractive (unlike charges).

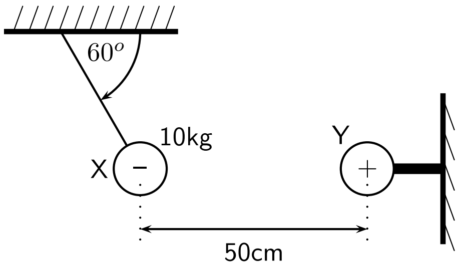
Therefore,

$$\begin{aligned} F_n et &= 4{,}50 \times 10^{-5} + 4{,}50 \times 10^{-5} \\ &= 6{,}19 \times 10^{-5}\text{N} \end{aligned}$$

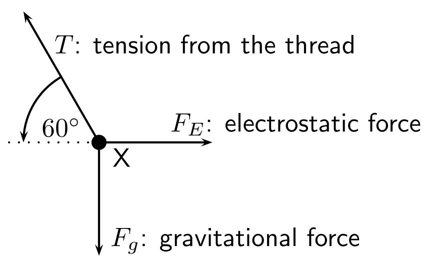
We mentioned in Chapter 9 that charge placed on a spherical conductor spreads evenly along the surface. As a result, if we are far enough from the charged sphere, electrostatically, it behaves as a point-like charge. Thus we can treat spherical conductors (e.g. metallic balls) as point-like charges, with all the charge acting at the centre.

**Worked Example 113: Coulomb's Law: challenging question**

**Question:** In the picture below, X is a small negatively charged sphere with a mass of 10kg. It is suspended from the roof by an insulating rope which makes an angle of $60°$ with the roof. Y is a small positively charged sphere which has the same magnitude of charge as X. Y is fixed to the wall by means of an insulating bracket. Assuming the system is in equilibrium, what is the magnitude of the charge on X?




**Answer**
How are we going to determine the charge on X? Well, if we know the force between X and Y we can use Coulomb's Law to determine their charges as we know the distance between them. So, firstly, we need to determine the magnitude of the electrostatic force between X and Y.
**Step 1 :**
Is everything in S.I. units? The distance between X and Y is $50\text{cm} = 0{,}5\text{m}$, and the mass of X is 10kg.
**Step 2 : Draw a force diagram**
Draw the forces on X (with directions) and label.




**Step 3 : Calculate the magnitude of the electrostatic force, $F_E$**
Since nothing is moving (system is in equilibrium) the vertical and horizontal components of the forces must cancel. Thus

$$F_E = T\cos(60°); \qquad F_g = T\sin(60°).$$

The only force we know is the gravitational force $F_g = mg$. Now we can calculate the magnitude of $T$ from above:

$$T = \frac{F_g}{\sin(60°)} = \frac{(10)(10)}{\sin(60°)} = 115{,}5\text{N}.$$

Which means that $F_E$ is:

$$F_E = T\cos(60°) = 115{,}5 \cdot \cos(60°) = 57{,}75\text{N}$$

**Step 4 :**
Now that we know the magnitude of the electrostatic force between X and Y, we can calculate their charges using Coulomb's Law. Don't forget that the magnitudes of the charges on X and Y are the same: $Q_\text{X} = Q_\text{Y}$. The magnitude of the electrostatic force is

$$\begin{aligned} F_E &= k\frac{Q_\text{X}Q_\text{Y}}{r^2} = k\frac{Q_\text{X}^2}{r^2} \\ Q_\text{X} &= \sqrt{\frac{F_E r^2}{k}} \\ &= \sqrt{\frac{(57.75)(0.5)^2}{8.99 \times 10^9}} \\ &= 5.66 \times 10^{-5}\text{C} \end{aligned}$$

Thus the charge on X is $-5.66 \times 10^{-5}$C.

?

**Exercise: Electrostatic forces**

1. Calculate the electrostatic force between two charges of $+6\text{nC}$ and $+1\text{nC}$ if they are separated by a distance of $2\text{mm}$.
2. Calculate the distance between two charges of $+4\text{nC}$ and $-3\text{nC}$ if the electrostaticforce between them is $0{,}005\text{N}$.
3. Calculate the charge on two identical spheres that are similiarly charged if they are separated by $20\text{cm}$ and the electrostatic force between them is $0{,}06\text{N}$.

## 17.3 Electric field around charges

We have learnt that objects that carry charge feel forces from all other charged objects. It is useful to determine what the effect of a charge would be at every point surrounding it. To do this we need some sort of reference. We know that the force that one charge feels due to another depends on both charges ($Q_1$ and $Q_2$). How then can we talk about forces if we only have one charge? The solution to this dilemma is to introduce a *test charge*. We then determine the force that would be exerted on it if we placed it at a certain location. If we do this for every point surrounding a charge we know what would happen if we put a test charge at any location.
This map of what would happen at any point we call an electric field map. It is a map of the electric field *due to* a charge. It tells us how large the force on a test charge would be and in what direction the force would be. Our map consists of the lines that tell us how the test charge would move if it were placed there.

**Definition: Electric field**
An electric field as a region of space in which an electric charge experiences a force. The direction of the electric field at a point is the direction that a positive test charge would move if placed at that point.

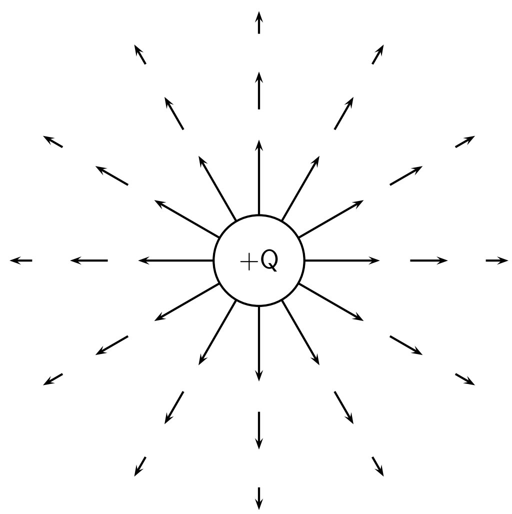
## 17.3.1 Electric field lines

The maps depend very much on the charge or charges that the map is being made for. We will start off with the simplest possible case. Take a single positive charge with no other charges around it. First, we will look at what effects it would have on a test charge at a number of points.
Electric field lines, like the magnetic field lines that were studied in Grade 10, are a way of representing the electric field at a point.

- Arrows on the field lines indicate the direction of the field, i.e. the direction a positive test charge would move.
- Electric field lines therefore point away from positive charges and towards negative charges.
- Field lines are drawn closer together where the field is stronger.

## 17.3.2 Positive charge acting on a test charge

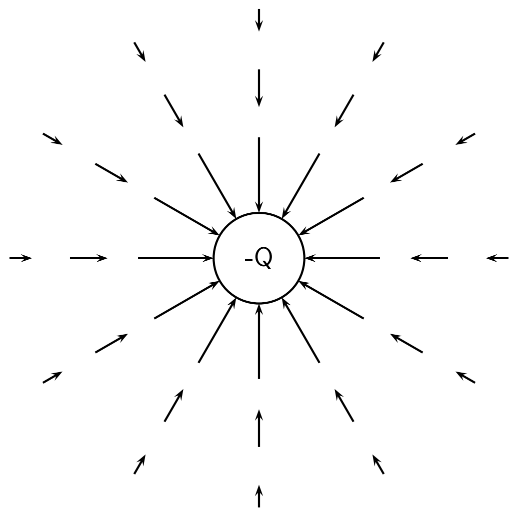
At each point we calculate the force on a test charge, $q$, and represent this force by a vector.




We can see that at every point the positive test charge, $q$, would experience a force pushing it away from the charge, $Q$. This is because both charges are positive and so they repel. Also notice that at points further away the vectors are shorter. That is because the force is smaller if you are further away.

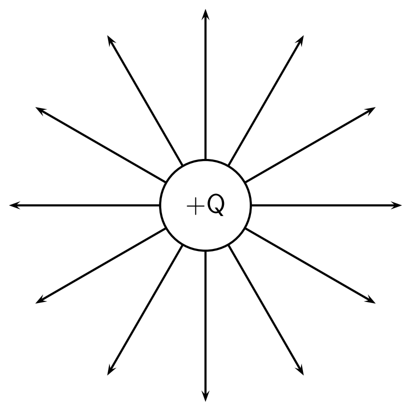
### Negative charge acting on a test charge

If the charge were negative we would have the following result.




Notice that it is **almost** identical to the positive charge case. This is important – the arrows are the same length because the magnitude of the charge is the same and so is the magnitude of the test charge. Thus the **magnitude** (size) of the force is the same. The arrows point in the opposite direction because the charges now have opposite sign and so the test charge is **attracted** to the charge. Now, to make things simpler, we draw continuous lines showing the path that the test charge would travel. This means we don't have to work out the magnitude of the force at many different points.

**Electric field map due to a positive charge**



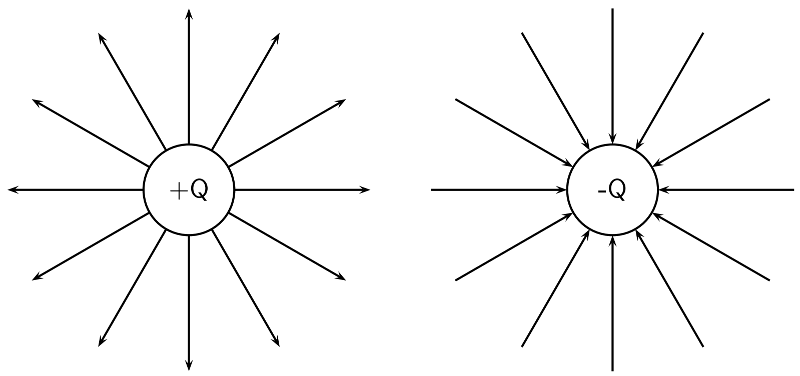

**Some important points to remember about electric fields:**

- There is an electric field at **every point** in space surrounding a charge.
- Field lines are merely a **representation** – they are not real. When we draw them, we just pick convenient places to indicate the field in space.
- Field lines always start at a **right-angle** ($90^o$) to the charged object causing the field.
- Field lines **never** cross.

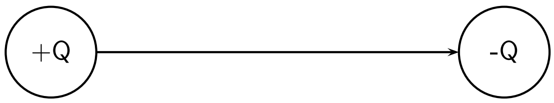
### 17.3.3 Combined charge distributions

We will now look at the field of a positive charge and a negative charge placed next to each other. The net resulting field would be the addition of the fields from each of the charges. To start off with let us sketch the field maps for each of the charges separately.

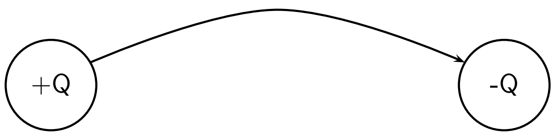
**Electric field of a negative and a positive charge in isolation**



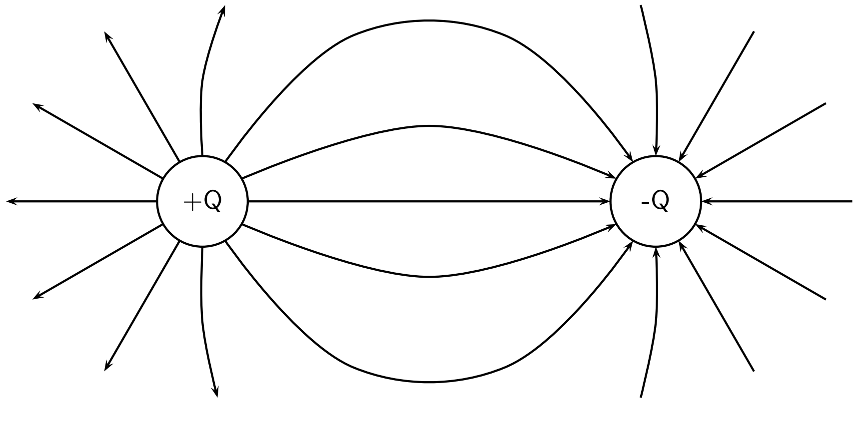

Notice that a test charge starting off directly between the two would be pushed away from the positive charge and pulled towards the negative charge in a straight line. The path it would follow would be a straight line between the charges.




Now let's consider a test charge starting off a bit higher than directly between the charges. If it starts closer to the positive charge the force it feels from the positive charge is greater, but the negative charge also attracts it, so it would move away from the positive charge with a tiny force attracting it towards the negative charge. As it gets further from the positive charge the force from the negative and positive charges change and they are equal in magnitude at equal distances from the charges. After that point the negative charge starts to exert a stronger force on the test charge. This means that the test charge moves towards the negative charge with only a small force away from the positive charge.

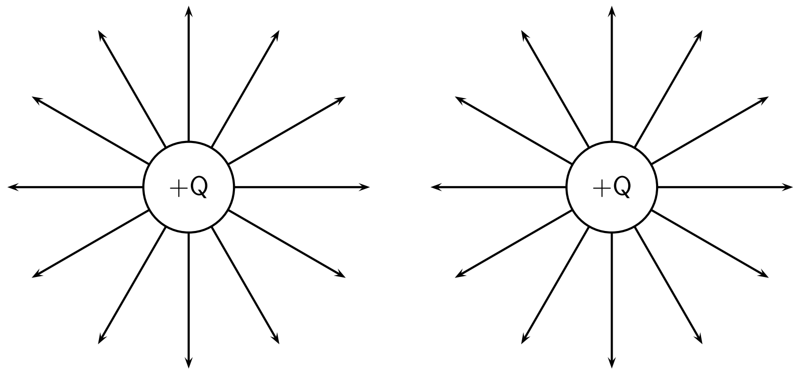



Now we can fill in the other lines quite easily using the same ideas. The resulting field map is:

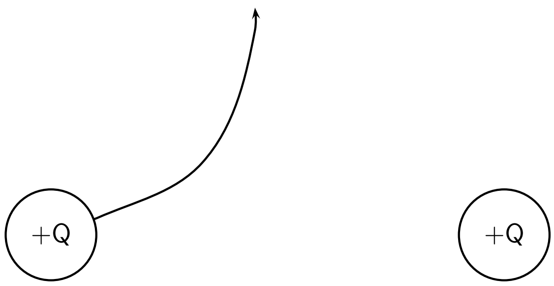


**Two like charges : both positive**

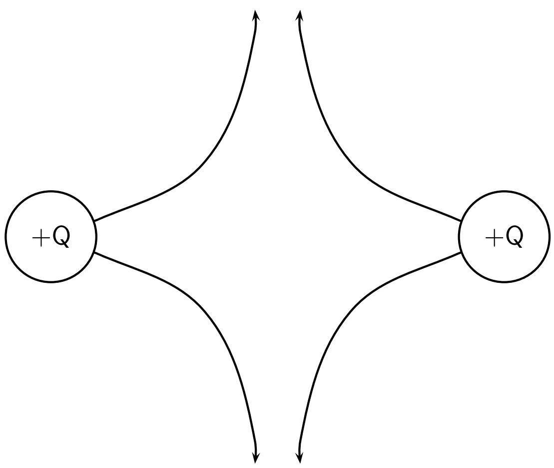
For the case of two positive charges things look a little different. We can't just turn the arrows around the way we did before. In this case the test charge is repelled by both charges. This tells us that a test charge will never cross half way because the force of repulsion from both charges will be equal in magnitude.



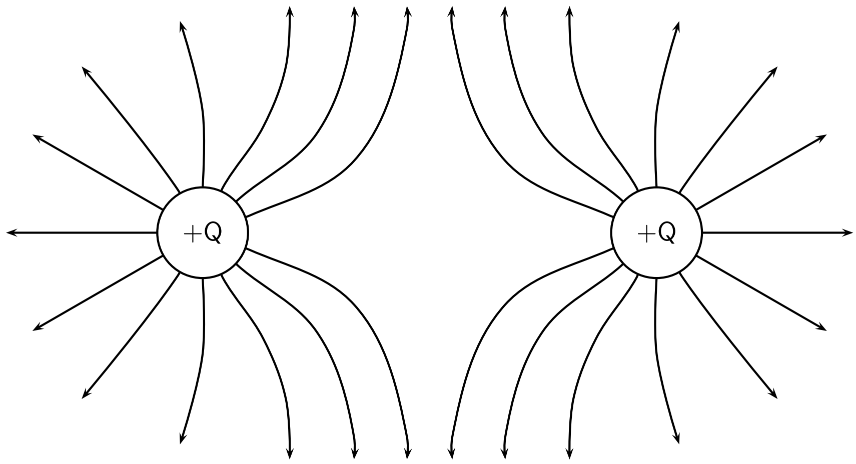

The field directly between the charges cancels out in the middle. The force has equal magnitude and opposite direction. Interesting things happen when we look at test charges that are not on a line directly between the two.




We know that a charge the same distance below the middle will experience a force along a reflected line, because the problem is symmetric (i.e. if we flipped vertically it would look the same). This is also true in the horizontal direction. So we use this fact to easily draw in the next four lines.

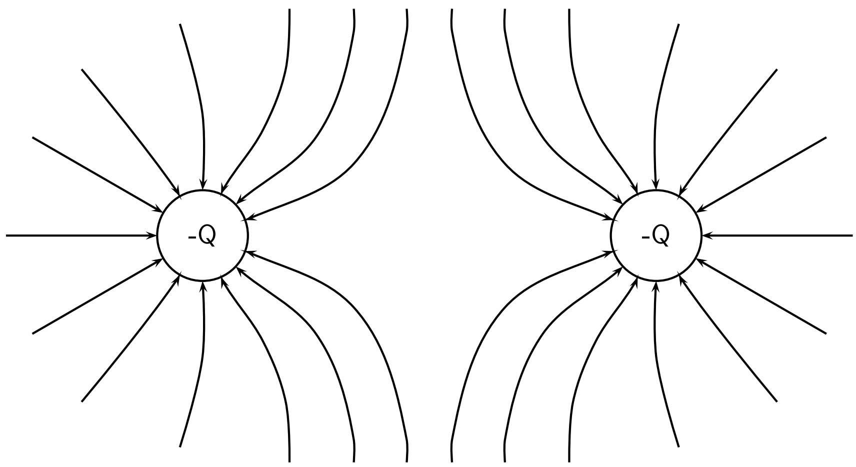



Working through a number of possible starting points for the test charge we can show the electric field map to be:




**Two like charges : both negative**

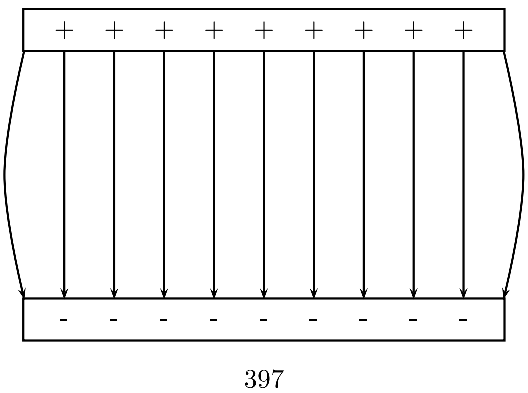
We can use the fact that the direction of the force is reversed for a test charge if you change the sign of the charge that is influencing it. If we change to the case where both charges are negative we get the following result:




### 17.3.4 Parallel plates

One very important example of electric fields which is used extensively is the electric field between two charged parallel plates. In this situation the electric field is constant. This is used for many practical purposes and later we will explain how Millikan used it to measure the charge on the electron.

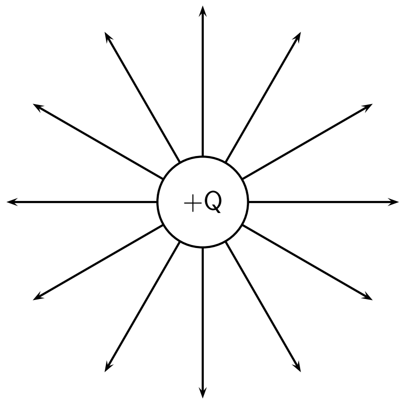
**Field map for oppositely charged parallel plates**



This means that the force that a test charge would feel at any point between the plates would be identical in magnitude and direction. The fields on the edges exhibit fringe effects, *i.e. they bulge outwards*. This is because a test charge placed here would feel the effects of charges only on one side (either left or right depending on which side it is placed). Test charges placed in the middle experience the effects of charges on both sides so they balance the components in the horizontal direction. This is clearly not the case on the edges.

### Strength of an electric field

When we started making field maps we drew arrows to indicate the strength of the field and the direction. When we moved to lines you might have asked "Did we forget about the field strength?". We did not. Consider the case for a single positive charge again:




Notice that as you move further away from the charge the field lines become more spread out. In field map diagrams the closer field lines are together the stronger the field. Therefore, the electric field is stronger closer to the charge (the electric field lines are closer together) and weaker further from the charge (the electric field lines are further apart).
The magnitude of the electric field at a point as the force per unit charge. Therefore,

$$E = \frac{F}{q}$$

E and F are vectors. From this we see that the force on a charge $q$ is simply:

$$F = E \cdot q$$

The force between two electric charges is given by:

$$F = k\frac{Qq}{r^2}.$$

(if we make the one charge $Q$ and the other $q$.) Therefore, the electric field can be written as:

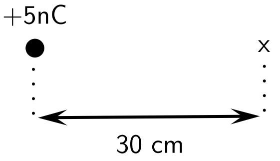
$$E = k\frac{Q}{r^2}$$

The electric field is the force per unit of charge and hence has units of newtons per coulomb.

As with Coulomb's law calculations, do not substitute the sign of the charge into the equation for electric field. Instead, choose a positive direction, and then either add or subtract the contribution to the electric field due to each charge depending upon whether it points in the positive or negative direction, respectively.

**Worked Example 114: Electric field 1**

**Question:** Calculate the electric field strength $30/rmcm$ from a $5/rmnC$ charge.




**Answer**

**Step 1 : Determine what is required**

We need to calculate the electric field a distance from a given charge.

**Step 2 : Determine what is given**

We are given the magnitude of the charge and the distance from the charge.

**Step 3 : Determine how to approach the problem**

We will use the equation:

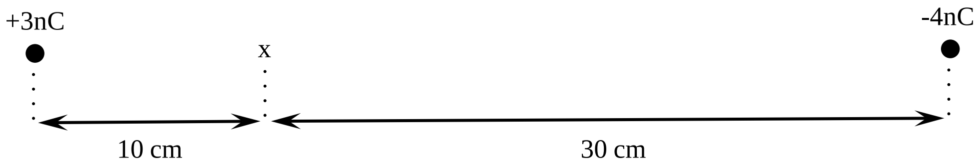
$$E = k\frac{Q}{r^2}.$$

**Step 4 : Solve the problem**

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= \frac{(8.99 \times 10^9)(5 \times 10^{-9})}{(0{,}3)^2} \\ &= 4{,}99 \times 102\text{N.C}^{-1} \end{aligned}$$

**Worked Example 115: Electric field 2**

**Question:** Two charges of $Q_1 = +3/rmnC$ and $Q_2 = -4/rmnC$ are separated by a distance of $50/rmcm$. What is the electric field strength at a point that is $20/rmcm$ from $Q_1$ and $50/rmcm$ from $Q_2$? The point lies beween $Q_1$ and $Q_2$.




**Answer**

**Step 1 : Determine what is required**

We need to calculate the electric field a distance from two given charges.

**Step 2 : Determine what is given**

We are given the magnitude of the charges and the distances from the charges.

**Step 3 : Determine how to approach the problem**

We will use the equation:

$$E = k\frac{Q}{r^2}.$$

We need to work out the electric field for each charge separately and then add them to get the resultant field.

**Step 4 : Solve the problem**

We first solve for $Q_1$:

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= \frac{(8.99 \times 10^9)(3 \times 10^{-9})}{(0{,}2)^2} \\ &= 6{,}74 \times 102\text{N.C}^{-1} \end{aligned}$$

Then for $Q_2$:

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= \frac{(8.99 \times 10^9)(4 \times 10^{-9})}{(0{,}3)^2} \\ &= 2{,}70 \times 102\mathrm{N.C}^{-1} \end{aligned}$$

We need to add the two electric field beacuse both are in the same direction. The field is away from $Q_1$ and towards $Q_2$. Therefore,
$E_total = 6{,}74 \times 102 + 2{,}70 \times 102 = 9{,}44 \times 102\mathrm{N.C}^{-1}$

## 17.4 Electrical potential energy and potential

The *electrical potential energy* of a charge is the energy it has because of its position relative to other charges that it interacts with. The potential energy of a charge $Q_1$ relative to a charge $Q_2$ a distance $r$ away is calculated by:

$$U = \frac{kQ_1Q_2}{r}$$

**Worked Example 116: Electrical potential energy 1**

**Question:** What is the electric potential energy of a $7\mathrm{nC}$ charge that is 2 cm from a $20\mathrm{nC}$?
**Answer**
**Step 1 : Determine what is required**
We need to calculate the electric potential energy (U).
**Step 2 : Determine what is given**
We are given both charges and the distance between them.
**Step 3 : Determine how to approach the problem**
We will use the equation:

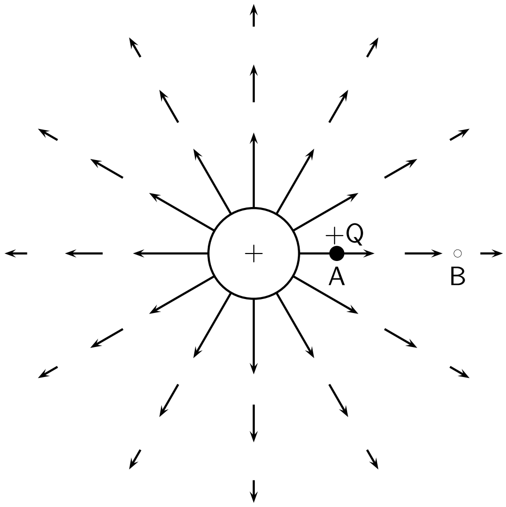
$$U = \frac{kQ_1Q_2}{r}$$

**Step 4 : Solve the problem**

$$\begin{aligned} U &= \frac{kQ_1Q_2}{r} \\ &= \frac{(8.99 \times 10^9)(7 \times 10^{-9})(20 \times 10^{-9})}{(0{,}02)} \\ &= 6{,}29 \times 10{-}5\mathrm{J} \end{aligned}$$

### 17.4.1 Electrical potential

The electric potential at a point is the electrical potential energy per unit charge, i.e. the potential energy a positive test charge would have if it were placed at that point.
Consider a positive test charge $+Q$ placed at A in the electric field of another positive point charge.




The test charge moves towards B under the influence of the electric field of the other charge. In the process the test charge loses electrical potential energy and gains kinetic energy. Thus, at A, the test charge has more potential energy than at B – **A is said to have a higher electrical potential than B**.

The potential energy of a charge at a point in a field is defined as the work required to move that charge from infinity to that point.

**Definition: Potential difference**

The **potential difference between two points** in an electric field is defined as the **work required to move a unit positive test charge from the point of lower potential to that of higher potential**.

If an amount of work $W$ is required to move a charge $Q$ from one point to another, then the potential difference between the two points is given by,

$$V = \frac{W}{Q} \qquad \text{unit : J.C}^{-1} \text{ or V (the volt)}$$

From this equation we can define the volt.

**Definition: The Volt**

One volt is the potential difference between two points in an electric field if one joule of work is done in moving one coulomb of charge from the one point to the other.

**Worked Example 117: Potential difference**

**Question:** What is the potential difference between two point in an electric field if it takes $600\text{J}$ of energy to move a charge of $2\text{C}$ between these two points.

**Answer**

**Step 5 : Determine what is required**

We need to calculate the potential difference (V) between two points in an electric field.

**Step 6 : Determine what is given**

We are given both the charges and the energy or work done to move the charge between the two points.

**Step 7 : Determine how to approach the problem**

We will use the equation:

$$V = \frac{W}{Q}$$

**Step 8 : Solve the problem**

$$\begin{aligned} V &= \frac{W}{Q} \\ &= \frac{600}{2} \\ &= 300\text{V} \end{aligned}$$

### 17.4.2 Real-world application: lightning

Lightning is an atmospheric discharge of electricity, usually, but not always, during a rain storm. An understanding of lightning is important for power transmission lines as engineers who need to know about lightning in order to adequately protect lines and equipment.

*Extension: Formation of lightning*

1. **Charge separation**
   The first process in the generation of lightning is charge separation. The mechanism by which charge separation happens is still the subject of research. One theory is that opposite charges are driven apart and energy is stored in the electric field between them. Cloud electrification appears to require strong updrafts which carry water droplets upward, supercooling them to $-10$ to $-20$ $^{o}$C. These collide with ice crystals to form a soft ice-water mixture called graupel. The collisions result in a slight positive charge being transferred to ice crystals, and a slight negative charge to the graupel. Updrafts drive lighter ice crystals upwards, causing the cloud top to accumulate increasing positive charge. The heavier negatively charged graupel falls towards the middle and lower portions of the cloud, building up an increasing negative charge. Charge separation and accumulation continue until the electrical potential becomes sufficient to initiate lightning discharges, which occurs when the gathering of positive and negative charges forms a sufficiently strong electric field.
2. **Leader formation**
   As a thundercloud moves over the Earth's surface, an equal but opposite charge is induced in the Earth below, and the induced ground charge follows the movement of the cloud. An initial bipolar discharge, or path of ionized air, starts from a negatively charged mixed water and ice region in the thundercloud. The discharge ionized channels are called leaders. The negative charged leaders, called a "stepped leader", proceed generally downward in a number of quick jumps, each up to 50 metres long. Along the way, the stepped leader may branch into a number of paths as it continues to descend. The progression of stepped leaders takes a comparatively long time (hundreds of milliseconds) to approach the ground. This initial phase involves a relatively small electric current (tens or hundreds of amperes), and the leader is almost invisible compared to the subsequent lightning channel. When a step leader approaches the ground, the presence of opposite charges on the ground enhances the electric field. The electric field is highest on trees and tall buildings. If the electric field is strong enough, a conductive discharge (called a positive streamer) can develop from these points. As the field increases, the positive streamer may evolve into a hotter, higher current leader which eventually connects to the descending stepped leader from the cloud. It is also possible for many streamers to develop from many different

objects simultaneously, with only one connecting with the leader and forming the main discharge path. Photographs have been taken on which non-connected streamers are clearly visible. When the two leaders meet, the electric current greatly increases. The region of high current propagates back up the positive stepped leader into the cloud with a "return stroke" that is the most luminous part of the lightning discharge.

3. **Discharge** When the electric field becomes strong enough, an electrical discharge (the bolt of lightning) occurs within clouds or between clouds and the ground. During the strike, successive portions of air become a conductive discharge channel as the electrons and positive ions of air molecules are pulled away from each other and forced to flow in opposite directions. The electrical discharge rapidly superheats the discharge channel, causing the air to expand rapidly and produce a shock wave heard as thunder. The rolling and gradually dissipating rumble of thunder is caused by the time delay of sound coming from different portions of a long stroke.

**Important:** Estimating distance of a lightning strike

The flash of a lightning strike and resulting thunder occur at roughly the same time. But light travels at 300 000 kilometres in a second, almost a million times the speed of sound. Sound travels at the slower speed of 330 m/s in the same time, so the flash of lightning is seen before thunder is heard. By counting the seconds between the flash and the thunder and dividing by 3, you can estimate your distance from the strike and initially the actual storm cell (in kilometres).

## 17.5 Capacitance and the parallel plate capacitor

### 17.5.1 Capacitors and capacitance

A parallel plate capacitor is a device that consists of two oppositely charged conducting plates separated by a small distance, which stores charge. When voltage is applied to the capacitor, electric charge of equal magnitude, but opposite polarity, build up on each plate.

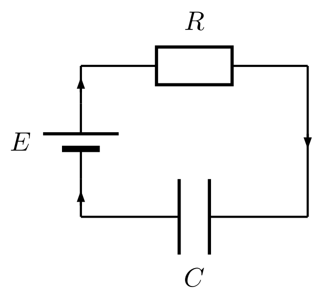



Figure 17.1: A capacitor (C) connected in series with a resistor (R) and an energy source (E).

**Definition: Capacitance**
Capacitance is the charge stored per volt and is measured in farad (F)

Mathematically, capacitance is the ratio of the charge on a single plate to the voltage across the plates of the capacitor:

$$C = \frac{Q}{V}.$$

Capacitance is measured in farads (F). Since capacitance is defined as $C = \frac{Q}{V}$, the units are in terms of charge over potential difference. The unit of charge is the coulomb and the unit of the potential difference is the volt. One farad is therefore the capacitance if one coulomb of charge was stored on a capacitor for every volt applied.
1 C of charge is a very large amount of charge. So, for a small amount of voltage applied, a 1 F capacitor can store a enormous amount of charge. Therefore, capacitors are often denoted in terms of microfarads ($1 \times 10^{-6}$), nanofarads ($1 \times 10^{-9}$), or picofarads ($1 \times 10^{-12}$).

**Important:** $Q$ is the magnitude of the charge stored on either plate, not on both plates added together. Since one plate stores positive charge and the other stores negative charge, the total charge on the two plates is zero.

**Worked Example 118: Capacitance**

**Question:** Suppose that a 5 V battery is connected in a circuit to a 5 pF capacitor. After the battery has been connected for a long time, what is the charge stored on each of the plates?

**Answer**

To begin remember that after a voltage has been applied for a long time the capacitor is fully charged. The relation between voltage and the maximum charge of a capacitor is found in equation **??**.

$$CV = Q$$

Inserting the given values of $C = 5\text{F}$ and $V = 5\text{V}$, we find that:

$$\begin{aligned} Q &= CV \\ &= (5 \times 10^{-12} F)(5V) \\ &= 2{,}5 \times 10^{-11} C \end{aligned}$$

### 17.5.2 Dielectrics

The electric field between the plates of a capacitor is affected by the substance between them. The substance between the plates is called a dielectric. Common substances used as dielectrics are mica, perspex, air, paper and glass.
When a dielectric is inserted between the plates of a parallel plate capacitor the dielectric becomes polarised so an electric field is induced in the dielectric that opposes the field between the plates. When the two electric fields are superposed, the new field between the plates becomes smaller. Thus the voltage between the plates decreases so the capacitance increases.
In every capacitor, the dielectric keeps the charge on one plate from travelling to the other plate. However, each capacitor is different in how much charge it allows to build up on the electrodes per voltage applied. When scientists started studying capacitors they discovered the property that the voltage applied to the capacitor was proportional to the maximum charge that would accumulate on the electrodes. The constant that made this relation into an equation was called the capacitance, C. The capacitance was different for different capacitors. But, it stayed constant no matter how much voltage was applied. So, it predicts how much charge will be stored on a capacitor when different voltages are applied.

### 17.5.3 Physical properties of the capacitor and capacitance

The capacitance of a capacitor is proportional to the surface area of the conducting plate and inversely proportional to the distance between the plates. It is also proportional to the

permittivity of the *dielectric*. The dielectric is the non-conducting substance that separates the plates. As mentioned before, dielectrics can be air, paper, mica, perspex or glass.
The capacitance of a parallel-plate capacitor is given by:

$$C = \epsilon_0 \frac{A}{d}$$

where $\epsilon_0$ is the permittivity of air, $A$ is the area of the plates and $d$ is the distance between the plates.

**Worked Example 119: Capacitance**

**Question:** What is the capacitance of a capacitor in which the dielectric is air, the area of the plates is $0{,}001\text{m}^2$ and the distance between the plates is $0{,}02\text{m}$?
**Answer**
**Step 1 : Determine what is required**
We need to determine the capacitance of the capacitor.
**Step 2 : Determine how to approach the problem**
We can use the formula:

$$C = \epsilon_0 \frac{A}{d}$$

**Step 3 : Determine what is given.**
We are given the area of the plates, the distance between the plates and that the dielectric is air.
**Step 4 : Determine the capacitance**

$$\begin{aligned} C &= \epsilon_0 \frac{A}{d} && (17.1) \\ &= \frac{(8{,}9 \times 10{-}12)(0{,}001)}{0{,}02} && (17.2) \\ &= 4{,}45 \times 10{-}13\text{F} && (17.3) \end{aligned}$$

## 17.5.4 Electric field in a capacitor

The electric field strength between the plates of a capacitor can be calculated using the formula:
$E = \frac{V}{d}$ where $E$ is the electric field in $\text{J.C}^{-1}$, $V$ is the potential difference in $\text{V}$ and $d$ is the distance between the plates in $\text{m}$.

**Worked Example 120: Electric field in a capacitor**

**Question:** What is the strength of the electric field in a capacitor which has a potential difference of $300\text{V}$ between its parallel plates that are $0{,}02\text{m}$ apart?
**Answer**
**Step 1 : Determine what is required**
We need to determine the electric field between the plates of the capacitor.
**Step 2 : Determine how to approach the problem**
We can use the formula:
$E = \frac{V}{d}$
**Step 3 : Determine what is given.**
We are given the potential difference and the distance between the plates.
**Step 4 : Determine the electric field**

$$
\begin{aligned}
E &= \frac{V}{d} && (17.4)\\
&= \frac{300}{0{,}02} && (17.5)\\
&= 1{,}50 \times 104\text{J.C}^{-1} && (17.6)\\
& && (17.7)
\end{aligned}
$$

?

**Exercise: Capacitance and the parallel plate capacitor**

1. Determine the capacitance of a capacitor which stores $9 \times 10^{-9}\text{C}$ when a potential difference of 12 V is applied to it.
2. What charge will be stored on a $5\mu\text{F}$ capacitor if a potential difference of $6\text{V}$ is maintained between its plates?
3. What is the capacitance of a capacitor that uses air as its dielectric if it has an area of $0{,}004\text{m}^2$ and a distance of $0{,}03\text{m}$ between its plates?
4. What is the strength of the electric field between the plates of a charged capacitor if the plates are $2\text{mm}$ apart and have a potential difference of $200\text{V}$ across them?

## 17.6 Capacitor as a circuit device

### 17.6.1 A capacitor in a circuit

When a capacitor is connected in a DC circuit, current will flow until the capacitor is fully charged. After that, no further current will flow. If the charged capacitor is connected to another circuit with no source of emf in it, the capacitor will discharge through the circuit, creating a potential difference for a short time. This is useful, for example, in a camera flash. Initially, the electrodes have no net charge. A voltage source is applied to charge a capacitor. The voltage source creates an electric field, causing the electrons to move. The charges move around the circuit stopping at the left electrode. Here they are unable to travel across the dielectric, since electrons cannot travel through an insulator. The charge begins to accumulate, and an electric field forms pointing from the left electrode to the right electrode. This is the opposite direction of the electric field created by the voltage source. When this electric field is equal to the electric field created by the voltage source, the electrons stop moving. The capacitor is then fully charged, with a positive charge on the left electrode and a negative charge on the right electrode.
If the voltage is removed, the capacitor will discharge. The electrons begin to move because in the absence of the voltage source, there is now a net electric field. This field is due to the imbalance of charge on the electrodes–the field across the dielectric. Just as the electrons flowed to the positive electrode when the capacitor was being charged, during discharge, the electrons flow to negative electrode. The charges cancel, and there is no longer an electric field across the dielectric.

### 17.6.2 Real-world applications: capacitors

Capacitors are used in many different types of circuitry. In car speakers, capacitors are often used to aid the power supply when the speaker require more power than the car battery can provide. Capacitors are also used to in processing electronic signals in circuits, such as smoothing voltage spikes due to inconsistent voltage sources. This is important for protecting sensitive electronic compoments in a circuit.

## 17.7 Summary

1. Objects can be **positively**, **negatively** charged or **neutral**.
2. Charged objects feel a force with a magnitude:

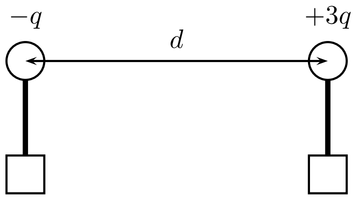
$$F = k\frac{Q_1 Q_2}{r^2}$$

3. The force is attractive for unlike charges and repulsive for like charges.
4. A test charge is $+1\text{C}$
5. Electric fields start on positive charges and end on negative charges
6. The electric field is constant between equally charged parallel plates
7. A charge in an electric field, just like a mass under gravity, has potential energy which is related to the work to move it.
8. A capacitor is a device that stores charge in a circuit.

## 17.8 Exercises - Electrostatics

1. Two charges of $+3\text{nC}$ and $-5\text{nC}$ are separated by a distance of $40\text{cm}$. What is the electrostatic force between the two charges?
2. Two insulated metal spheres carrying charges of $+6\text{nC}$ and $-10\text{nC}$ are separated by a distance of 20 mm.
   A What is the electrostatic force between the spheres?
   B The two spheres are touched and then separated by a distance of $60\text{mm}$. What are the new charges on the spheres?
   C What is new electrostatic force between the spheres at this distance?
3. The electrostatic force between two charged spheres of $+3\text{nC}$ and $+4\text{nC}$ respectively is $0{,}04\text{N}$. What is the distance between the spheres?
4. Calculate the potential difference between two parallel plates if it takes $5000\text{J}$ of energy to move $25\text{C}$ of charge between the plates?
5. Draw the electric field pattern lines between:
   A two equal positive point charges.
   B two equal negative point charges.
6. Calculate the electric field between the plates of a capacitor if the plates are $20\text{mm}$ apart and the potential difference between the plates is $300\text{V}$.
7. Calculate the electrical potential energy of a $6\text{nC}$ charge that is $20\text{cm}$ from a $10\text{nC}$ charge.
8. What is the capacitance of a capacitor if it has a charge of $0{,}02\text{C}$ on each of its plates when the potential difference between the plates is $12\text{V}$?

9. [SC 2003/11] Two small identical metal spheres, on insulated stands, carry charges -$q$ and $+3q$ respectively. When the centres of the spheres are separated by a distance $d$ the one exerts an electrostatic force of magnitude $F$ on the other.




The spheres are now made to touch each other and are then brought back to the same distance $d$ apart. What will be the magnitude of the electrostatic force which one sphere now exerts on the other?

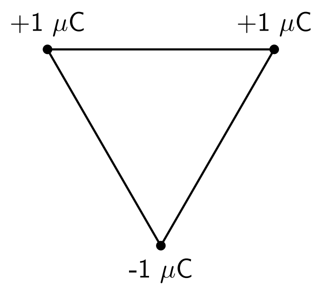
A $\frac{1}{4}F$

B $\frac{1}{3}F$

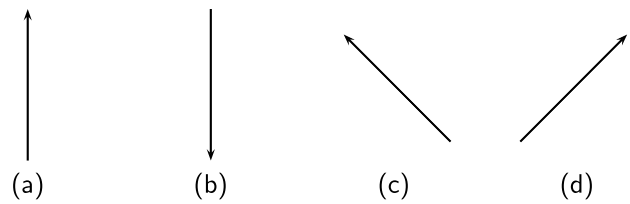
C $\frac{1}{2}F$

D $3F$

10. [SC 2003/11] Three point charges of magnitude $+1$ $\mu$C, $+1$ $\mu$C and -1 $\mu$C respectively are placed on the three corners of an equilateral triangle as shown.




Which vector best represents the direction of the resultant force acting on the -1 $\mu$C charge as a result of the forces exerted by the other two charges?




11. [IEB 2003/11 HG1 - Force Fields] **Electric Fields**

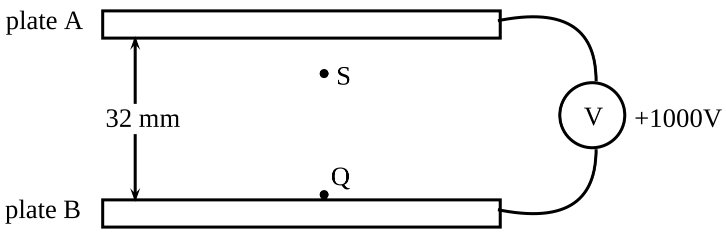
A Write a statement of Coulomb's law.

B Calculate the magnitude of the force exerted by a point charge of $+2$ nC on another point charge of -3 nC separated by a distance of 60 mm.

C Sketch the electric field between two point charges of $+2$ nC and -3 nC, respectively, placed 60 mm apart from each other.

12. [IEB 2003/11 HG1 - Electrostatic Ping-Pong]

Two charged parallel metal plates, X and Y, separated by a distance of 60 mm, are connected to a d.c. supply of emf 2 000 V in series with a microammeter. An initially uncharged conducting sphere (a graphite-coated ping pong ball) is suspended from an insulating thread between the metal plates as shown in the diagram.




When the ping pong ball is moved to the right to touch the positive plate, it acquires a charge of $+9$ nC. It is then released. The ball swings to and fro between the two plates, touching each plate in turn.

A How many electrons are removed from the ball when it acquires a charge of $+9$ nC?

B Explain why a current is established in the circuit.

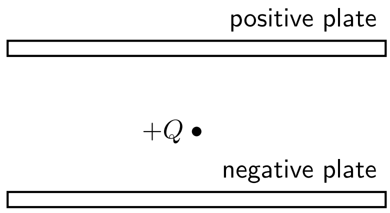
C Determine the current if the ball takes 0,25 s to swing from Y to X.

D Using the same graphite-coated ping pong ball, and the same two metal plates, give TWO ways in which this current could be increased.

E Sketch the electric field between the plates X and Y.

F How does the electric force exerted on the ball change as it moves from Y to X?

13. [IEB 2005/11 HG] A positive charge $Q$ is released from rest at the centre of a uniform electric field.




How does $Q$ move immediately after it is released?

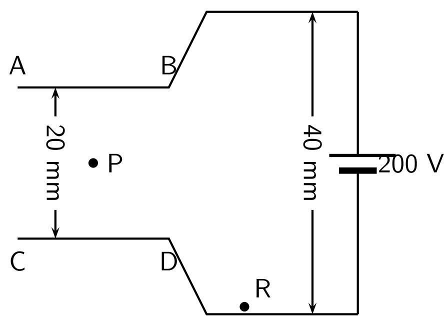
A It accelerates uniformly.

B It moves with an increasing acceleration.

C It moves with constant speed.

D It remains at rest in its initial position.

14. [SC 2002/03 HG1] The sketch below shows two sets of parallel plates which are connected together. A potential difference of 200 V is applied across both sets of plates. The distances between the two sets of plates are 20 mm and 40 mm respectively.




When a charged particle Q is placed at point R, it experiences a force of magnitude $F$. Q is now moved to point P, halfway between plates AB and CD. Q now experiences a force of magnitude ...

A $\frac{1}{2}F$

B $F$

C $2F$

D $4F$

15. [SC 2002/03 HG1] The electric field strength at a distance $x$ from a point charge is $E$. What is the magnitude of the electric field strength at a distance $2x$ away from the point charge?

A $\frac{1}{4}E$

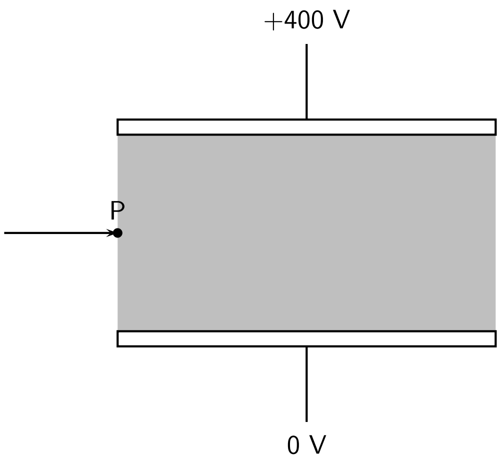
B $\frac{1}{2}E$

C $2E$

D $4E$

16. [IEB 2005/11 HG1]

An electron (mass 9,11 $\times$ $10^{-31}$ kg) travels horizontally in a vacuum. It enters the shaded regions between two horizontal metal plates as shown in the diagram below.




A potential difference of 400 V is applied across the places which are separated by 8,00 mm.
The electric field intensity in the shaded region between the metal plates is uniform. Outside this region, it is zero.

A Explain what is meant by the phrase **"the electric field intensity is uniform"**.

B Copy the diagram and draw the following:

i. The electric field between the metal plates.

ii. An arrow showing the direction of the electrostatic force on the electron when it is at **P**.

C Determine the magnitude of the electric field intensity between the metal plates.

D Calculate the magnitude of the electrical force on the electron during its passage through the electric field between the plates.

E Calculate the magnitude of the acceleration of the electron (due to the electrical force on it) during its passage through the electric field between the plates.

F After the electron has passed through the electric field between these plates, it collides with phosphorescent paint on a TV screen and this causes the paint to glow. What energy transfer takes place during this collision?

17. [IEB 2004/11 HG1] A positively-charged particle is placed in a uniform electric field. Which of the following pairs of statements correctly describes the potential energy of the charge, and the force which the charge experiences in this field?

Potential energy — Force

A Greatest near the negative plate — Same everywhere in the field

B Greatest near the negative plate — Greatest near the positive and negative plates

C Greatest near the positive plate — Greatest near the positive and negative plates

D Greatest near the positive plate — Same everywhere in the field

18. [IEB 2004/11 HG1 - TV Tube]

    A speck of dust is attracted to a TV screen. The screen is negatively charged, because this is where the electron beam strikes it. The speck of dust is neutral.

    A What is the name of the electrostatic process which causes dust to be attracted to a TV screen?

    B Explain why a neutral speck of dust is attracted to the negatively-charged TV screen?

    C Inside the TV tube, electrons are accelerated through a uniform electric field. Determine the magnitude of the electric force exerted on an electron when it accelerates through a potential difference of 2 000 V over a distance of 50 mm.

    D How much kinetic energy (in J) does one electron gain while it accelerates over this distance?

    E The TV tube has a power rating of 300 W. Estimate the maximum number of electrons striking the screen per second.

19. [IEB 2003/11 HG1] A point charge is held stationary between two charged parallel plates that are separated by a distance d. The point charge experiences an electrical force F due to the electric field E between the parallel plates.
    What is the electrical force on the point charge when the plate separation is increased to 2d?

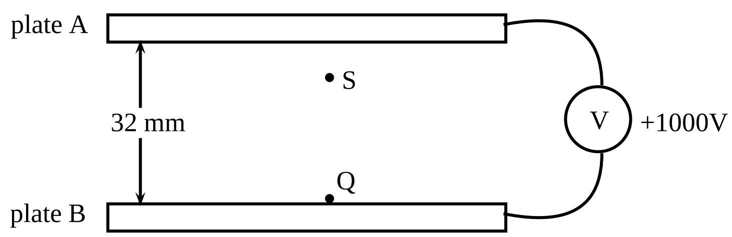
    A $\frac{1}{4}$ F

    B $\frac{1}{2}$ F

    C 2 F

    D 4 F

20. [IEB 2001/11 HG1] - **Parallel Plates**
    A distance of 32 mm separates the horizontal parallel plates A and B.
    B is at a potential of $+1$ 000 V.




    A Draw a sketch to show the electric field lines between the plates A and B.

    B Calculate the magnitude of the electric field intensity (strength) between the plates. A tiny charged particle is stationary at S, 8 mm below plate A that is at zero electrical potential. It has a charge of 3,2 $\times$ $10^{-12}$ C.

    C State whether the charge on this particle is positive or negative.

    D Calculate the force due to the electric field on the charge.

    E Determine the mass of the charged particle.
    The charge is now moved from S to Q.

    F What is the magnitude of the force exerted by the electric field on the charge at Q?

    G Calculate the work done when the particle is moved from S to Q.

# Chapter 18

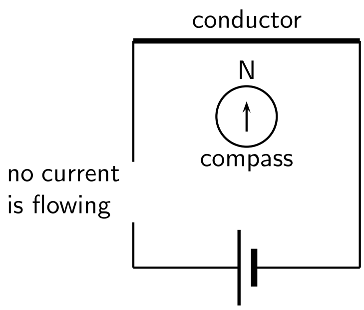
# Electromagnetism - Grade 11

## 18.1 Introduction

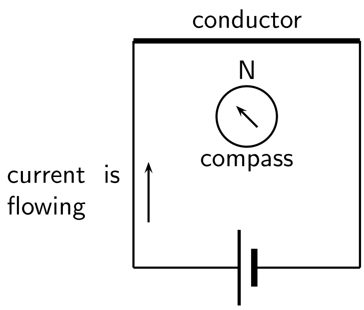
Electromagnetism is the science of the properties and relationship between electric currents and magnetism. An electric current creates a magnetic field and a moving magnetic field will create a flow of charge. This relationship between electricity and magnetism has resulted in the invention of many devices which are useful to humans.

## 18.2 Magnetic field associated with a current

If you hold a compass near a wire through which current is flowing, the needle on the compass will be deflected.




There is no deflection on the compass when there is no current flowing in the conductor.




The compass needle deflects when there *is* current flowing in the conductor.

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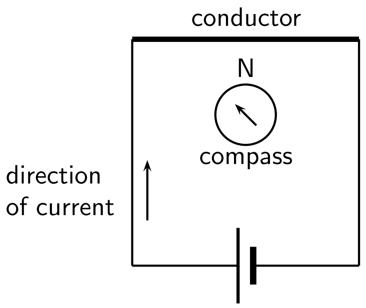
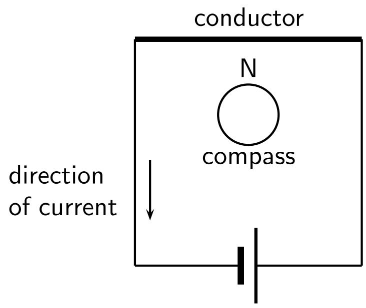
**Activity :: Case Study : Magnetic field near a current carrying conductor**




When the battery is connected as shown, the compass needle is deflected to the left.




What do you think will happen if the direction of the current is reversed as shown?

**Interesting Fact**

The discovery of the relationship between magnetism and electricity was, like so many other scientific discoveries, stumbled upon almost by accident. The Danish physicist Hans Christian Oersted was lecturing one day in 1820 on the possibility of electricity and magnetism being related to one another, and in the process demonstrated it conclusively by experiment in front of his whole class. By passing an electric current through a metal wire suspended above a magnetic compass, Oersted was able to produce a definite motion of the compass needle in response to the current. What began as a guess at the start of the class session was confirmed as fact at the end. Needless to say, Oersted had to revise his lecture notes for future classes. His discovery paved the way for a whole new branch of science - electromagnetism.

The magnetic field produced by an electric current is always oriented perpendicular to the direction of the current flow. When we are drawing directions of magnetic fields and currents, we use the symbol $\odot$ and $\otimes$. The symbol

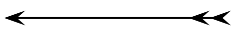
$$\odot$$

for an arrow that is coming out of the page and the symbol

$$\otimes$$

for an arrow that is going into the page.

It is easy to remember the meanings of the symbols if you think of an arrow with a head and a tail.



When the arrow is coming out of the page, you see the head of the arrow ($\odot$). When the arrow is going into the page, you see the tail of the arrow ($\otimes$).

The direction of the magnetic field around the current carrying conductor is shown in Figure 18.1.

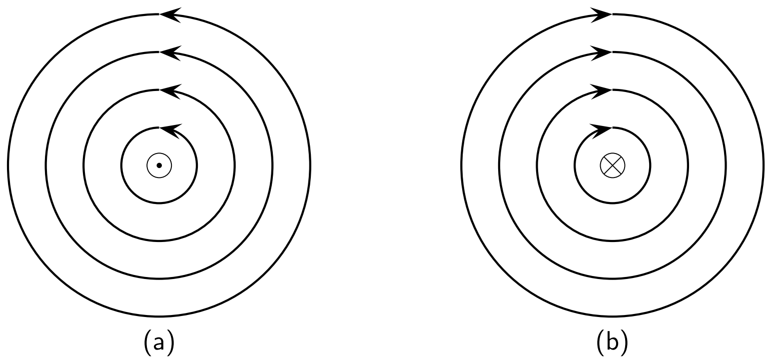



Figure 18.1: Magnetic field around a conductor when you look at the conductor from one end. (a) Current flows into the page and the magnetic field is counter clockwise. (b) Current flows out of the page and the magnetic field is clockwise.

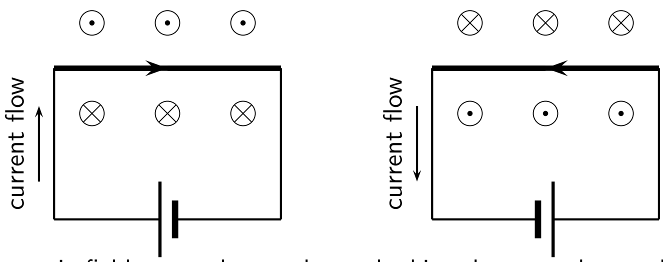
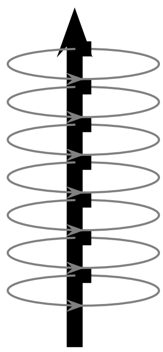



Figure 18.2: Magnetic fields around a conductor looking down on the conductor, for current in a conductor that is flowing to the right and to the left.

**Activity :: Case Study : Direction of a magnetic field**

Using the directions given in Figure 18.1 and Figure 18.2 and try to find a rule that easily tells you the direction of the magnetic field.

Hint: Use your fingers. Hold the wire in your hands and try to find a link between the direction of your thumb and the direction in which your fingers curl.



The magnetic field around a current carrying conductor.

There is a simple method of showing the relationship between the direction of the current flowing in a conductor and the direction of the magnetic field around the same conductor. The method is called the *Right Hand Rule*. Simply stated, the right hand rule says that the magnetic flux lines produced by a current-carrying wire will be oriented the same direction as the curled fingers of a person's right hand (in the "hitchhiking" position), with the thumb pointing in the direction of the current flow.

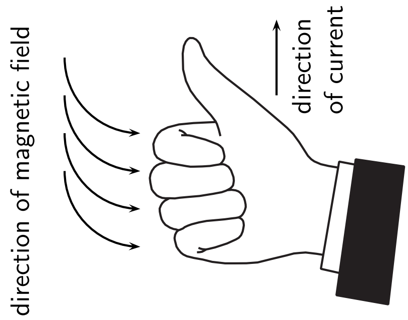
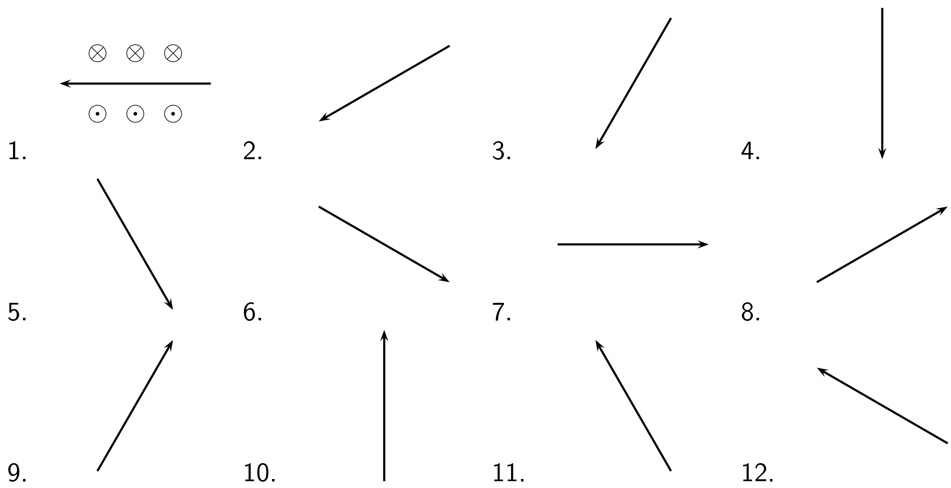



Figure 18.3: The Right Hand Rule.

**Activity :: Case Study : The Right Hand Rule**
Use the Right Hand Rule and draw in the directions of the magnetic field for the following conductors with the currents flowing in the directions shown by the arrow. The first problem has been completed for you.




**Activity :: Experiment : Magnetic field around a current carrying conductor**

**Apparatus:**

1. 1 9V battery with holder
2. 2 hookup wires with alligator clips
3. compass
4. stop watch

**Method:**

1. Connect your wires to the battery leaving one end unconnected so that the circuit is not closed.
2. One student should be in charge of limiting the current flow to 10 seconds. This is to preserve battery life as well as to prevent overheating of wires and battery contacts.
3. Place the compass close to the wire.
4. Close the circuit and observe what happens to the compass.

5. Reverse the polarity of the battery and close the circuit. Observe what happens to the compass.

**Conclusions:**

Use your observations to answer the following questions:

1. Does a current flowing in a wire generate a magnetic field?
2. Is the magnetic field present when the current is not flowing?
3. Does the direction of the magnetic field produced by a current in a wire depend on the direction of the current flow?
4. How does the direction of the current affect the magnetic field?

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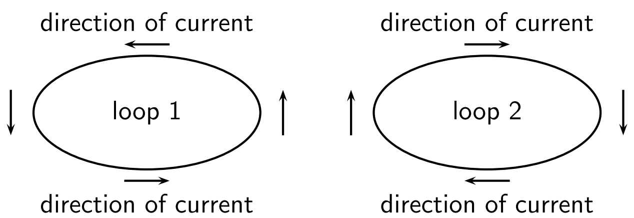
**Activity :: Case Study : Magnetic field around a loop of conductor**

Consider two loops of current carrying conductor that are placed in the plane of the page. Draw what you think the magnetic field would look like, by using the Right Hand Rule at different points of the two loops shown. Loop 1 has the current flowing in a counter-clockwise direction, while loop 2 has the current flowing in a clockwise direction.

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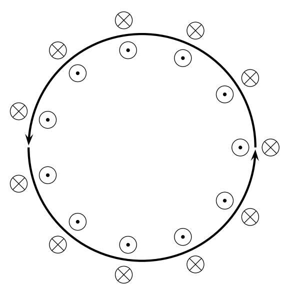
If you make a loop of current carrying conductor, then the direction of the magnetic field is obtained by applying the Right Hand Rule to different points in the loop.



The directions of the magnetic field around a loop of current carrying conductor with the current flowing in a counter-clockwise direction is shown.

If we know add another loop then the magnetic field around each loop joins to create a stronger field. As more loops are added, the magnetic field gets a definite magnetic (north and south) polarity. Such a coil is more commonly known as a *solenoid*. The magnetic field pattern of a solenoid is similar to the magnetic field pattern of a bar magnet that you studied in Grade 10.

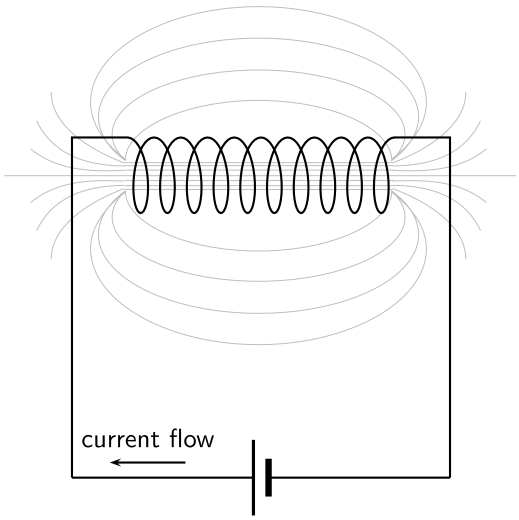



Figure 18.4: Magnetic field around a solenoid.

### 18.2.1 Real-world applications

#### Electromagnets

An *electromagnet* is a piece of wire intended to generate a magnetic field with the passage of electric current through it. Though all current-carrying conductors produce magnetic fields, an electromagnet is usually constructed in such a way as to maximize the strength of the magnetic field it produces for a special purpose. Electromagnets find frequent application in research, industry, medical, and consumer products.

As an electrically-controllable magnet, electromagnets find application in a wide variety of "electromechanical" devices: machines that effect mechanical force or motion through electrical power. Perhaps the most obvious example of such a machine is the *electric motor* which will be described in detail in Grade 12. Other examples of the use of electromagnets are electric bells, relays, loudspeakers and scrapyard cranes.

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**Activity :: Experiment : Electromagnets**

**Aim:**
A magnetic field is created when an electric current flows through a wire. A single wire does not produce a strong magnetic field, but a coiled wire around an iron core does. We will investigate this behaviour.

**Apparatus:**

1. a battery and holder
2. a length of wire
3. a compass
4. a few nails
5. a few paper clips

**Method:**

1. Bend the wire into a series of coils before attaching it to the battery. Observe what happens to the deflection on the compass. Has the deflection of the compass grown stronger?
2. Repeat the experiment by changing the number and size of the coils in the wire. Observe what happens to the deflection on the compass.
3. Coil the wire around an iron nail and then attach the coil to the battery. Observe what happens to the deflection on the compass.

**Conclusions:**

1. Does the number of coils affect the strength of the magnetic field?
2. Does the iron nail increase or decrease the strength of the magnetic field?

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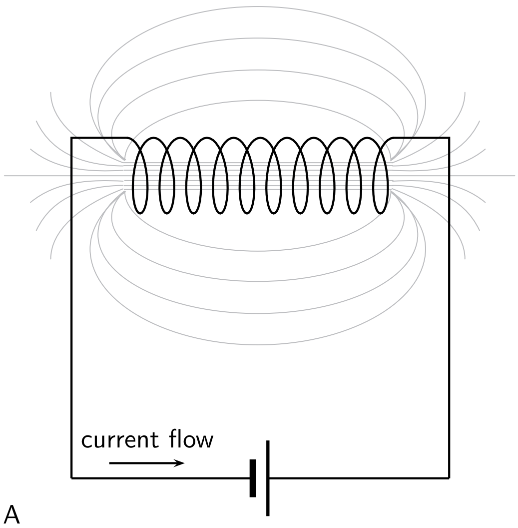
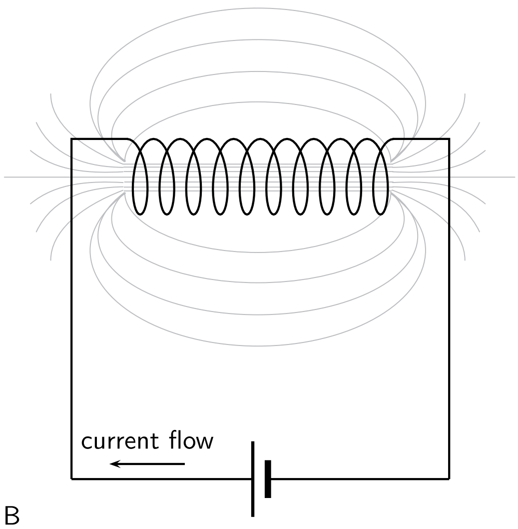
**Exercise: Magnetic Fields**

1. Give evidence for the existence of a magnetic field near a current carrying wire.
2. Describe how you would use your right hand to determine the direction of a magnetic field around a current carrying conductor.
3. Use the right hand rule to determine the direction of the magnetic field for the following situations.

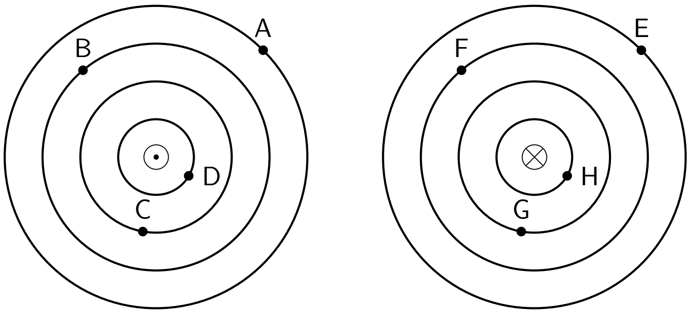



A




B

4. Use the Right Hand Rule to find the direction of the magnetic fields at each of the labelled points in the diagrams.




## 18.3 Current induced by a changing magnetic field

While Oersted's surprising discovery of electromagnetism paved the way for more practical applications of electricity, it was Michael Faraday who gave us the key to the practical generation of electricity: **electromagnetic induction**.

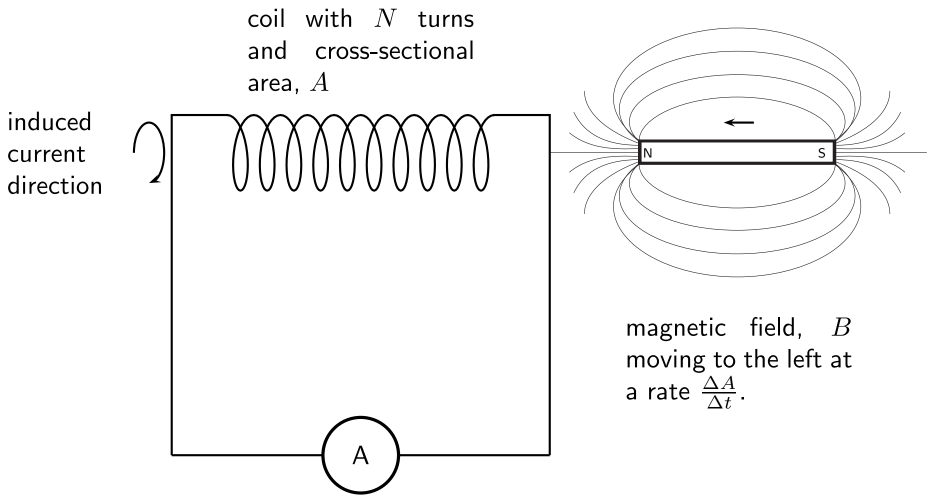
Faraday discovered that a voltage was generated across a length of wire while moving a magnet nearby, such that the distance between the two changed. This meant that the wire was exposed to a magnetic field flux of changing intensity. Furthermore, the voltage also depended on the orientation of the magnet; this is easily understood again in terms of the magnetic flux. The flux will be at its maximum as the magnet is aligned perpendicular to the wire. The magnitude of the changing flux and the voltage are linked. In fact, if the lines of flux are parallel to the wire, there will be no induced voltage.

**Definition: Faraday's Law**
The emf, $\epsilon$, produced around a loop of conductor is proportional to the rate of change of the magnetic flux, $\phi$, through the area, $A$, of the loop. This can be stated mathematically as:

$$\epsilon = -N\frac{\Delta\phi}{\Delta t}$$

where $\phi = B \cdot A$ and $B$ is the strength of the magnetic field.

Faraday's Law relates induced emf to the rate of change of flux, which is the product of the magnetic field and the cross-sectional area the field lines pass through.




When the north pole of a magnet is pushed into a solenoid, the flux in the solenoid increases so the induced current will have an associated magnetic field pointing out of the solenoid (opposite to the magnet's field). When the north pole is pulled out, the flux decreases, so the induced current will have an associated magnetic field pointing into the solenoid (same direction as the magnet's field) to try to oppose the change. The directions of currents and associated magnetic fields can all be found using only the Right Hand Rule. When the fingers of the right hand are pointed in the direction of the current, the thumb points in the direction of the magnetic field. When the thumb is pointed in the direction of the magnetic field, the fingers point in the direction of the current.

**Important:** An easy way to create a magnetic field of changing intensity is to move a permanent magnet next to a wire or coil of wire. The magnetic field must increase or decrease in intensity *perpendicular* to the wire (so that the lines of flux "cut across" the conductor), or else no voltage will be induced.

**Important:** Finding the direction of the induced current

The induced current generates a magnetic field. The induced magnetic field is in a direction that cancels out the magnetic field in which the conductor is moving. So, you can use the Right Hand Rule to find the direction of the induced current by remembering that the induced magnetic field is opposite in direction to the magnetic field causing the change.

Electromganetic induction is put into practical use in the construction of electrical generators, which use mechanical power to move a magnetic field past coils of wire to generate voltage. However, this is by no means the only practical use for this principle.

If we recall that the magnetic field produced by a current-carrying wire was always perpendicular to that wire, and that the flux intensity of that magnetic field varied with the amount of current through it, we can see that a wire is capable of inducing a voltage *along its own length* simply due to a change in current through it. This effect is called *self-induction*. Self-induction is when a changing magnetic field is produced by changes in current through a wire inducing voltage along the length of that same wire.

If the magnetic field flux is enhanced by bending the wire into the shape of a coil, and/or wrapping that coil around a material of high permeability, this effect of self-induced voltage will be more intense. A device constructed to take advantage of this effect is called an *inductor*, and will be discussed in greater detail in the next chapter.

*Extension: Lenz's Law*

The induced current will create a magnetic field that opposes the change in the magnetic flux.

**Worked Example 121: Faraday's Law**

**Question:** Consider a flat square coil with 5 turns. The coil is 0,50 m on each side, and has a magnetic field of 0,5 T passing through it. The plane of the coil is perpendicular to the magnetic field: the field points out of the page. Use Faraday's Law to calculate the induced emf if the magnetic field is increases uniformly from 0,5 T to 1 T in 10 s. Determine the direction of the induced current.

**Answer**

**Step 1 : Identify what is required**

We are required to use Faraday's Law to calculate the induced emf.

**Step 2 : Write Faraday's Law**

$$\epsilon = -N\frac{\Delta\phi}{\Delta t}$$

**Step 3 : Solve Problem**

$$\begin{aligned}
\epsilon &= -N\frac{\Delta\phi}{\Delta t} \\
&= -N\frac{\phi_f - \phi_i}{\Delta t} \\
&= -N\frac{B_f \cdot A - B_i \cdot A}{\Delta t} \\
&= -N\frac{A(B_f - B_i)}{\Delta t} \\
&= -(5)\frac{(0{,}5)^2(1 - 0{,}5)}{10} \\
&= 0{,}0625 \text{ V}
\end{aligned}$$

### 18.3.1 Real-life applications

The following devices use Faraday's Law in their operation.

- induction stoves
- tape players
- metal detectors
- transformers

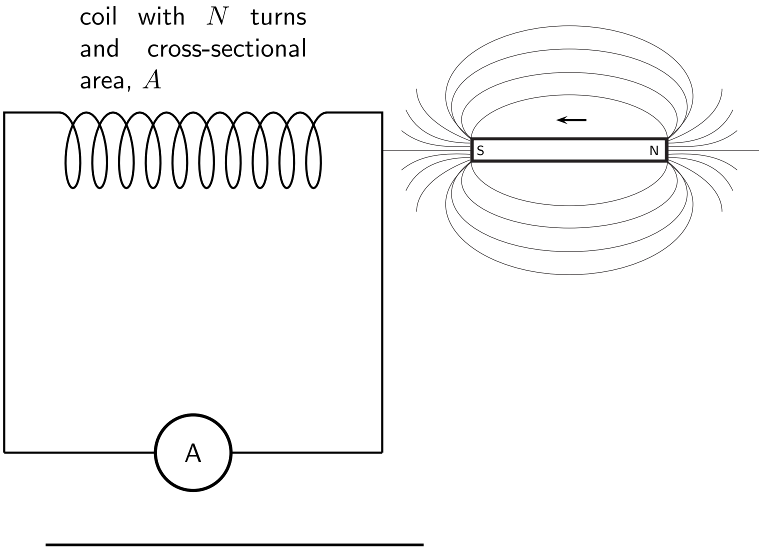
**Activity :: Research Project : Real-life applications of Faraday's Law**
Choose one of the following devices and do some research on the internet or in a library how your device works. You will need to refer to Faraday's Law in your explanation.

- induction stoves
- tape players
- metal detectors
- transformers

?

**Exercise: Faraday's Law**

1. State Faraday's Law in words and write down a mathematical relationship.
2. Describe what happens when a bar magnet is pushed into or pulled out of a solenoid connected to an ammeter. Draw pictures to support your description.
3. Use the right hand rule to determine the direction of the induced current in the solenoid below.



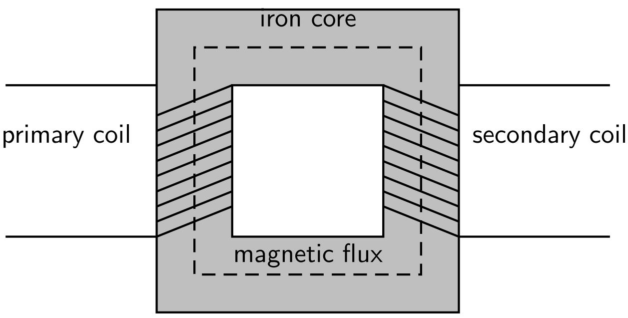

## 18.4 Transformers

One of the real-world applications of Faraday's Law is in a *transformer*.

Eskom generates electricity at around 22 000 V. When you plug in a toaster, the mains voltage is 220 V. A transformer is used to *step-down* the high voltage to the lower voltage that is used as mains voltage.

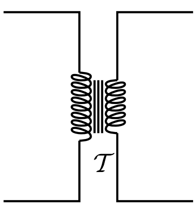
**Definition: Transformer**
A transformer is an electrical device that uses the principle of induction between the primary coil and the secondary coil to either step-up or step-down voltage.

The essential features of a transformer are two coils of wire, called the primary coil and the secondary coil, which are wound around different sections of the same iron core.




When an alternating voltage is applied to the primary coil it creates an alternating current in that coil, which induces an alternating magnetic field in the iron core. This changing magnetic field induces an emf, which creates a current in the secondary coil.

The circuit symbol for a transformer is:



A very useful property of transformers is the ability to transform voltage and current levels according to a simple ratio, determined by the ratio of input and output coil turns. We can derive a mathematical relationship by using Faraday's law.

Assume that an alternating voltage $V_p$ is applied to the primary coil (which has $N_p$ turns) of a transformer. The current that results from this voltage generates a magnetic flux $\phi_p$. We can then describe the emf in the primary coil by:

$$V_p = N_p \frac{\Delta \phi_p}{\Delta t}$$

Similarly, for the secondary coil,

$$V_s = N_s \frac{\Delta \phi_s}{\Delta t}$$

If we assume that the primary and secondary windings are perfectly coupled, then:

$$\phi_p = \phi_s$$

which means that:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

**Worked Example 122: Transformer specifications**

**Question:** Calculate the voltage on the secondary coil if the voltage on the primary coil is 120 V and the ratio of primary windings to secondary windings is 10:1.

**Answer**

**Step 1 : Determine how to approach the problem**

Use

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

with

- $V_p = 120$
- $\frac{N_p}{N_s} = \frac{10}{1}$

**Step 2 : Rearrange equation to solve for $V_s$**

$$\begin{aligned} \frac{V_p}{V_s} &= \frac{N_p}{N_s} \\ \frac{1}{V_s} &= \frac{N_p}{N_s}\frac{1}{V_p} \\ \therefore V_s &= \frac{1}{\frac{N_p}{N_s}} V_p \end{aligned}$$

**Step 3 : Substitute values and solve for $V_s$**

$$\begin{aligned} V_s &= \frac{1}{\frac{N_p}{N_s}} V_p \\ &= \frac{1}{\frac{10}{1}} 120 \\ &= 12 \text{ V} \end{aligned}$$

A transformer designed to output more voltage than it takes in across the input coil is called a *step-up* transformer. A step-up transformer has more windings on the secondary coil than on the primary coil. This means that:

$$N_s > N_p$$

Similarly, a transformer designed to output less than it takes in across the input coil is called a *step-down* transformer. A step-down transformer has more windings on the primary coil than on the primary coil. This means that:

$$N_p > N_s$$

We use a step-up transformer to increase the voltage from the primary coil to the secondary coil. It is used at power stations to increase the voltage for the transmission lines. A step-down transformer decreases the voltage from the primary coil to the secondary coil. It is particularly used to decrease the voltage from the transmission lines to a voltage which can be used in factories and in homes.

Transformer technology has made long-range electric power distribution practical. Without the ability to efficiently step voltage up and down, it would be cost-prohibitive to construct power systems for anything but close-range (within a few kilometres) use.

As useful as transformers are, they only work with AC, not DC. This is because the phenomenon of mutual inductance relies on *changing* magnetic fields, and direct current (DC) can only produce steady magnetic fields, transformers simply will not work with direct current.

Of course, direct current may be interrupted (pulsed) through the primary winding of a transformer to create a changing magnetic field (as is done in automotive ignition systems to produce high-voltage spark plug power from a low-voltage DC battery), but pulsed DC is not that different from AC. Perhaps more than any other reason, this is why AC finds such widespread application in power systems.

### 18.4.1 Real-world applications

Transformers are very important in the supply of electricity nationally. In order to reduce energy losses due to heating, electrical energy is transported from power stations along power lines at high voltage and low current. Transformers are used to step the voltage up from the power station to the power lines, and step it down from the power lines to buildings where it is needed.

---

?

**Exercise: Transformers**

1. Draw a sketch of the main features of a transformer
2. Use Faraday's Law to explain how a transformer works in words and pictures.
3. Use the equation for Faraday's Law to derive an expression involving the ratio between the voltages and number of windings in the primary and secondary coils.
4. If we have $N_p = 100$ and $N_s = 50$ and we connect the primary winding to a 230 V, 50Hz supply then calculate the voltage on the secondary winding.
5. State the difference between a step-up and a step-down transformer in both structure and function.
6. Give an example of the use of transformers.

---

## 18.5 Motion of a charged particle in a magnetic field

When a charged particle moves through a magnetic field it experiences a force. For a particle that is moving at right angles to the magnetic field, the force is given by:

$$F = qvB$$

where $q$ is the charge on the particle, $v$ is the velocity of the particle and $B$ is the magnetic field through which the particle is moving.


**Worked Example 123: Charged particle moving in a magnetic field**

**Question:** An electron travels at $150\text{m.s}^{-1}$ at right angles to a magnetic field of 80 000 T. What force is exerted on the electron?

**Answer**

**Step 1 : Determine what is required**

We are required to determine the force on a moving charge in a magnetic field

**Step 2 : Determine how to approach the problem**

We can use the formula:

$$F = qvB$$

**Step 3 : Determine what is given**

We are given

- $q = 1{,}6 \times 10^{-19}\text{C}$ (The charge on an electron)
- $v = 150\text{m.s}^{-1}$
- $B = 80\ 000\text{T}$

**Step 4 : Determine the force**

$$\begin{aligned} F &= qvB \\ &= (1{,}6 \times 10^{-19}\text{C})(150\text{m.s}^{-1})(80\ 000\text{T}) \\ &= 1{,}92 \times 10^{-12}\text{N} \end{aligned}$$

**Important:** The direction of the force exerted on a charged particle moving through a magnetic field is determined by using the Right Hand Rule.
Point your fingers in the direction of the velocity of the charge and turn them (as if turning a screwdriver) towards the direction of the magnetic field. Your thumb will point in the direction of the force. If the charge is negative, the direction of the force will be opposite to the direction of your thumb.

### 18.5.1 Real-world applications

The following devices use the movement of charge in a magnetic field

- televisions
- oscilloscope

**Activity :: Research Project : Real-life applications of charges moving in a magnetic field**

Choose one of the following devices and do some research on the internet or in a library how your device works.

- oscilloscope
- television

?

**Exercise: Lorentz Force**

1. What happens to a charged particle when it moves through a magnetic field?
2. Explain how you would use the Right Hand Rule to determine the direction of the force experienced by a charged particle as it moves in a magnetic field.
3. Explain how the force exerted on a charged particle moving through a magnetic field is used in a television.

## 18.6 Summary

1. Electromagnetism is the study of the properties and relationship between electric current and magnetism.
2. A current carrying conductor will produce a magnetic field around the conductor.
3. The direction of the magnetic field is found by using the Right Hand Rule.
4. Electromagnets are temporary magnets formed by current-carrying conductors.
5. Electromagnetic induction occurs when a moving magnetic field induces a voltage in a current-carrying conductor.
6. Transformers use electromagnetic induction to alter the voltage.
7. A charged particle will experience a force in a magnetic field.

## 18.7 End of chapter exercises

1. State the Right Hand Rule.
2. What did Hans Oersted discover about the relationship between electricity and magnetism?
3. List two uses of electromagnetism.
4. Draw a labelled diagram of an electromagnet and show the poles of the electromagnet on your sketch.
5. Transformers are useful electrical devices.

   A What is a transformer?

   B Draw a sketch of a step-down transformer?

   C What is the difference between a step-down and step-up transformer?

D When would you use a step-up transformer?

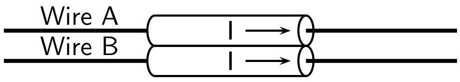
6. Calculate the voltage on the secondary coil of a transformer if the voltage on the primary coil is 22 000 V and the ratio of secondary windings to secondary windings is 500:1.

7. You find a transformer with 1000 windings on the primary coil and 200 windinds on the secondary coil.

   A What type of transformer is it?

   B What will be the voltage on the secondary coil if the voltage on the primary coil is 400 V?

IEB 2005/11 HG An electric cable consists of two long straight parallel wires separated by plastic insulating material. Each wire carries a current $I$ in the same direction (as shown in the diagram below).




Which of the following is **true** concerning the force of Wire A on Wire B?

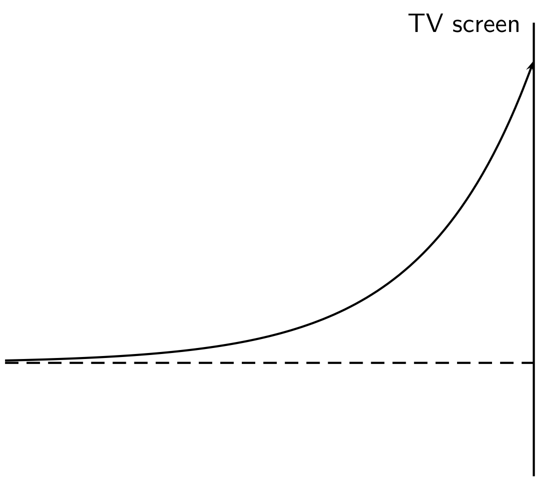
| | **Direction of Force** | **Origin of Force** |
|---|---|---|
| (a) | towards A (attraction) | electrostatic force between opposite charges |
| (b) | towards B (repulsion) | electrostatic force between opposite charges |
| (c) | towards A (attraction) | magnetic force on current-carrying conductor |
| (d) | towards B (repulsion) | magnetic force on current-carrying conductor |

IEB 2005/11 HG1 **Force of parallel current-carrying conductors**
Two long straight parallel current-carrying conductors placed 1 m apart from each other in a vacuum each carry a current of 1 A in the same direction.

A What is the magnitude of the force of 1 m of one conductor on the other?

B How does the force compare with that in the previous question when the current in one of the conductors is halved, and their distance of separation is halved?

IEB 2005/11 HG An electron moving horizontally in a TV tube enters a region where there is a uniform magnetic field. This causes the electron to move along the path (shown by the solid line) because the magnetic field exerts a constant force on it. What is the direction of this magnetic field?




A upwards (towards the top of the page)

B downwards (towards the bottom of the page)

C into the page

D out of the page

# Chapter 19

# Electric Circuits - Grade 11

## 19.1 Introduction

The study of electrical circuits is essential to understand the technology that uses electricity in the real-world. This includes electricity being used for the operation of electronic devices like computers.

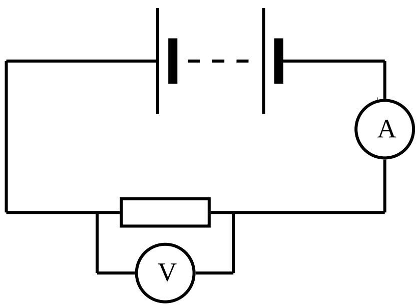
## 19.2 Ohm's Law

### 19.2.1 Definition of Ohm's Law

**Activity :: Experiment : Ohm's Law**

**Aim:**

In this experiment we will look at the relationship between the current going through a resistor and the potential difference (voltage) across the same resistor.




**Method:**

1. Set up the circuit according to the circuit diagram.
2. Draw the following table in your lab book.

| Voltage, $V$ (V) | Current, $I$ (A) |
|---|---|
| 1,5 | |
| 3,0 | |
| 4,5 | |
| 6,0 | |

3. Get your teacher to check the circuit before turning the power on.
4. Measure the current.
5. Add one more 1,5 V battery to the circuit and measure the current again.

6. Repeat until you have four batteries and you have completed your table.
7. Draw a graph of voltage versus current.

**Results:**

1. Does your experimental results verify Ohm's Law? Explain.
2. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor $R_0$?

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**Activity :: Activity : Ohm's Law**

If you do not have access to the equipment necessary for the Ohm's Law experiment, you can do this activity.

| Voltage, $V$ (V) | Current, $I$ (A) |
|---|---|
| 3,0 | 0,4 |
| 6,0 | 0,8 |
| 9,0 | 1,2 |
| 12,0 | 1,6 |

1. Plot a graph of voltage (on the $y$-axis) and current (on the $x$-axis).

**Conclusions:**

1. What type of graph do you obtain (straight line, parabola, other curve)
2. Calculate the gradient of the graph.
3. Does your experimental results verify Ohm's Law? Explain.
4. How would you go about finding the resistance of an unknown resistor using only a power supply, a voltmeter and a known resistor $R_0$?

---

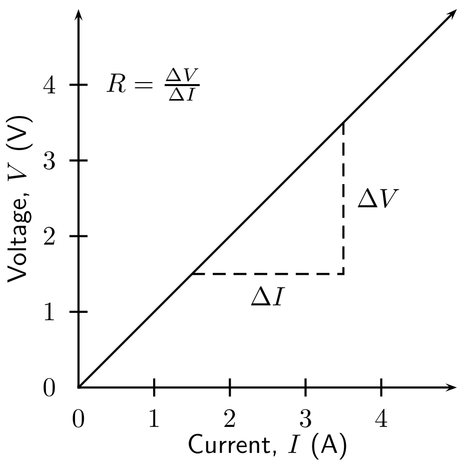
An important relationship between the current, voltage and resistance in a circuit was discovered by Georg Simon Ohm and is called **Ohm's Law**.

**Definition: Ohm's Law**

The amount of electric current through a metal conductor, at a constant temperature, in a circuit is proportional to the voltage across the conductor. Mathematically, Ohm's Law is written:

$$V = R \cdot I.$$

Ohm's Law tells us that if a conductor is at a constant temperature, the voltage across the ends of the conductor is proportional to the current. This means that if we plot voltage on the $y$-axis of a graph and current on the $x$-axis of the graph, we will get a straight-line. The gradient of the straight-line graph is then the resistance of the conductor.



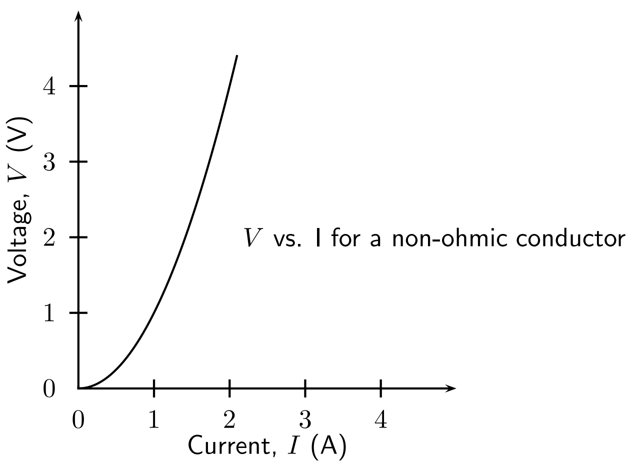

### 19.2.2 Ohmic and non-ohmic conductors

As you have seen, there is a mention of *constant temperature* when we talk about Ohm's Law. This is because the resistance of some conductors change as their temperature changes. These types of conductors are called *non-ohmic* conductors, because they do not obey Ohm's Law. As can be expected, the conductors that obey Ohm's Law are called *ohmic* conductors. A light bulb is a common example of a non-ohmic conductor. Nichrome wire is an ohmic conductor.

In a light bulb, the resistance of the filament wire will increase dramatically as it warms from room temperature to operating temperature. If we increase the supply voltage in a real lamp circuit, the resulting increase in current causes the filament to increase in temperature, which increases its resistance. This effectively limits the increase in current. In this case, voltage and current do not obey Ohm's Law.

The phenomenon of resistance changing with variations in temperature is one shared by almost all metals, of which most wires are made. For most applications, these changes in resistance are small enough to be ignored. In the application of metal lamp filaments, which increase a lot in temperature (up to about 1000°C, and starting from room temperature) the change is quite large.

In general non-ohmic conductors have plots of voltage against current that are curved, indicating that the resistance is not constant over all values of voltage and current.

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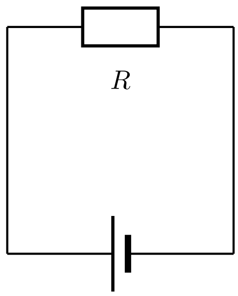
**Activity :: Experiment : Ohmic and non-ohmic conductors**
Repeat the experiment as decribed in the previous section. In this case use a light bulb as a resistor. Compare your results to the ohmic resistor.

### 19.2.3 Using Ohm's Law

We are now ready to see how Ohm's Law is used to analyse circuits.

Consider the circuit with an ohmic resistor, $R$. If the resistor has a resistance of 5 $\Omega$ and voltage across the resistor is 5V, then we can use Ohm's law to calculate the current flowing through the resistor.




Ohm's law is:

$$V = R \cdot I$$

which can be rearranged to:

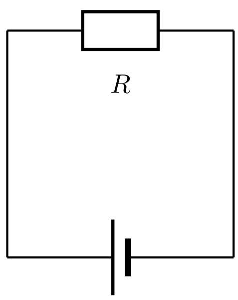
$$I = \frac{V}{R}$$

The current flowing in the resistor is:

$$\begin{aligned} I &= \frac{V}{R} \\ &= \frac{5\ \text{V}}{5\ \Omega} \\ &= 1\ \text{A} \end{aligned}$$

**Worked Example 124: Ohm's Law**

**Question:**




The resistance of the above resistor is 10 $\Omega$ and the current going through the resistor is 4 A. What is the potential difference (voltage) across the resistor?

**Answer**

**Step 1 : Determine how to approach the problem**
It is an Ohm's Law problem. So we use the equation:

$$V = R \cdot I$$

**Step 2 : Solve the problem**

$$\begin{aligned} V &= R \cdot I \\ &= (10)(4) \\ &= 40\ V \end{aligned}$$

**Step 3 : Write the final answer**
The voltage across the resistor is 40 V.

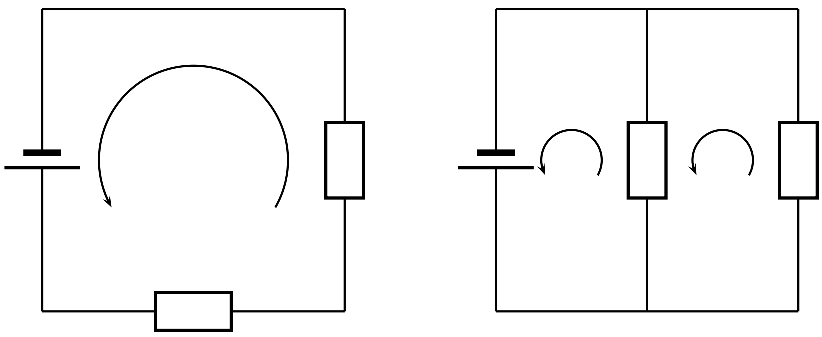
?

**Exercise: Ohm's Law**

1. Calculate the resistance of a resistor that has a potential difference of 8 V across it when a current of 2 A flows through it.
2. What current will flow through a resistor of 6 Ω when there is a potential difference of 18 V across its ends?
3. What is the voltage acroos a 10 Ω resistor when a current of 1,5 A flows though it?

## 19.3 Resistance

In Grade 10, you learnt about resistors and were introduced to circuits where resistors were connected in series and circuits where resistors were connected in parallel. In a series circuit there is one path for the current to flow through. In a parallel circuit there are multiple paths for the current to flow through.



series circuit
one current path

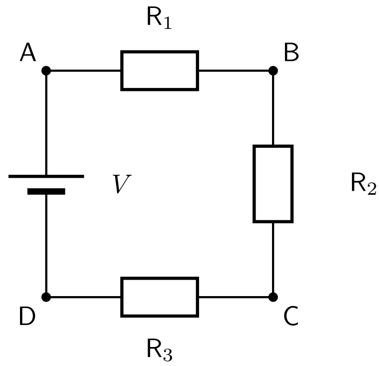
parallel circuit
multiple current paths

### 19.3.1 Equivalent resistance

When there is more than one resistor in a circuit, we are usually able to replace all resistors with a single resistor whose effect is the same as all the resistors put together. The resistance of the single resistor is known as *equivalent resistance*. We are able to calculate equivalent resistance for resistors connected in series and parallel.

**Equivalent Series Resistance**

Consider a circuit consisting of three resistors and a single battery connected in series.




The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit. From the way that the battery is connected, we can tell which direction the current will flow. We know that charge flows from positive to negative, by convention. Current in this circuit will flow in a clockwise direction, from point A to B to C to D and back to A.

So, how do we use this knowledge to calculate the value of a single resistor that can replace the three resistors in the circuit and still have the same current?

We know that in a series circuit the current has to be the same in all components. So we can write:

$$I = I_1 = I_2 = I_3$$

We also know that total voltage of the circuit has to be equal to the sum of the voltages over all three resistors. So we can write:

$$V = V_1 + V_2 + V_3$$

Finally, we know that Ohm's Law has to apply for each resistor individually, which gives us:

$$\begin{aligned} V_1 &= I_1 \cdot R_1 \\ V_2 &= I_2 \cdot R_2 \\ V_3 &= I_3 \cdot R_3 \end{aligned}$$

Therefore:

$$V = I_1 \cdot R_1 + I_2 \cdot R_2 + I_3 \cdot R_3$$

However, because

$$I = I_1 = I_2 = I_3$$

, we can further simplify this to:

$$\begin{aligned} V &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

Further, we can write an Ohm's Law relation for the entire circuit:

$$V = I \cdot R$$

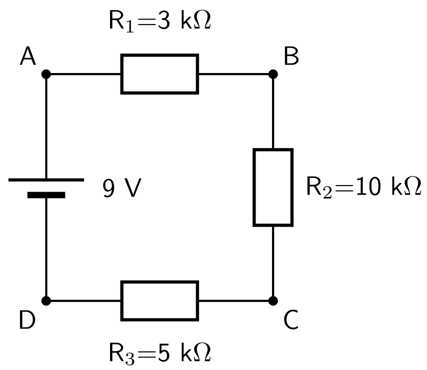
Therefore:

$$\begin{aligned} V &= I(R_1 + R_2 + R_3) \\ I \cdot R &= I(R_1 + R_2 + R_3) \\ \therefore \quad R &= R_1 + R_2 + R_3 \end{aligned}$$

**Definition: Equivalent resistance in a series circuit, $R_s$**
For $n$ resistors in series the equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n$$

Let us apply this to the following circuit.




The resistors are in series, therefore:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 3\,\Omega + 10\,\Omega + 5\,\Omega \\ &= 18\,\Omega \end{aligned}$$

**Worked Example 125: Equivalent series resistance I**

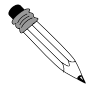
**Question:** Two 10 k$\Omega$ resistors are connected in series. Calculate the equivalent resistance.
**Answer**
**Step 1 : Determine how to approach the problem**
Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

**Step 2 : Solve the problem**

$$\begin{aligned} R_s &= R_1 + R_2 \\ &= 10\,\text{k}\,\Omega + 10\,\text{k}\,\Omega \\ &= 20\,\text{k}\,\Omega \end{aligned}$$

**Step 3 : Write the final answer**
The equivalent resistance of two 10 k$\Omega$ resistors connected in series is 20 k$\Omega$.



**Worked Example 126: Equivalent series resistance II**

**Question:** Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.

**Answer**

**Step 1 : Determine how to approach the problem**

Since the resistors are in series we can use:

$$R_s = R_1 + R_2$$

We are given the value of $R_s$ and $R_1$.

**Step 2 : Solve the problem**

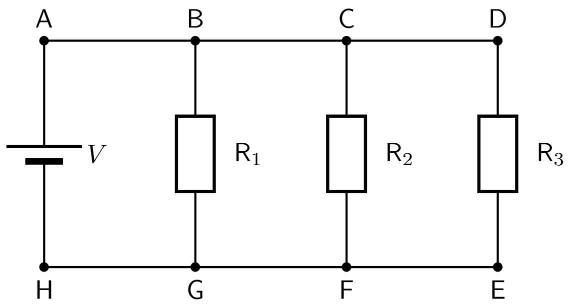
$$\begin{aligned} R_s &= R_1 + R_2 \\ \therefore \quad R_2 &= R_s - R_1 \\ &= 100\,\Omega - 10\,\Omega \\ &= 90\,\Omega \end{aligned}$$

**Step 3 : Write the final answer**

The second resistor has a resistance of 90Ω.

### Equivalent parallel resistance

Consider a circuit consisting of a single battery and three resistors that are connected in parallel.




The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. So, for the circuit shown, the following is true:

$$V = V_1 = V_2 = V_3$$

The second principle for a parallel circuit is that all the currents through each resistor must add up to the total current in the circuit.

$$I = I_1 + I_2 + I_3$$

Also, from applying Ohm's Law to the entire circuit, we can write:

$$V = \frac{I}{R_p}$$

where $R_p$ is the equivalent resistance in this parallel arrangement.

We are now ready to apply Ohm's Law to each resistor, to get:

$$\begin{aligned} V_1 &= R_1 \cdot I_1 \\ V_2 &= R_2 \cdot I_2 \\ V_3 &= R_3 \cdot I_3 \end{aligned}$$

This can be also written as:

$$\begin{aligned} I_1 &= \frac{V_1}{R_1} \\ I_2 &= \frac{V_2}{R_2} \\ I_3 &= \frac{V_3}{R_3} \end{aligned}$$

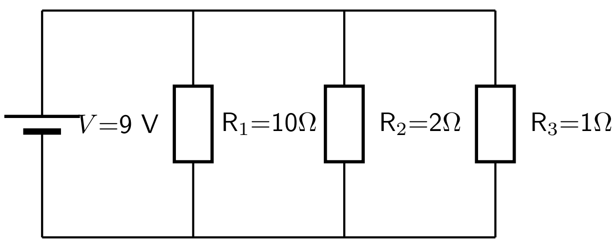
Now we have:

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ \frac{V}{R_p} &= \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \\ &= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ \text{because} \quad & V = V_1 = V_2 = V_3 \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ \therefore \quad \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \end{aligned}$$

**Definition: Equivalent resistance in a parallel circuit, $R_p$**
For $n$ resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)$$

Let us apply this formula to the following circuit.

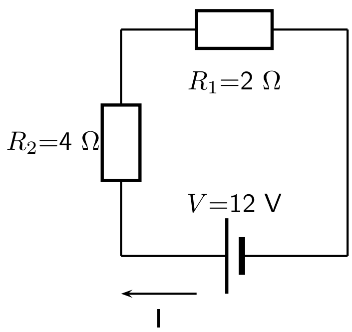



$$\begin{aligned} \frac{1}{R_p} &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \\ &= \left(\frac{1}{10\,\Omega} + \frac{1}{2\,\Omega} + \frac{1}{1\,\Omega}\right) \\ &= \left(\frac{1+5+10}{10}\right) \\ &= \left(\frac{16}{10}\right) \\ \therefore \quad R_p &= \frac{10}{16}\,\Omega \end{aligned}$$

### 19.3.2 Use of Ohm's Law in series and parallel Circuits

**Worked Example 127: Ohm's Law**

**Question:** Calculate the current ($I$) in this circuit if the resistors are both ohmic in nature.
**Answer**




**Step 1 : Determine what is required**
We are required to calculate the current flowing in the circuit.
**Step 2 : Determine how to approach the problem**
Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
**Step 3 : Find total resistance in circuit**
Since the resistors are connected in series, the total resistance $R$ is:

$$R = R_1 + R_2$$

Therefore,

$$R = 2 + 4 = 6\ \Omega$$

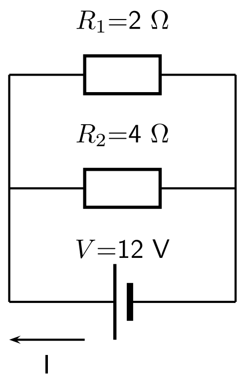
**Step 4 : Apply Ohm's Law**

$$\begin{aligned} V &= R \cdot I \\ \therefore\quad I &= \frac{V}{R} \\ &= \frac{12}{6} \\ &= 2\ \mathrm{A} \end{aligned}$$

**Step 5 : Write the final answer**
A 2 A current is flowing in the circuit.

**Worked Example 128: Ohm's Law I**

**Question:** Calculate the current ($I$) in this circuit if the resistors are both ohmic in nature.
**Answer**




**Step 1 : Determine what is required**
We are required to calculate the current flowing in the circuit.
**Step 2 : Determine how to approach the problem**
Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
**Step 3 : Find total resistance in circuit**
Since the resistors are connected in parallel, the total resistance $R$ is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Therefore,

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2+1}{4} \\ &= \frac{3}{4} \\ Therefore, R &= \frac{4}{3}\ \Omega \end{aligned}$$

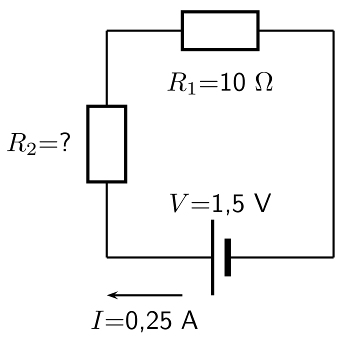
**Step 4 : Apply Ohm's Law**

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{\frac{4}{3}} \\ &= 9\ \mathrm{A} \end{aligned}$$

**Step 5 : Write the final answer**
A 9 A current is flowing in the circuit.

**Worked Example 129: Ohm's Law II**

**Question:** Two ohmic resistors ($R_1$ and $R_2$) are connected in series with a battery. Find the resistance of $R_2$, given that the current flowing through $R_1$ and $R_2$ is 0,25 A and that the voltage across the battery is 1,5 V. $R_1$=1 Ω.
**Answer**
**Step 6 : Draw the circuit and fill in all known values.**




**Step 7 : Determine how to approach the problem.**

We can use Ohm's Law to find the total resistance $R$ in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2$$

in a series circuit.

**Step 8 : Find the total resistance**

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad R &= \frac{V}{I} \\ &= \frac{1{,}5}{0{,}25} \\ &= 6\,\Omega \end{aligned}$$

**Step 9 : Find the unknown resistance**

We know that:

$$R = 6\,\Omega$$

and that

$$R_1 = 1\,\Omega$$

Since

$$R = R_1 + R_2$$

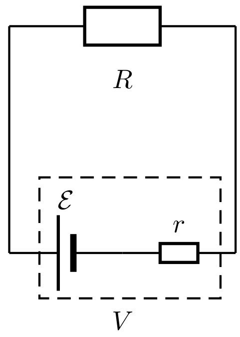
$$R_2 = R - R_1$$

Therefore,

$$R_2 = 5\,\Omega$$

## 19.3.3 Batteries and internal resistance

Real batteries are made from materials which have resistance. This means that real batteries are not just sources of potential difference (voltage), but they also possess internal resistances. If the pure voltage source is referred to as the emf, $\mathcal{E}$, then a real battery can be represented as an emf connected in series with a resistor $r$. The internal resistance of the battery is represented by the symbol $r$.




**Definition: Load**
The external resistance in the circuit is referred to as the load.

Suppose that the battery (or cell) with emf $\mathcal{E}$ and internal resistance $r$ supplies a current $I$ through an external load resistor $R$. Then the voltage drop across the load resistor is that supplied by the battery:

$$V = I \cdot R$$

Similarly, from Ohm's Law, the voltage drop across the internal resistance is:

$$V_r = I \cdot r$$

The voltage $V$ of the battery is related to its emf $\mathcal{E}$ and internal resistance $r$ by:

$$\begin{aligned} \mathcal{E} &= V + Ir; or \\ V &= \mathcal{E} - Ir \end{aligned}$$

The emf of a battery is essentially constant because it only depends on the chemical reaction (that converts chemical energy into electrical energy) going on inside the battery. Therefore, we can see that the voltage across the terminals of the battery is dependent on the current drawn by the load. The higher the current, the lower the voltage across the terminals, because the emf is constant. By the same reasoning, the voltage only equals the emf when the current is very small.

The maximum current that can be drawn from a battery is limited by a critical value $I_c$. At a current of $I_c$, $V$=0 V. Then, the equation becomes:

$$\begin{aligned} 0 &= \mathcal{E} - I_c r \\ I_c r &= \mathcal{E} \\ I_c &= \frac{\mathcal{E}}{r} \end{aligned}$$

The maximum current that can be drawn from a battery is less than $\frac{\mathcal{E}}{r}$.

**Worked Example 130: Internal resistance**

**Question:** What is the internal resistance of a battery if its emf is 12 V and the voltage drop across its terminals is 10 V when a current of 4 A flows in the circuit when it is connected across a load?

**Answer**

**Step 1 : Determine how to approach the problem**

It is an internal resistance problem. So we use the equation:

$$\mathcal{E} = V + Ir$$

**Step 2 : Solve the problem**

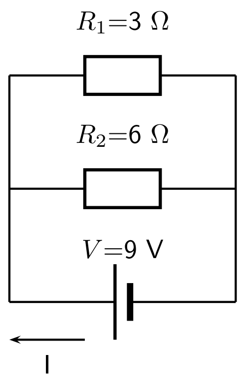
$$\begin{aligned} \mathcal{E} &= V + Ir \\ 12 &= 10 + 4(r) \\ &= 0.5 \end{aligned}$$

**Step 3 : Write the final answer**
The internal resistance of the resistor is 0.5 Ω.

?

**Exercise: Resistance**

1. Calculate the equivalent resistance of:
   A three 2 Ω resistors in series;
   B two 4 Ωresistors in parallel;
   C a 4 Ω resistor in series with a 8 Ω resistor;
   D a 6 Ω resistor in series with two resistors (4 Ω and 2Ω ) in parallel.
2. Calculate the current in this circuit if both resistors are ohmic.




3. Two ohmic resistors are connected in series. The resistance of the one resistor is 4 ΩẆhat is the resistance of the other resistor if a current of 0,5 A flows through the resistors when they are connected to a voltage supply of 6 V.
4. Describe what is meant by the *internal resistance* of a real battery.
5. Explain why there is a difference between the emf and terminal voltage of a battery if the load (external resistance in the circuit) is comparable in size to the battery's internal resistance
6. What is the internal resistance of a battery if its emf is 6 V and the voltage drop across its terminals is 5,8 V when a current of 0,5 A flows in the circuit when it is connected across a load?

## 19.4 Series and parallel networks of resistors

Now that you know how to handle simple series and parallel circuits, you are ready to tackle problems like this:

It is relatively easy to work out these kind of circuits because you use everything you have already learnt about series and parallel circuits. The only difference is that you do it in stages.

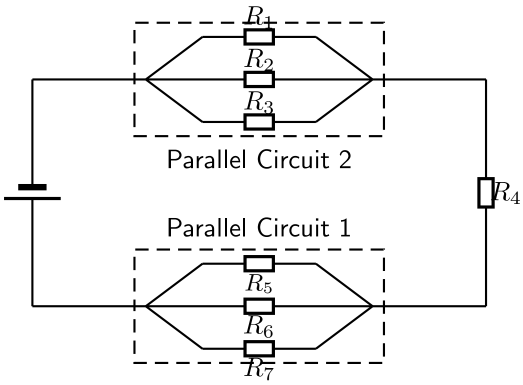
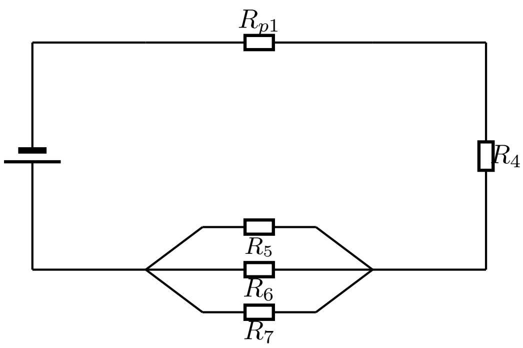



Figure 19.1: An example of a series-parallel network. The dashed boxes indicate parallel sections of the circuit.

In Figure 19.1, the circuit consists of 2 parallel portions that are then in series with 1 resistor. So, in order to work out the equivalent resistance, you start by reducing the parallel portions to a single resistor and then add up all the resistances in series. If all the resistors in Figure 19.1 had resistances of 10 Ω, we can calculate the equivalent resistance of the entire circuit.

We start by reducing *Parallel Circuit 1* to a single resistor.

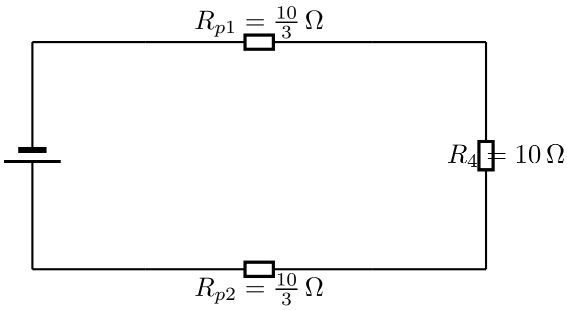



The value of $R_{p1}$ is:

$$\begin{aligned}
\frac{1}{R_{p1}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
R_{p1} &= \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10}\right)^{-1} \\
&= \left(\frac{1+1+1}{10}\right)^{-1} \\
&= \left(\frac{3}{10}\right)^{-1} \\
&= \frac{10}{3}\,\Omega
\end{aligned}$$

We can similarly replace *Parallel Circuit 2* with $R_{p2}$ which has a value given by:

$$\begin{aligned}\frac{1}{R_{p2}} &= \frac{1}{R_5}+\frac{1}{R_6}+\frac{1}{R_7}\\ R_{p2} &= \left(\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)^{-1}\\ &= \left(\frac{1+1+1}{10}\right)^{-1}\\ &= \left(\frac{3}{10}\right)^{-1}\\ &= \frac{10}{3}\,\Omega\end{aligned}$$




This is now a simple series circuit and the equivalent resistance is:

$$\begin{aligned}R &= R_{p1}+R_4+R_{p2}\\ &= \frac{10}{3}+10+\frac{10}{3}\\ &= \frac{100+30+100}{30}\\ &= \frac{230}{30}\\ &= 7\frac{2}{3}\,\Omega\end{aligned}$$

The equivalent resistance of the circuit in Figure 19.1 is $7\frac{2}{3}\,\Omega$.

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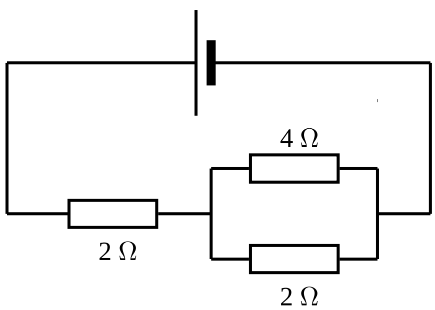
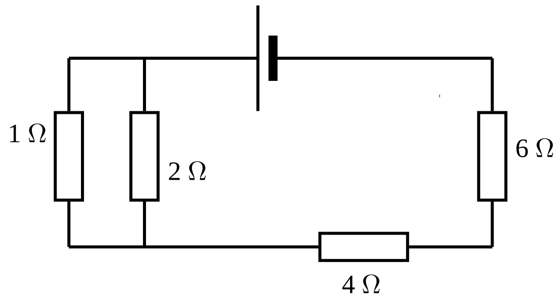
?

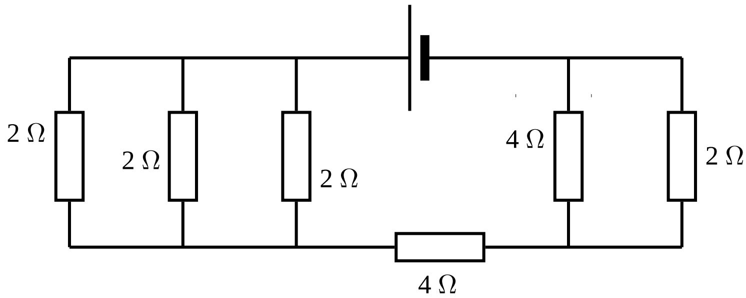
**Exercise: Series and parallel networks**

Determine the equivalent resistance of the following circuits:




1. Hello

2. 

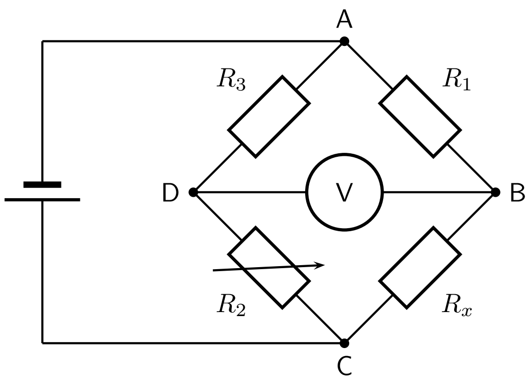

3. 


## 19.5 Wheatstone bridge

Another method of finding an unknown resistance is to use a *Wheatstone bridge*. A Wheatstone bridge is a measuring instrument that is used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component. Its operation is similar to the original potentiometer except that in potentiometer circuits the meter used is a sensitive galvanometer.

**Interesting Fact**

The Wheatstone bridge was invented by Samuel Hunter Christie in 1833 and improved and popularized by Sir Charles Wheatstone in 1843.

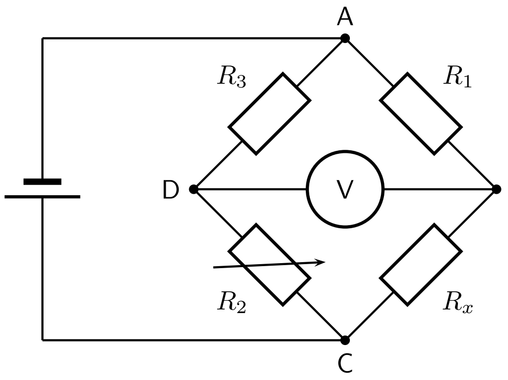



Circuit for Wheatstone bridge

In the circuit of the Wheatstone bridge, $R_x$ is the unknown resistance. $R_1$, $R_2$ and $R_3$ are resistors of known resistance and the resistance of $R_2$ is adjustable. If the ratio of $R_2$:$R_1$ is equal to the ratio of $R_x$:$R_3$, then the voltage between the two midpoints will be zero and no current will flow between the midpoints. In order to determine the unknown resistance, $R_2$ is varied until this condition is reached. That is when the voltmeter reads 0 V.

**Worked Example 131: Wheatstone bridge**

**Question:**
**Answer**
What is the resistance of the unknown resistor $R_x$ in the diagram below if $R_1$=4Ω $R_2$=8Ω and $R_3$=6Ω.



Circuit for Wheatstone bridge

**Step 1 : Determine how to approach the problem**
The arrangement is a Wheatstone bridge. So we use the equation:

$$R_x : R_3 = R_2 : R_1$$

**Step 2 : Solve the problem**

$$\begin{aligned} R_x : R_3 &= R_2 : R_1 \\ R_x : 6 &= 8 : 4 \\ R_x &= 12\ \Omega \end{aligned}$$

**Step 3 : Write the final answer**
The resistance of the unknown resistor is 12 Ω.

*Extension: Power in electric circuits*

In addition to voltage and current, there is another measure of free electron activity in a circuit: *power*. Power is a measure of how rapidly a standard amount of *work* is done. In electric circuits, power is a function of both voltage and current:

**Definition: Electrical Power**
Electrical power is calculated as:

$$P = I \cdot V$$

Power ($P$) is exactly equal to current ($I$) multiplied by voltage ($V$) and there is no extra constant of proportionality. The unit of measurement for power is the *Watt* (abbreviated W).

It was James Prescott Joule, not Georg Simon Ohm, who first discovered the mathematical relationship between power dissipation and current through a resistance. This discovery, published in 1841, followed the form of the equation:

$$P = I^2R$$

and is properly known as Joule's Law. However, these power equations are so commonly associated with the Ohm's Law equations relating voltage, current, and resistance that they are frequently credited to Ohm.

---

**Activity :: Investigation : Equivalence**
Use Ohm's Law to show that:

$$P = VI$$

is identical to

$$P = I^2R$$

and

$$P = \frac{V^2}{R}$$

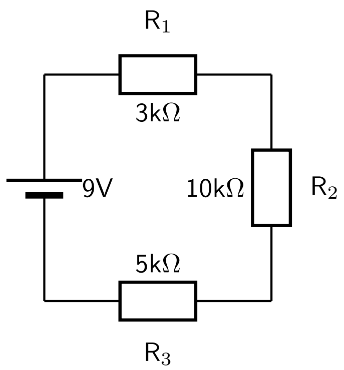
---

## 19.6 Summary

1. Ohm's Law states that the amount of current through a conductor, at constant temperature, is proportional to the voltage across the resistor. Mathematically we write $V = R/cdotI$
2. Conductors that obey Ohm's Law are called ohmic conductors; those who do not are called non-ohmic conductors.
3. We use Ohm's Law to calculate the resistance of a resistor. $(R = \frac{V}{I}$
4. The equivalent resistance of resistors in series $(R_s)$ can be calculated as follows: $R_s = R_1 + R_2 + R_3 + ... + R_n$
5. The equivalent resistance of resistors in parallel $(R_p)$ can be calculated as follows: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$
6. Real batteries have an internal resistance.
7. Wheatstone bridges can be used to accurately determine the resistance of an unknown resistor.

## 19.7 End of chapter exercise

1. Calculate the current in the following circuit and then use the current to calculate the voltage drops across each resistor.




2. Explain why a voltmeter is placed in parallel with a resistor.

3. Explain why an ammeter is placed in series with a resistor.

4. [IEB 2001/11 HG1] - **Emf**

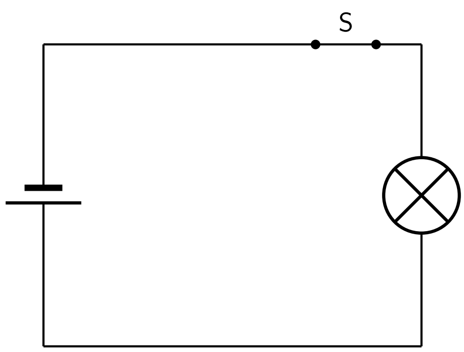
   A Explain the meaning of each of these two statements:

      i. "The current through the battery is 50 mA."

      ii. "The emf of the battery is 6 V."

   B A battery tester measures the current supplied when the battery is connected to a resistor of 100 Ω. If the current is less than 50 mA, the battery is "flat" (it needs to be replaced). Calculate the maximum internal resistance of a 6 V battery that will pass the test.

5. [IEB 2005/11 HG] The electric circuit of a torch consists of a cell, a switch and a small light bulb.



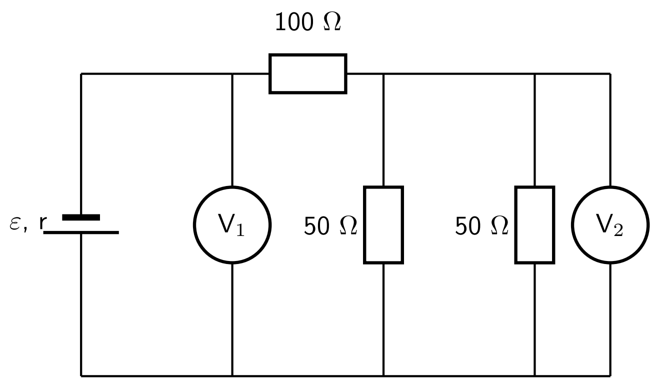

The electric torch is designed to use a D-type cell, but the only cell that is available for use is an AA-type cell. The specifications of these two types of cells are shown in the table below:

| **Cell** | **emf** | **Appliance for which it is designed** | **Current drawn from cell when connected to the appliance for which it is designed** |
|---|---|---|---|
| D | 1,5 V | torch | 300 mA |
| AA | 1,5 V | TV remote control | 30 mA |

What is likely to happen and why does it happen when the AA-type cell replaces the D-type cell in the electric torch circuit?

| | What happens | Why it happens |
|---|---|---|
| (a) | the bulb is dimmer | the AA-type cell has greater internal resistance |
| (b) | the bulb is dimmer | the AA-type cell has less internal resistance |
| (c) | the brightness of the bulb is the same | the AA-type cell has the same internal resistance |
| (d) | the bulb is brighter | the AA-type cell has less internal resistance |

6. [IEB 2005/11 HG1] A battery of emf $\varepsilon$ and internal resistance r = 25 Ω is connected to this arrangement of resistors.




The resistances of voltmeters $V_1$ and $V_2$ are so high that they do not affect the current in the circuit.

A Explain what is meant by "the emf of a battery".
   The power dissipated in the 100 Ω resistor is 0,81 W.
B Calculate the current in the 100 Ω resistor.
C Calculate the reading on voltmeter $V_2$.
D Calculate the reading on voltmeter $V_1$.
E Calculate the emf of the battery.

7. [SC 2003/11] A kettle is marked 240 V; 1 500 W.

A Calculate the resistance of the kettle when operating according to the above specifications.
B If the kettle takes 3 minutes to boil some water, calculate the amount of electrical energy transferred to the kettle.

8. [IEB 2001/11 HG1] - **Electric Eels**

Electric eels have a series of cells from head to tail. When the cells are activated by a nerve impulse, a potential difference is created from head to tail. A healthy electric eel can produce a potential difference of 600 V.

A What is meant by "a potential difference of 600 V"?
B How much energy is transferred when an electron is moved through a potential difference of 600 V?

# Chapter 20

# Electronic Properties of Matter - Grade 11

## 20.1 Introduction

We can study many different features of solids. Just a few of the things we could study are how hard or soft they are, what are their magnetic properties or how well do they conduct heat. The thing that we are interested in, in this chapter are their electronic properties. Simply how well do they conduct electricity and how do they do it.

We are only going to discuss materials that form a 3-dimensional lattice. This means that the atoms that make up the material have a regular pattern (carbon, silicon, etc.). We won't discuss materials where the atoms are jumbled together in a irregular way (plastic, glass, rubber etc.).

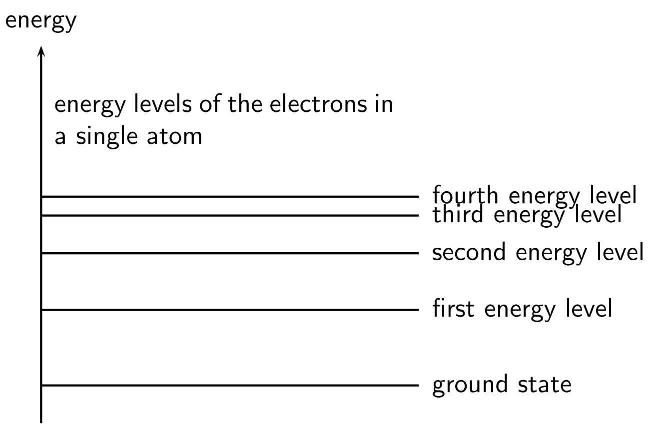
## 20.2 Conduction

We know that there are materials that do conduct electricity, called conductors, like the copper wires in the circuits you build. There are also materials that do not conduct electricity, called insulators, like the plastic covering on the copper wires.

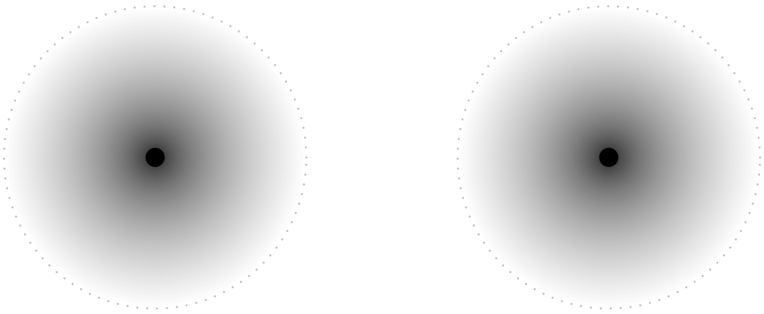
Conductors come in two major categories: metals (e.g. copper) and semi-conductors (e.g. silicon). Metals conduct very well and semi-conductors don't. One very interesting difference is that metals conduct less as they become hotter but semi-conductors conduct more.

What is different about these substances that makes them conduct differently? That is what we are about to find out.

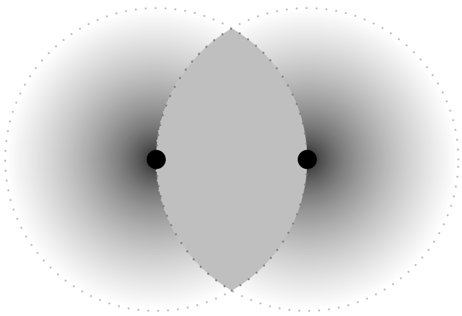
We have learnt that electrons in an atom have discrete energy levels. When an electron is given the right amount of energy, it can jump to a higher energy level, while if it loses the right amount of energy it can drop to a lower energy level. The lowest energy level is known as the ground state.



When two atoms are far apart from each other they don't influence each other. Look at the picture below. There are two atoms depicted by the black dots. When they are far apart their electron clouds (the gray clouds) are distinct. The dotted line depicts the distance of the outermost electron energy level that is occupied.

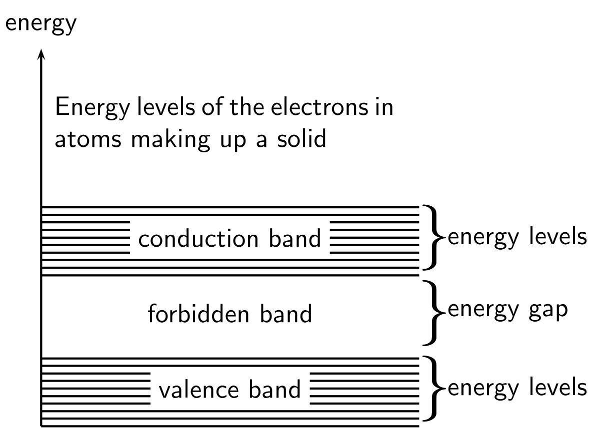


In some lattice structures the atoms would be closer together. If they are close enough their electron clouds, and therefore electron energy levels start to overlap. Look at the picture below. In this picture the two atoms are closer together. The electron clouds now overlap. The overlapping area is coloured in solid gray to make it easier to see.



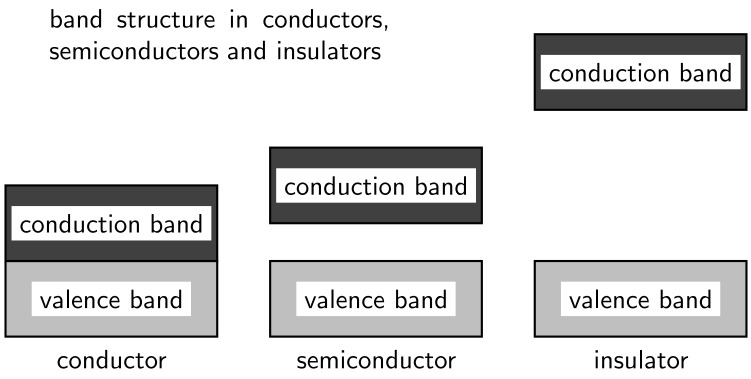
When this happens we might find two electrons with the same energy and spin in the same space. We know that this is not allowed from the Pauli exclusion principle. Something must change to allow the overlapping to happen. The change is that the energies of the energy levels change a tiny bit so that the electrons are not in exactly the same spin and energy state at the same time.

So if we have 2 atoms then in the overlapping area we will have twice the number of electrons and energy levels but the energy levels from the different atoms will be very very close in energy. If we had 3 atoms then there would be 3 energy levels very close in energy and so on. In a solid there may be very many energy levels that are very close in energy. These groups of energy levels are called bands. The spacing between these bands determines whether the solid is a conductor or an insulator.




In a gas, the atoms are spaced far apart and they do not influence each other. However, the atoms in a solid greatly influence each other. The forces that bind these atoms together in a solid affect how the electrons of the atoms behave, by causing the individual energy levels of an atom to break up and form energy bands. The resulting energy levels are more closely spaced than those in the individual atoms. The energy bands still contain discrete energy levels, but there are now many more energy levels than in the single atom.

In crystalline solids, atoms interact with their neighbors, and the energy levels of the electrons in isolated atoms turn into bands. Whether a material conducts or not is determined by its band structure.




Electrons follow the Pauli exclusion principle, meaning that two electrons cannot occupy the same state. Thus electrons in a solid fill up the energy bands up to a certain level (this is called the Fermi energy). Bands which are completely full of electrons cannot conduct electricity, because there is no state of nearby energy to which the electrons can jump. Materials in which all bands are full are insulators.

## 20.2.1 Metals

Metals are good conductors because they have unfilled space in the valence energy band. In the absence of an electric field, there are electrons traveling in all directions. When an electric field is applied the mobile electrons flow. Electrons in this band can be accelerated by the electric field because there are plenty of nearby unfilled states in the band.

## 20.2.2 Insulator

The energy diagram for the insulator shows the insulator with a very wide energy gap. The wider this gap, the greater the amount of energy required to move the electron from the valence band to the conduction band. Therefore, an insulator requires a large amount of energy to obtain a small amount of current. The insulator "insulates" because of the wide forbidden band or energy gap.

#### Breakdown

A solid with filled bands is an insulator. If we raise the temperature the electrons gain thermal energy. If there is enough energy added then electrons can be thermally excited from the valence band to the conduction band. The fraction of electrons excited in this way depends on:

- the temperature and
- the band gap, the energy difference between the two bands.

Exciting these electrons into the conduction band leaves behind positively charged holes in the valence band, which can also conduct electricity.

### 20.2.3 Semi-conductors

A semi-conductor is very similar to an insulator. The main difference between semiconductors and insulators is the size of the band gap between the conduction and valence bands. The band gap in insulators is larger than the band gap in semiconductors.

In semi-conductors at room temperature, just as in insulators, very few electrons gain enough thermal energy to leap the band gap, which is necessary for conduction. For this reason, pure semi-conductors and insulators, in the absence of applied fields, have roughly similar electrical properties. The smaller band gaps of semi-conductors, however, allow for many other means besides temperature to control their electrical properties. The most important one being that for a certain amount of applied voltage, more current will flow in the semiconductor than in the insulator.

---

**Exercise: Conduction**

1. Explain how energy levels of electrons in an atom combine with those of other atoms in the formation of crystals.
2. Explain how the resulting energy levels are more closely spaced than those in the individual atoms, forming energy bands.
3. Explain the existence of energy bands in metal crystals as the result of superposition of energy levels.
4. Explain and contrast the conductivity of conductors, semi-conductors and insulators using energy band theory.
5. What is the main difference in the energy arrangement between an isolated atom and the atom in a solid?
6. What determines whether a solid is an insulator, a semiconductor, or a conductor?

---

## 20.3 Intrinsic Properties and Doping

We have seen that the size of the energy gap between the valence band and the conduction band determines whether a solid is a conductor or an insulator. However, we have seen that there is a material known as a semi-conductor. A semi-conductor is a solid whose band gap is smaller than that of an insulator and whose electrical properties can be modified by a process known as *doping*.

**Definition: Doping**
Doping is the deliberate addition of impurities to a pure semiconductor material to change its electrical properties.

Semiconductors are often the Group IV elements in the periodic table. The most common semiconductor elements are silicon (Si) and germanium (Ge). The most important property of Group IV elements is that they 4 valence electrons.

*Extension: Band Gaps of Si and Ge*
Si has a band gap of $1.744 \times 10^{-19}$ J while Ge has a band gap of $1.152 \times 10^{-19}$ J.

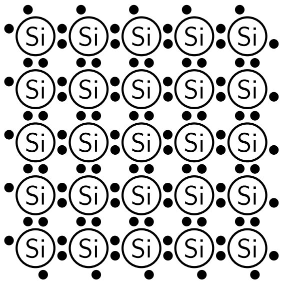



Figure 20.1: Arrangement of atoms in a Si crystal.

So, if we look at the arrangement of for example Si atoms in a crystal, they would look like that shown in Figure 20.1.

The main aim of doping is to make sure there are either too many (surplus) or too few electrons (deficiency). Depending on what situation you want to create you use different elements for the doping.

## 20.3.1 Surplus

A surplus of electrons is created by adding an element that has more valence electrons than Si to the Si crystal. This is known as *n-type* doping and elements used for n-type doping usually come from Group V in the periodic table. Elements from Group V have 5 valence electrons, one more than the Group IV elements.

A common n-type dopant (substance used for doping) is arsenic (As). The combination of a semiconductor and an n-type dopant is known as an n-type semiconductor. A Si crystal doped with As is shown in Figure 20.2. When As is added to a Si crystal, the 4 of the 5 valence electrons in As bond with the 4 Si valence electrons. The fifth As valence electron is free to move around.

It takes only a few As atoms to create enough free electrons to allow an electric current to flow through the silicon. Since n-type dopants 'donate' their free atoms to the semiconductor, they are known as *donor atoms*.

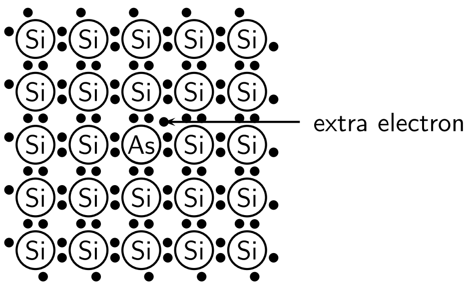



Figure 20.2: Si crystal doped with As. For each As atom present in the Si crystal, there is one extra electron. This combination of Si and As is known as an n-type semiconductor, because of its overall surplus of electrons.

## 20.3.2 Deficiency

A deficiency of electrons is created by adding an element that has less valence electrons than Si to the Si crystal. This is known as *p-type* doping and elements used for p-type doping usually come from Group III in the periodic table. Elements from Group III have 3 valence electrons, one less than the semiconductor elements that come from Group IV. A common p-type dopant is boron (B). The combination of a semiconductor and a p-type dopant is known as an p-type semiconductor. A Si crystal doped with B is shown in Figure 20.3. When B is mixed into the silicon crystal, there is a Si valence electron that is left unbonded.

The lack of an electron is known as a *hole* and has the effect of a positive charge. Holes can conduct current. A hole happily accepts an electron from a neighbor, moving the hole over a space. Since p-type dopants 'accept' electrons, they are known as *acceptor atoms*.

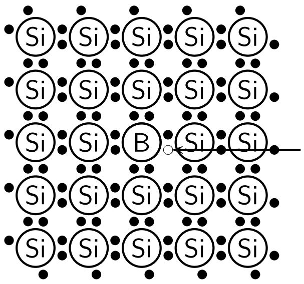
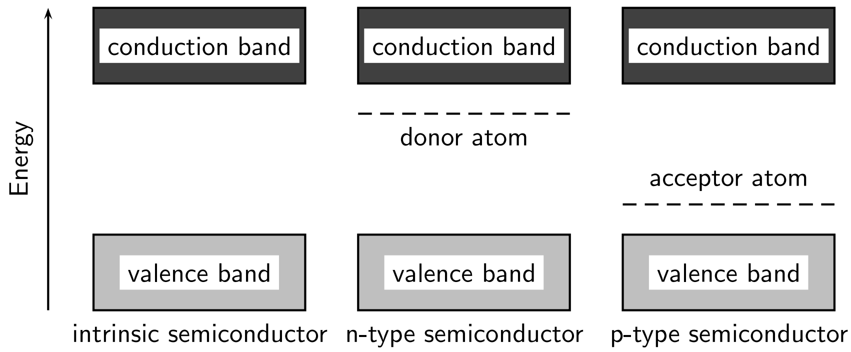



Figure 20.3: Si crystal doped with B. For each B atom present in the Si crystal, there is one less electron. This combination of Si and B is known as a p-type semiconductor, because of its overall deficiency of electrons.

Donor (n-type) impurities have extra valence electrons with energies very close to the conduction band which can be easily thermally excited to the conduction band. Acceptor (p-type) impurities capture electrons from the valence band, allowing the easy formation of holes.




The energy level of the donor atom is close to the conduction band and it is relatively easy for electrons to enter the conduction band. The energy level of the acceptor atom is close to the valence band and it is relatively easy for electrons to leave the valence band and enter the vacancies left by the holes.

---

**Exercise: Intrinsic Properties and Doping**

1. Explain the process of doping using detailed diagrams for p-type and n-type semiconductors.
2. Draw a diagram showing a Ge crystal doped with As. What type of semiconductor is this?
3. Draw a diagram showing a Ge crystal doped with B. What type of semiconductor is this?
4. Explain how doping improves the conductivity of semi-conductors.
5. Would the following elements make good p-type dopants or good n-type dopants?

   A B
   B P
   C Ga
   D As

E In
F Bi

## 20.4 The p-n junction

### 20.4.1 Differences between p- and n-type semi-conductors

We have seen that the addition of specific elements to semiconductor materials turns them into p-type semiconductors or n-type semiconductors. The differences between n- and p-type semiconductors are summarised in Table **??**.

### 20.4.2 The p-n Junction

When p-type and n-type semiconductors are placed in contact with each other, a p-n junction is formed. Near the junction, electrons and holes combine to create a depletion region.

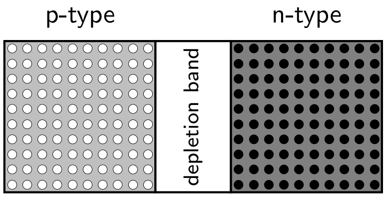



Figure 20.4: The p-n junction forms between p- and n-type semiconductors. The free electrons from the n-type material combine with the holes in the p-type material near the junction. There is a small potential difference across the junction. The area near the junction is called the depletion region because there are few holes and few free electrons in this region.

Electric current flows more easily across a p-n junction in one direction than in the other. If the positive pole of a battery is connected to the p-side of the junction, and the negative pole to the n-side, charge flows across the junction. If the battery is connected in the opposite direction, very little charge can flow.

This might not sound very useful at first but the p-n junction forms the basis for computer chips, solar cells, and other electronic devices.

### 20.4.3 Unbiased

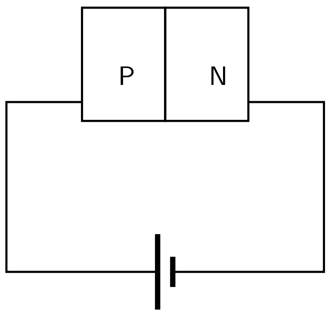
In a p-n junction, without an external applied voltage (no bias), an equilibrium condition is reached in which a potential difference is formed across the junction.

P-type is where you have more "holes"; N-type is where you have more electrons in the material. Initially, when you put them together to form a junction, holes near the junction tends to "move" across to the N-region, while the electrons in the N-region drift across to the p-region to "fill" some holes. This current will quickly stop as the potential barrier is built up by the migrated charges. So in steady state no current flows.

Then now when you put a potential different across the terminals you have two cases: forward biased and reverse biased.

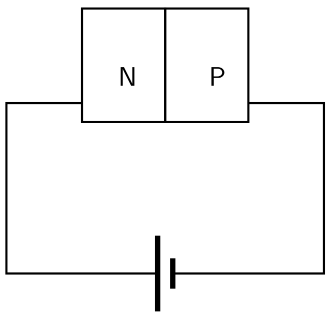
### 20.4.4 Forward biased

Forward-bias occurs when the p-type semiconductor material is connected to the positive terminal of a battery and the n-type semiconductor material is connected to the negative terminal.




The electric field from the external potential different can easily overcome the small internal field (in the so-called depletion region, created by the initial drifting of charges): usually anything bigger than 0.6V would be enough. The external field then attracts more e- to flow from n-region to p-region and more holes from p-region to n-region and you have a forward biased situation. the diode is ON.

### 20.4.5 Reverse biased




in this case the external field pushes e- back to the n-region while more holes into the p-region, as a result you get no current flow. Only the small number of thermally released minority carriers (holes in the n-type region and e- in the p-type region) will be able to cross the junction and form a very small current, but for all practical purposes, this can be ignored

of course if the reverse biased potential is large enough you get avalanche break down and current flow in the opposite direction. In many cases, except for Zener diodes, you most likely will destroy the diode.

### 20.4.6 Real-World Applications of Semiconductors

Semiconductors form the basis of modern electronics. Every electrical appliance usually has some semiconductor-based technology inside it. The fundamental uses of semiconductors are in microchips (also known as integrated circuits) and microprocessors.

Integrated circuits are miniaturised circuits. The use of integrated circuits makes it possible for electronic devices (like a cellular telephone or a hi-fi) to get smaller.

Microprocessors are a special type of integrated circuit. (NOTE TO SELF: more is needed but I'm not that knowledgable and I'm tired of Googling...)

---

**Activity :: Research Project : Semiconductors**
Assess the impact on society of the invention of transistors, with particular reference to their use in microchips (integrated circuits) and microprocessors.

?

**Exercise: The p-n junction**

1. Compare p- and n-type semi-conductors.
2. Explain how a p-n junction works using a diagram.
3. Give everyday examples of the application.

## 20.5 End of Chapter Exercises

1. What is a conductor?
2. What is an insulator?
3. What is a semiconductor?

# Appendix A

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